Metamorphosis between polyhedra

Harder to compute, but has many advantages

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Correspondence by projection

- 1992, Kent et al
- 1997, Kanai et al
- 1998, Shapiro et al.
- 1999, Gregory et al.
- 2000, Alexa
- 2002, Shlafman et al.
- 2004, Kraevoy et al

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Genus-0 polyhedra (Kent et al.)

Concepts:

- Topology (1-skeleton) = vertex / edge / face connectivity graph
- Geometry = specific instance of the topology
- Objects homeomorphic = continuous, invertible, one-to-one mapping exists

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Outline of the algorithm

1. Project the polyhedra into a unit sphere
2. Merge the two topologies
3. Map the merged topology onto the surfaces of the original polyhedra
4. Interpolate the coordinates

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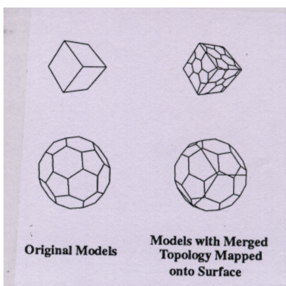
Example



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Projection methods

- Convex and star-shaped objects
- Methods using model knowledge, in particular objects of revolution and extruded objects
- Physically-based methods

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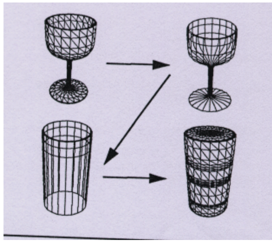
Example



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The merging algorithm

1. Read the topology and geometry of the objects and their projected vertices
2. Translate their centers to the origin
3. Intersect each edge of $(M_a)_p$ with a subset of the edges of $(M_b)_p$
4. Find vertex locations using barycentric coordinates
5. Identify the faces

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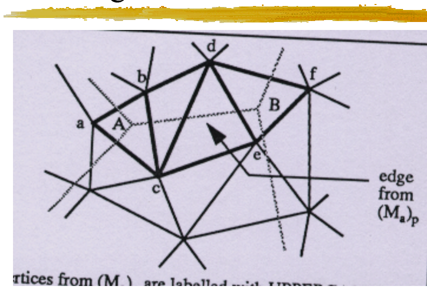
Calculating the intersections of an edge



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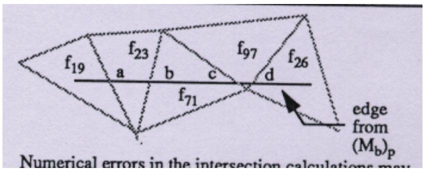
Sorting the intersections



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Interpolation issues

- Interpolation of each pair of corresponding vertex location (linear, Hermite,…)
- Self-intersection is not avoided
- Interpolation of other attributes

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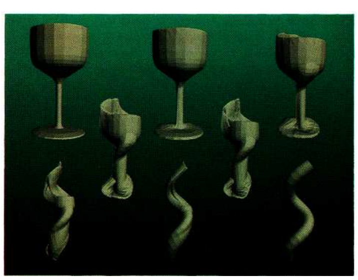
Results



CH-snap technique into tabular object

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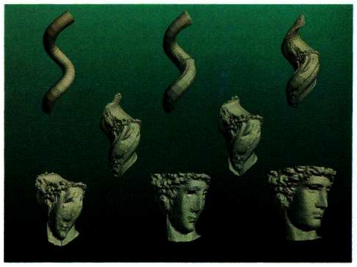
Results



Tabular object into object of revolution

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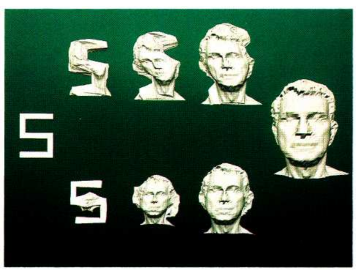
Results



Different projection methods

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Results

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Problems

- Not general
- Only genus 0
- Self intersection during morph
- User control is limited

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Finding correspondence (Shapiro)

1. Create a convex polyhedron *P'* which realizes the given polyhedron *P*
2. Similarly, find *Q'* for Q
3. Merge the vertex/edge connectivity graphs of *P*' and *Q*'
4. Induce the merged graph onto *P*, forming a congruent polyhedron *P''*
5. Similarly form *Q''*

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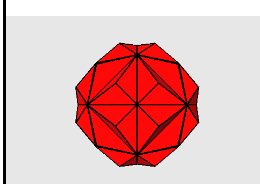
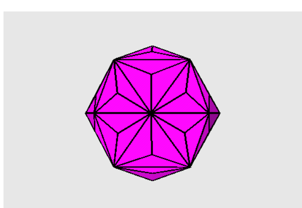
Polyhedron realization





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Polyhedron realization

A polyhedron *P'* is said to realize a polyhedron *P*, if *P*' is convex and the vertex/edge connectivity graphs of *P*' and *P* are isomorphic

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The realization algorithm

- *Simplification* - Simplify the vertex/edge connectivity graph by removing vertices and triangulating

  The result is a sequence of graphs $G_n\ G_{n-1} \ldots G_4$
- *Reattachment* - Construct a convex polyhedron bottom-up

