Mesh analysis

Effective techniques for representing, analyzing, searching, and reusing

Why now?

Large repositories of 3D data more accessible

- Data storage
- Computing power
- Modeling techniques

Why "Shape Extraction"

Examining human image understanding many works indicate that recognition and shape understanding are based on structural decomposition of the shape into smaller parts

HOFFMAN D., RICHARDS W.: Parts of recognition. Cognition 18, 1-3 (December 1984), 65–96. BIEDERMAN I.: Recognition-by-Components: A theory of human image understanding. Psychological Review 94 (1987), 115–147. HOFFMAN D., SIGNH M.: Salience of visual parts. Cognition 63 (1997), 29-78.

For instance Modeling by example (Siggraph 2004)

Sub-problems

- ☐ Shape-based search
- ☐ Alignment
- □ Segmentation
- ☐ "Stitching"



Segmentation

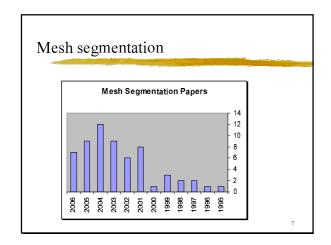
Applies in different domains:

- ☐ Images "segmentation"
- □ Polyhedra "triangulation" or "convex pieces"
- ☐ Meshes "decomposition" or "segmentation"

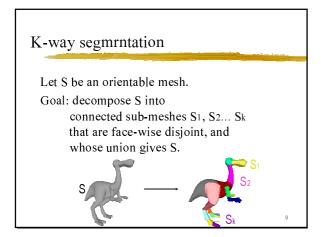


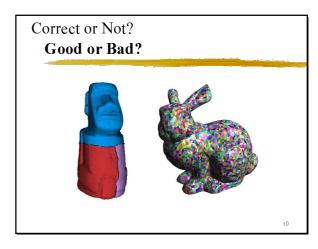


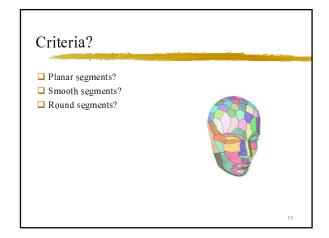


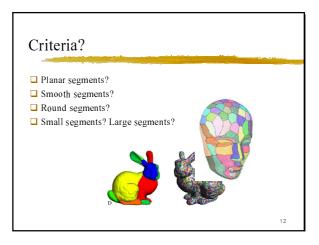


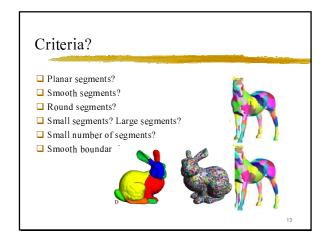
Today's class 1. Definitions 2. Criteria 3. Applications 4. Algorithms

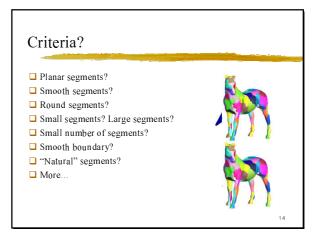












How to choose criteria?

- ☐ What you want / need
- ☐ Application in mind

15

Segmentation types

- ☐ Geometry-based
- ☐ "Meaningful" components

16

Segmentation as optimization definition

Given a mesh $M = \{V, E, F\}$ and the set of elements $S \in \{V, E, F\}$, find a disjoint partitioning of S into S_1, \ldots, S_k such that the criterion function

 $J = J(S_1, ..., S_k)$

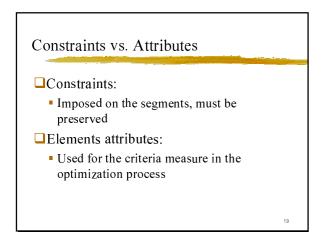
Be minimized under a set of constraints C.

