Student Project:
Data Fusion in MMSE Problems

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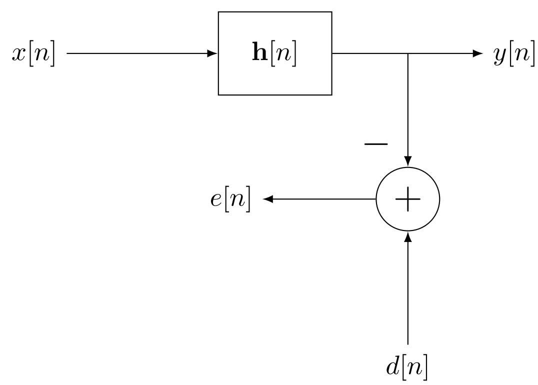
1 Background

In this project, we will revisit a well-known signal processing problem: Minimizing the mean-square error using a linear filter. In many cases, filters are designed using frequency-domain specifications, which lead to low-pass/band-pass/high-pass/band-stop filters. The goal in these cases is to extract the desired information from an input signal. However, in other cases, which will be the focus of the project, we wish to filter a signal $x[n]$ in order to modify it so that it approximates some other signal $d[n]$ in some statistical sense. That is, the output of the filter, $y[n]$, is a good estimate of a reference signal $d[n]$. The output error $e[n]$ represents the mismatch between $y[n]$ and $d[n]$. This can be considered a time-domain specification of the filter. An illustration of the system is given in the following figure:



Where can we find such problems? Well, almost everywhere! For example:

1. *Noise cancellation*: Say, for example, that we have a clean version of a song, along with a corrupted version of it, at the output of an earphone. Our goal is to tune our earphones so that they "fix" the distortion. Here, $\mathbf{h}$ will converge to a noise-reducing filter.

2. *Reverse Engineering*: Say you have a system that you are not familiar with. In such cases, we can use the input and output signal to and from the system to identify the system itself. Here, $\mathbf{h}$ will converge to a linear approximation of the "black-box" system.

3. *Channel estimation*: Say you want to know how a radio signal was distorted by a physical channel between the transmitter and the receiver. You can send a reference signal and measure its distortion. Here, $\mathbf{h}$ will converge to a linear channel impulse response.

This is all very standard. In fact, we can solve the aforementioned problem using classical tools and derive an optimal filter, often referred to as the "Wiener Filter". In this project, we will try to generalize this classical result. Specifically, we will assume that the reference signal $d[n]$ is available to us for only a short period of time. We will then ask ourselves if there is anything we can learn from the signal during the blind times, i.e., the time intervals in which the reference is unknown, in order to obtain better overall performance.

2 Project Goals

The goals of this project are as follows.

1. Understand the Wiener Filter and its derivation.
2. Understand the disadvantages of a naive approach, often used to solve the filtering problem.
3. Use an alternative estimation approach, using the maximum likelihood (ML) estimator.
4. Approximate the Wiener Filter using the ML approach in both scalar and vector forms.

3 Prerequisite

- Random Signals (044202)