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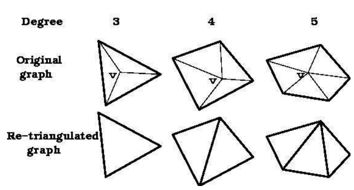
Simplification

- Find a vertex *v* of degree 3, 4, or 5 and remove it from the graph
- Re-triangulate the resulting face



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Simplification - correctness

Theorem: Given *Gi* as described above, *v* of degree 3, 4, or 5 can be found and removed and the graph can be re-triangulated and remain planar and triangular

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Simplification - correctness (Cont')

Lemma 1: A planar triangular graph must contain at least one vertex of degree three, four or five

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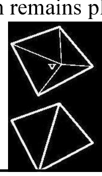
Simplification - correctness (Cont')

Lemma 2: If a vertex *v* of degree 4 is removed from a planar triangular graph *Gi* then

(1) the neighbors of *v* cannot all be inter-connected

(2) a diagonal can be added between two of *v*'s neighbors so that the graph remains planar and triangular



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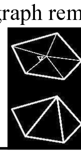
Simplification - correctness (Cont')

Lemma 3: If a vertex v of degree 5 is removed from a planar triangular graph G_i then

(1) there are at most 3 diagonals between the neighbors of v in G_i and

(2) two diagonals that share a common vertex can be added so that the graph remains planar and triangular.



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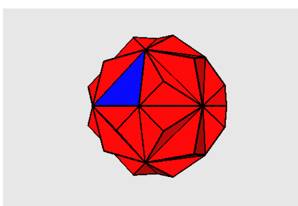
Simplification



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Re-attachment

Proceed bottom-up, realizing G_4 by a tetrahedron

The major consideration is how to locate vertex v, detached during simplification, to P_i, in order to form a convex polyhedron P_{i+1}.

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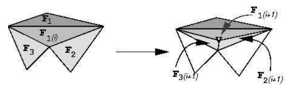
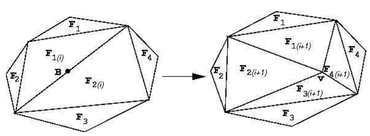
Case I: *v* of degree *3*

Lemma 4: It is possible to position *v* correctly. That is to say, *v* is above $F_{1(i)}$ and below F_j $\{0<j<4\}$



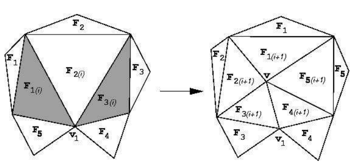
Case II: *v* of degree *4*

Lemma 5: It is possible to position *v* correctly. That is to say, *v* is above $F_{1(i)}$ and $F_{2(i)}$ and below F_j $\{0<j<5\}$



Case III: *v* of degree *5*

It is sometimes necessary to apply a transformation in order to position *v* correctly



Main theorem

Given a convex polyhedron P_i, v can be attached so that the resulting polyhedron P_{i+1} is convex

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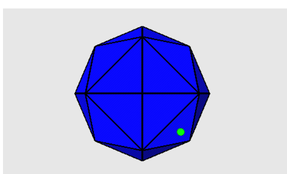
Re-attachment



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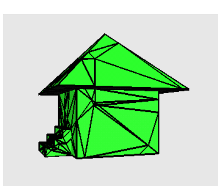
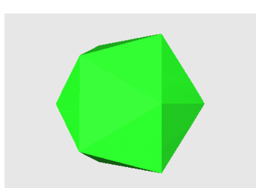
Results





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Advantages and disadvantages

+ General and be applied to other applications
+ Avoid circular arcs
+ Always convex
- Lack of user control
- Lack of finer correspondence
- Only genus zero

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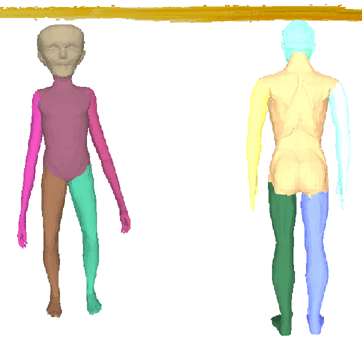
Finer correspondence



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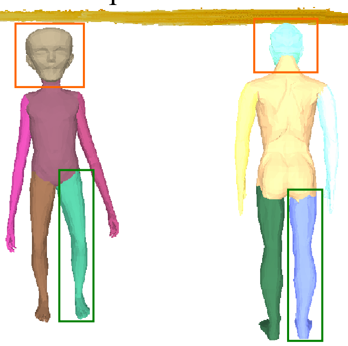
Surface decomposition



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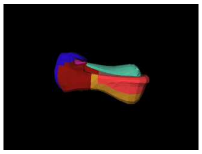
Results



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Results

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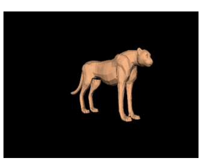
Results



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Results

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Summary

- A Compatible triangulation is useful for finding correspondence
- It is doable both in 2D and in 3D, both theoretically and practically
- Surface decomposition - useful for
  - fine control
  - non genus zero models

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Future challenges

- Handling non genus-zero polyhedra
- Non self-intersecting morph in 3D

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