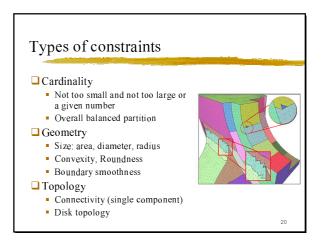
Optimal solution?

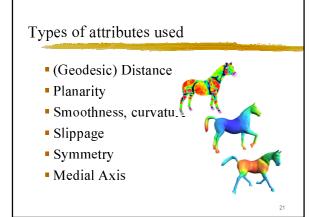
If |S| = n and $|\Sigma| = k$, then the search space is of order k^n .

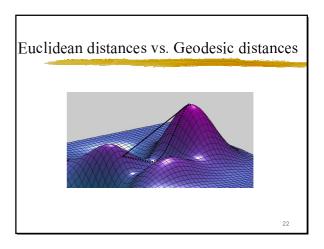
Segmentation must revert to some approximation algorithm:

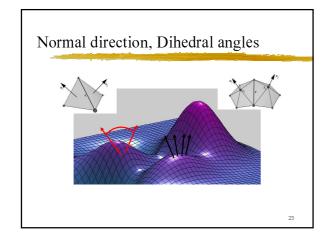
- Region growing (local greedy)
- Hierarchical clustering (global greedy)
- K-means (iterative)
- Graph Cut
- Spectral Analysis

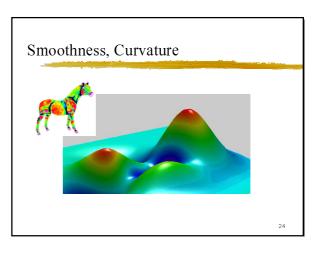


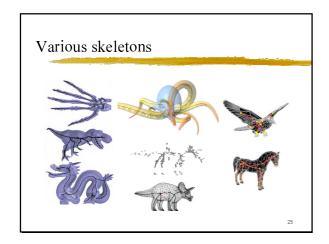


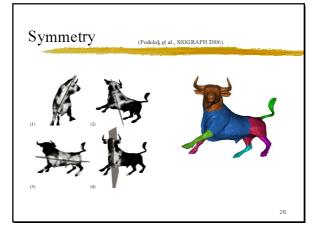


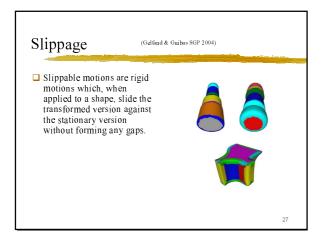


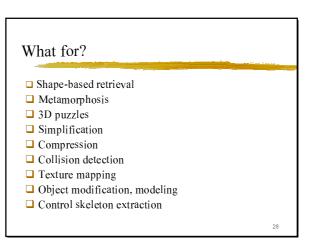








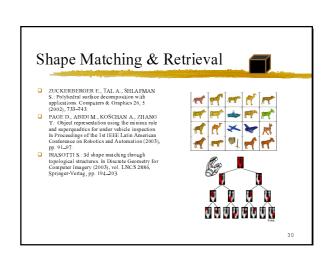


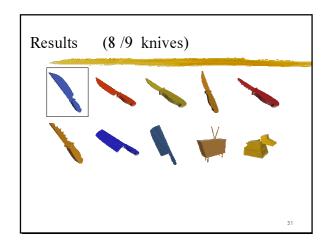


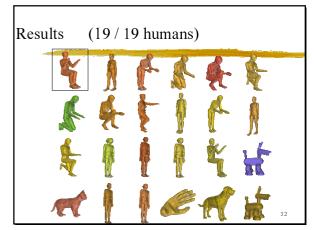
Application: Shape-based retrieval

Signature = decomposition graph with attributes
Retrieval = sub-graph isomorphism

Support - human visual perception
(Biederman, Marr)

