A Doppler Focusing Approach for Sub-Nyquist Radar

Omer Bar-Ilan and Yonina C. Eldar

Technion–Israel Institute of Technology

Motivation and Main Results

- Demand for high resolution radar requires high bandwidth signals
- Such signals are hard to sample and process digitally
- Previous CS works for this problem either do not address sampling, require a prohibitive dictionary size, or perform poorly with noise

We develop a sub-Nyquist sampling and recovery method implemented in hardware which provides both simple recovery and robustness to noise by performing beamforming on the low rate samples

Main Concept

- The sub-Nyquist recovery method is based on the following concepts:

- Finite Rate of Innovation (FRI) is the mathematical framework which enables modeling the analog signal with a small set of unknown parameters
- Xampling (Compressed Sampling) is the process of sampling a signal at a low rate in such a way that preserves the information required for recovery
- Doppler Focusing is a method of digitally beamforming the low rate samples which is both numerically efficient and robust to noise

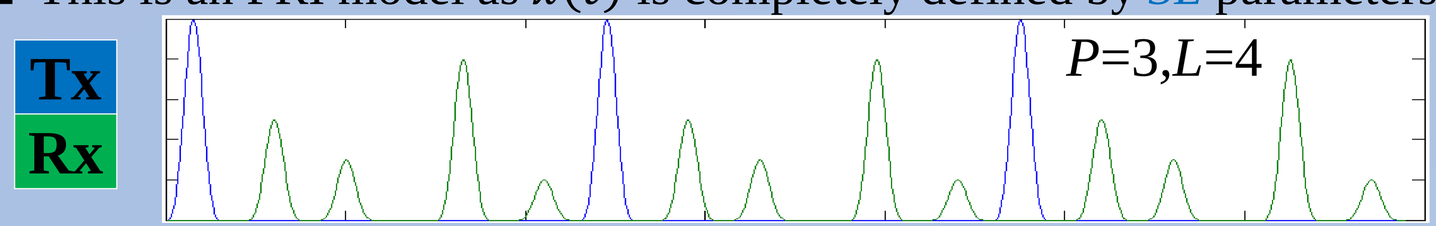
Radar FRI Model

- L targets, each defined by 3 degrees of freedom: amplitude α_ℓ, delay τ_ℓ, and Doppler frequency ν_ℓ
- After transmitting P equispaced high-bandwidth pulses $h(t)$, the received signal*:

$$x(t)=\sum_{p=0}^{P-1}\sum_{\ell=0}^{L-1}\alpha_\ell h(t-\tau_\ell-p\tau)e^{-j\nu_\ell p\tau}$$

(* some assumptions on target dynamics are needed for this model)

- This is an FRI model as $x(t)$ is completely defined by $3L$ parameters



- Signal's Fourier coefficients contain the required parameters:

$$c_p[k]=\frac{1}{\tau}\int_0^\tau x_p(t)e^{-j2\pi kt/\tau}dt=\frac{1}{\tau}H(2\pi k/\tau)\sum_{\ell=0}^{L-1}\alpha_\ell e^{-j2\pi k\tau_\ell/\tau}e^{-j\nu_\ell p\tau}$$

- Standard radar methods sample and process at the Nyquist rate

Our goal: break the link between signal bandwidth and sampling and processing rates

Previous Approaches

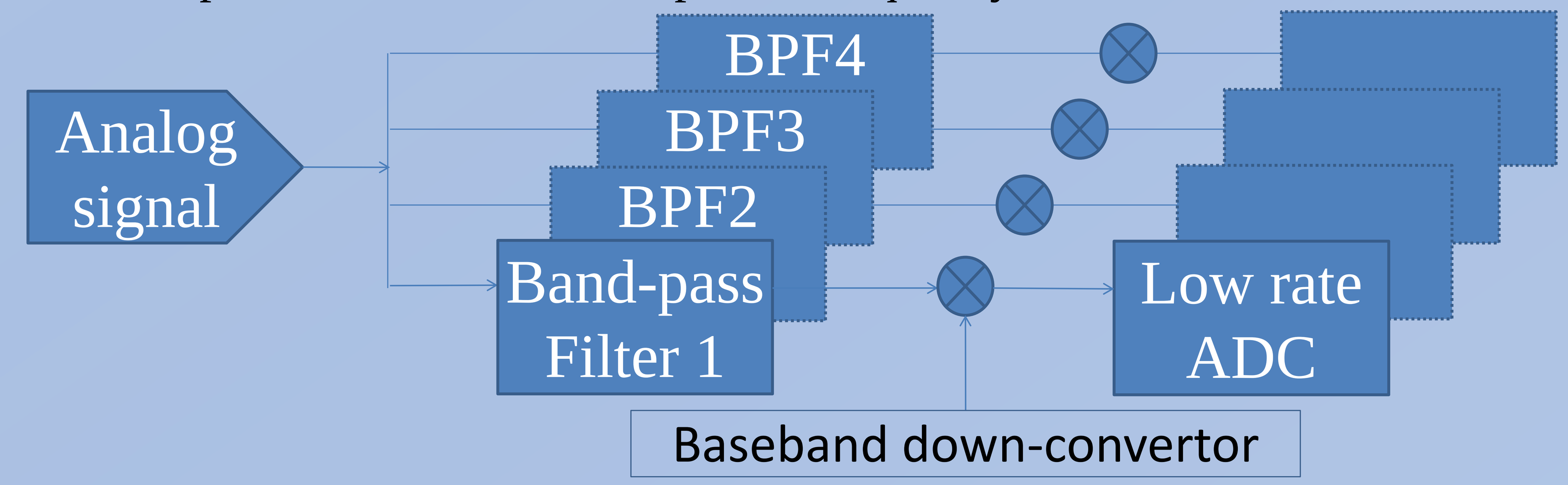
- Previous works do not address sample rate reduction feasible in hardware