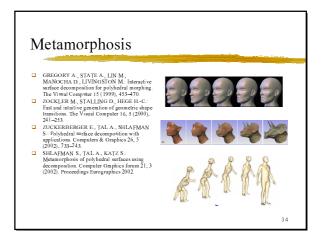


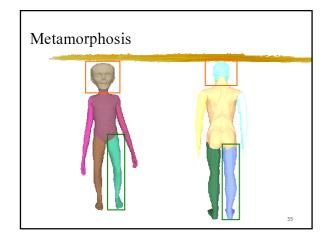


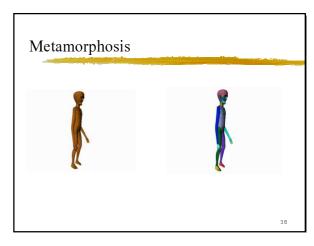


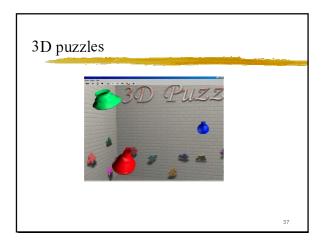
Benefits and drawbacks

- + Invariance to non-rigid transformations
- + No normalization
- + Small signatures
- + No restrictions on topology
- Computation time









Simplification

- ☐ We want to approximate a complex model (shape) with a simpler one.
- □ Replacing complex mathematical objects with simpler ones, while keeping the primal information content.

38

Segmentation context?

☐ Segment the mesh into regions which will be replaced by simpler elements (planes, cylinders etc.) while the geometric distance between the approximation elements and the original mesh will be



COHEN-STEINER D., ALLIEZ P., DESBRUN M.: Variational shape approximation. ACM Trans. Graph. 23, 3 (2004), 905–914.

39

Shape modeling

- FUNKHOUSER T., KAZHDAN M., SHILANE P., MIN P., KIEFER W., TAL A., RUSINKIEWICZ S., DOBKIN D.: Modeling by example. ACM Transactions on Graphics (Proceedings SIGGRAPH 2004) 23 (2004), 652-663
- ☐ Vladislav Kraevoy, Dan Julius, Alla Sheffer, Shuffler: Modeling with Interchangeable Parts, Technical sketch, Siggraph 2006

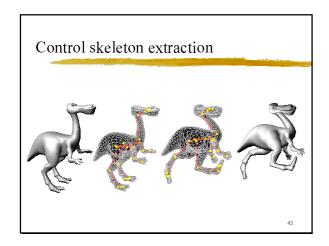


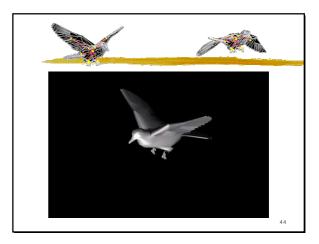


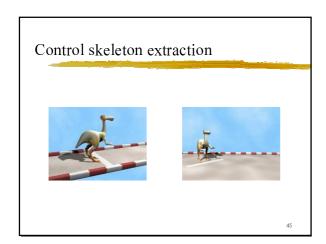
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Modeling

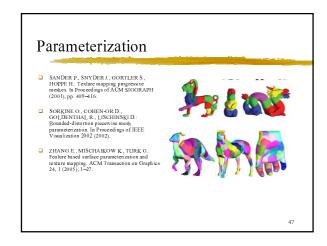
Skeleton extraction & Animation MORTARA M., PATAN. G., SPAGNUOLO M., FALCIDIENO B., ROSSIGNAC J: Blowing toables for multi-said analysis and decomposition of imagine meshes. Algorithmica 38. 1 (2003), 227-248. KATZ S., TAL. A: Hierarchical mesh decomposition using fuzzy clustering and cuts. A CM Transactions on Graphics (Proceedings SIGGRAPH 2003) 224, 260(3), 934-946. WU F.-C., MA W.-C., LIANG R.-H., CHEN B.-Y., OUHY O'ING M.: Domain connected graph: the skeleton of a closed 3d slape for animation. The V small Computer 22, 2 (2006), 117-133.

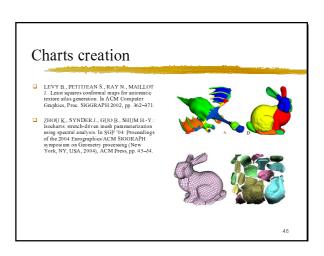












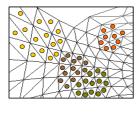
Goal – meaningful components

- Convexity
- Curvatures
- ☐ Geodesic distances

Segmentation as a clustering problem

- ☐ The basic segmentation problems can viewed as assigning primitive mesh elements to sub meshes.
- ☐ This is in fact a clustering problem of primitive elements into groups or clusters.
- ☐ This problem is well studied in Machine Learning.
- ☐ The different algorithms can be classified as variants of classic clustering algorithms.

Region growing

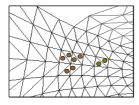


Region growing

Region Growing Algorithm

Region Growing Algorithm
Initialize a priority queue Q of elements
Loop until all elements are clustered
Choose a seed element and insert to QCreate a cluster C from seed
Loop until Q is empty
Get the next element s from QIf s can be clustered into CCluster s into CInsert s neighbors to QMerge small clusters into neighboring ones

Hierarchical clustering

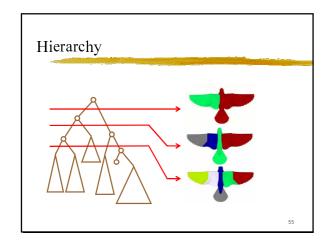


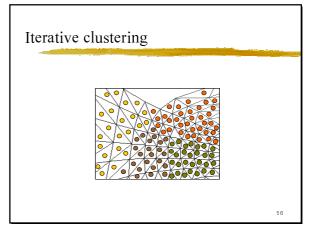


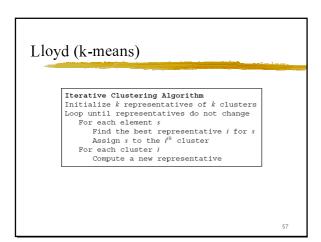
Hierarchical clustering

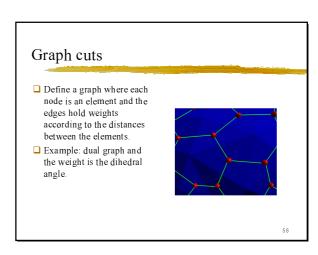
Hierarchical Clustering Algorithm

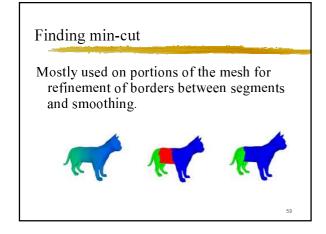
Hierarchical Clustering Algorithm Initialize a priority queue $\mathcal Q$ of pairs Insert all valid element pairs to $\mathcal Q$ Loop until $\mathcal Q$ is empty Get the next pair (u,v) from $\mathcal Q$ If (u,v) can be merged Merge (u,v) into w Insert all valid pairs of w to $\mathcal Q$











Today's algorithms

Convex decomposition – Chazelle et al, 97

Wateshed – Mangan & Whitaker, 99

Two-phase – Katz & Tal, 03

Feature-point & Core extraction, Katz et al, 05

Convex decomposition, (Chazelle et al, 97)

- ☐ Easiest to represent, manipulate and render
- ☐ The human visual system tends to segment complex objects at regions of deep concavities (Biederman)

Convex decomposition

- ☐ Goal: decompose into convex patches
- ☐ Convex patch lies entirely on the boundary of its convex hull

Algorithms

- 1. Space partitioning
- 2. Space sweeping
- 3. Flooding

63

Flooding algorithm

- 1 Let G be the dual graph
- 2 Traversing G, collecting vertices (faces), as long as a pre-defined property is not violated
- 3 When traversal cannot be continued, a new patch is started and the traversal is resumed.

64

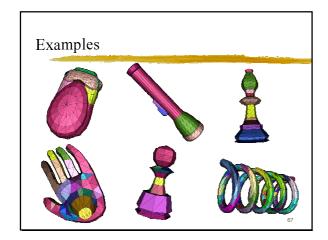
Failures

- ☐ Local failure the edge at which the facet is attached to the patch exhibits non-convexity
- ☐ Global failure the patch is locally convex everywhere, but some facet fails to be on the boundary of the convex hull

65

Flood & Retract

- 1. Flood the surface by covers patches might overlap
- 2. Transform the covers into partition retracting each patch



Drawbacks

- ☐ The optimization problem is NP-complete
- Over segmentation
- ☐ Jagged boundaries

68

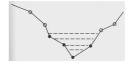
Watershed (Mangan and Whitaker, 99)

Extension to 3D a known 2D algorithm in image processing

Key idea - Regions are segmented into catchment-basins (watersheds)

Watersheds

- ☐ Catchment-basin set of points whose path of steepest descent terminates in the same local minimum of a height function
- ☐ Height function depends on the application



70

Watershed segmentation algorithm

- 1. Compute the height function (curvature) at each vertex
- 2. Find the local minima and assign each a unique label
- 3. Find each flat area and classify it as a minimum or a plateau
- 4. Loop through plateaus and allow each one to descend until a labeled region is encountered
- 5. Allow all remaining unlabeled vertices to similarly descend and join labeled regions
- Merge regions whose watershed depth is below a preset threshold

1

Initial labeling

- 1. Local minima consisted of single vertex
- 2. Flat minimum
- 3. Flat plateau

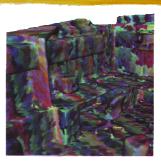


Descent

Imagine a drop of water placed at the starting vertex, flowing downhill on the height function



Region merging (why?)



7.4

Region merging

Metric – greatest depth of water that segment can hold before it "spills over"



Region merging algorithm

- 1. For each region, find its lowest point, neighbors, and lowest boundary point with each neighbor
- 2. Find depth of region, the difference between the lowest point to lowest boundary point
- 3. If depth is below predefined threshold, merge this region to region adjacent to lowest boundary point and update new region's information accordingly
- 4. Repeat until no regions exist that are below the minimum depth

3

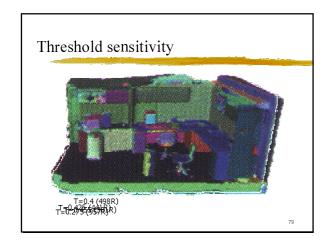
Curvature calculation

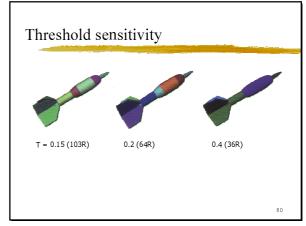
Depends on type of data and the level of noise

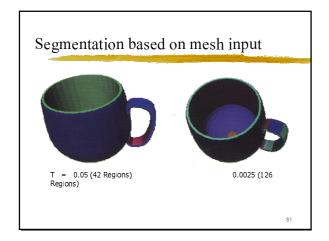
- ☐ Inputs
 - Volumes (voxels) data is used to compute curvature
 - Meshes several possibilities

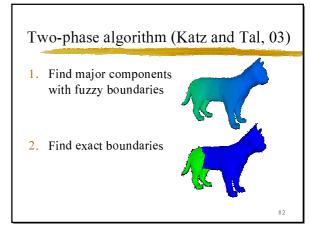
Results

- 1. Over-segmentation
- 2. Noise partitions might fail dramatically
- 3. Threshold sensibily

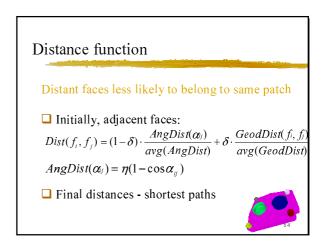








Algorithm outline 1. Construct fuzzy decomposition a. Assign distances to pairs of faces b. Assign probabilities of belonging to patches c. Compute a fuzzy decomposition 2. Construct exact boundaries



Probabilities

$$P_{B}(f_{i}) = \frac{Dist(f_{i}, REP_{A})}{Dist(f_{i}, REP_{A}) + Dist(f_{i}, REP_{B})} = \frac{a_{f_{i}}}{a_{f_{i}} + b_{f_{i}}}$$

Properties

- I. $\forall a_{fi} < b_{fi}, P_B(f_i) < 0.5$
- II. $\forall a_f > b_f P_B(f_i) > 0.5$
- III. $\forall a_{fi} = b_{fi}$, $P_B(f_i) = 0.5$
- IV. $P_B(f_i) = 1 P_A(f_i)$



Fuzzy K-means

Goal: optimize $F = \sum_{p} \sum_{f} \Pr(f \in patch(p)) \cdot Dist(f, p)$ 1. Initialization - select set of representatives

- 2. Compute probabilities
- 3. Re-compute the set of representatives V_k REP_B = min ∫ ∑ P_B(f_i) · Dist(f, f_i)
 4. If V_k is sufficiently different from V_k,
- set $V_k \longleftarrow V_{k'}$ and go back to 2

Fuzzy decomposition

The surface is decomposed into A, B, Fuzzy

$$\begin{split} A &= \left\{ f_i \mid P_B(f_i) < 0.5 - \varepsilon \right\} \\ B &= \left\{ f_i \mid P_B(f_i) > 0.5 + \varepsilon \right\} \\ Fuzzy &= \left\{ f_i \mid 0.5 - \varepsilon \le P_B(f_i) \le 0.5 + \varepsilon \right\} \end{split}$$



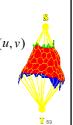
Problem: finding boundaries

- Given:
 - G=(V,E) the dual graph of the mesh
 - A,B,Fuzzy
- Partition V into VA' and VB' s.t.
 - $I. \quad V = V_{A'} \cup V_{B'}$
 - $\text{II.} \quad \mathsf{V}_{\mathsf{A}'} \cap \mathsf{V}_{\mathsf{B}'} = \phi$
 - ${\color{red} III.} \ \, V_{A} \subseteq V_{A'}, V_{B} \subseteq V_{B'}$
 - IV. Good cut!



Algorithm for finding boundaries

- Assign capacities
- ☐ Construct a flow network on *Fuzzy*
- ☐ Find the minimum cut $weight(Cut(V_{A'}, V_{B'})) = \sum_{u \in V_{A'}, v \in V_{B'}} w(u, v)$

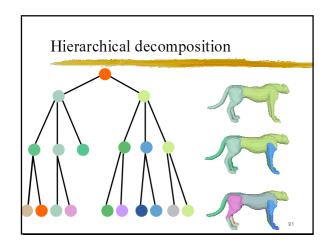


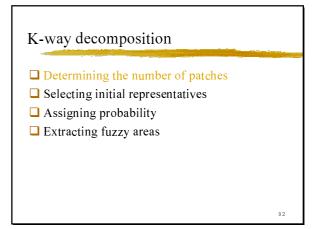
Assigning capacities

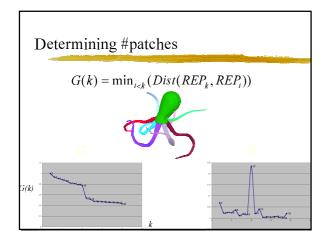
Cuts should pass at regions of deep concavities (Biederman)

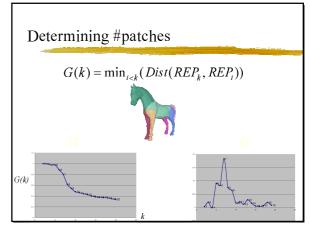
$$Cap(i, j) = \frac{1}{1 + \frac{AngDist(\alpha_{ij})}{avg(AngDist)}}$$

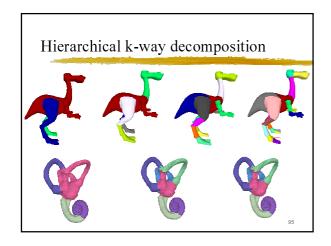


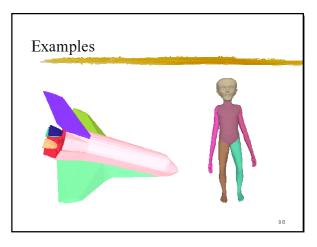


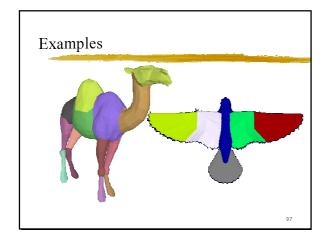


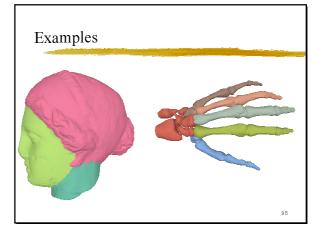












Segmentation by feature point & core extraction

For each hierarchical-level

- 1. Mesh coarsening
- 2. Pose invariant representation
- 3. Feature point detection
- 4. Core component extraction
- 5. Mesh segmentation
- 6. Coarse mesh cut refinement
- 7. Fine mesh cut refinement

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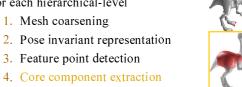
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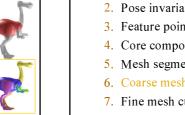
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Focus

For each hierarchical-level

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Pose invariant representation



Multi-Dimensional Scaling (MDS)

Pose invariant representation by MDS

- ☐ Transform the vertices such that Geodesic dist. in $S \cong Euclidean dist.$ in SMDs $\delta_{ii} = dissimilarity = GeodesicDist(v_i, v_i)$ in S
- $d_{ij} = EuclideanDist(v_i, v_j)$ in SMDS
- Using MDS optimize $F_S = \frac{\sum_{i < j} (f(\delta_{ij}) d_{ij})^2}{\sum_{i < j} d_{ij}^2}$



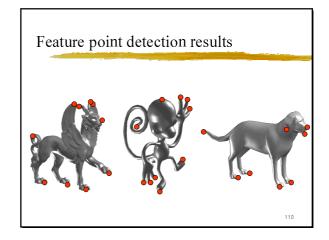


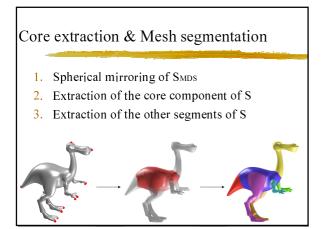
Feature points

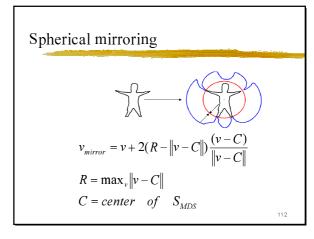
- ☐ Should reside on tips of prominent components
- ☐ Useful for:
 - Deformation transfer
 - Mesh retrieval
 - Texture mapping
 - Metamorphosis (cross-parameterization)

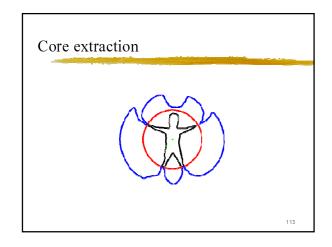


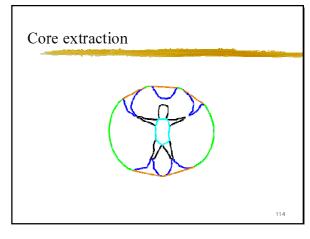
Feature point detection Local maximum of sum of geodesic distances $\sum_{v_i \in S} GeodDist(v, v_i) > \sum_{v_i \in S} GeodDist(v_n, v_i)$ Resides on the convex-hull of SMDS Insensitive to noise, does not require user

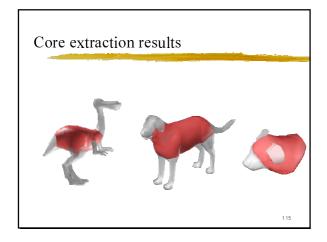


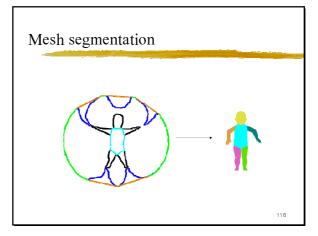






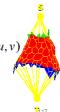






Refining the boundaries

- ☐ Construct a flow network on *the search* region
- ☐ Assign capacities
- Find the minimum cut $weight(Cut(V_{A'}, V_{B'})) = \sum_{u \in V_{A'}, v \in V_{B'}} w(u, v)$

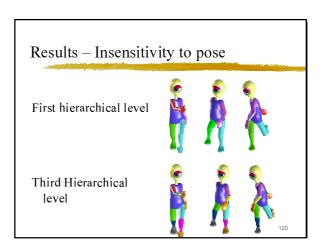


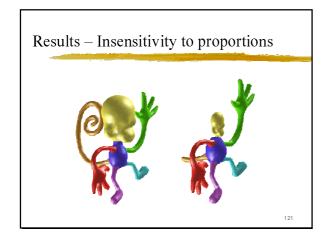
Cut refinement – Minimum cut

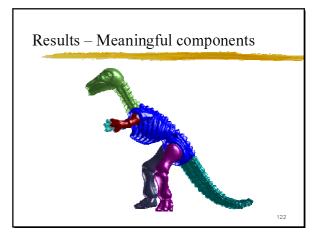
For each coarse boundary between segments:

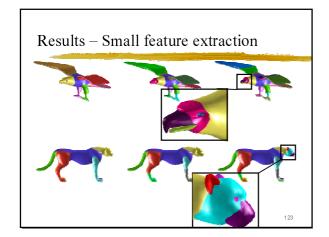
- ☐ Search region the faces whose distance to the coarse boundary is small
- Arc capacities

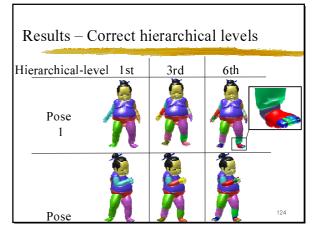
$$\omega_{ij} = \alpha \left(\frac{angW_{ij}}{AVG_{angW}}\right) + (1 - \alpha)\left(\frac{edge_{ij}}{AVG_{edge}}\right)$$











Advanced Issues

- ☐ Can we say it is "correct"?
- ☐ What is the notion of "shape"?
- ☐ Some interpretation would be that our notion and perception of the shape (our "segmentation") would not change under certain transformations:
 - Rigid body invariant
 - Generally NOT Affine invariant
 - Pose invariant?

Final Remarks

- ☐ Many applications use mesh segmentation
- ☐ Segmentation usually has more effect on the results than seem to be realized
- □ 3D segmentation is still a very difficult problem and still in its infancy, e.g. compared to image segmentation
- ☐ More advanced coherency issues should be addressed pose invariance, extracting similar parts and more... 128