Various other works suffer from the following shortcomings:

- Impose constraints on the radar transmitter and do not treat noise (e.g. Baraniuk & Steeghs, "Compressive Radar Imaging")
- Construct a CS dictionary with a column for each two dimensional grid point causes dictionary explosion for any practical problem size (e.g. Herman and Strohmer, "High-Resolution Radar via CS")
- Perform non-coherent integration over pulses, obtaining a sub-linear SNR improvement with P (e.g. Bajwa, Gedalyahu & Eldar, "Identification of Parametric Underspread Linear Systems and Super-Resolution Radar")

Xampling Scheme – Acquiring Fourier Coefficients

- We've seen the signal's parameters are embodied in its Fourier coefficients
- We use the following multichannel analog processing and low rate sampling to extract spectral information for specific frequency bands:



- Calculating the Fourier coefficients is performed digitally after sampling

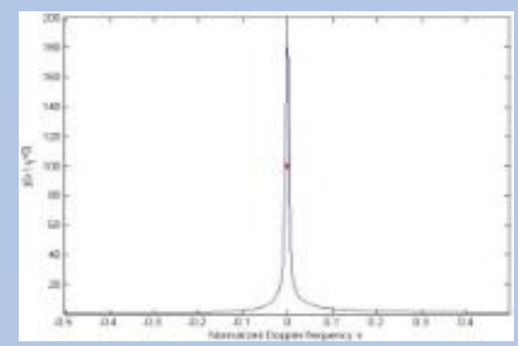
Digital Recovery Using Doppler Focusing

- Transforms a simultaneous delay-Doppler estimation problem into a set of delay-only problems with specific Doppler frequency
- Focusing on Doppler frequency ν for sampled Fourier coefficients:

$$\Psi_\nu[k]=\sum_{p=0}^{P-1}c_p[k]e^{j\nu p\tau}=\frac{1}{\tau}H(2\pi k/\tau)\sum_{\ell=0}^{L-1}\alpha_\ell e^{-j2\pi k\tau_\ell/\tau}\sum_{p=0}^{P-1}e^{j(\nu-\nu_\ell)p\tau}$$

Spectral analysis problem — Focusing term

- Advantages:
  - Beamforming on the low rate samples
  - Robust performance with noise
  - Fast to compute (FFT)
  - Can use any known spectral analysis methods (CS, matrix pencil, MUSIC, etc.)

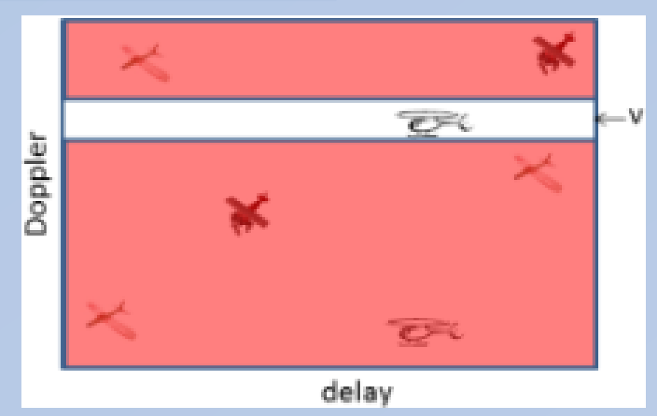


$$\cong\begin{cases}P, & |\nu-\nu_\ell|<\pi/P\tau\\ 0, & otherwise\end{cases}$$

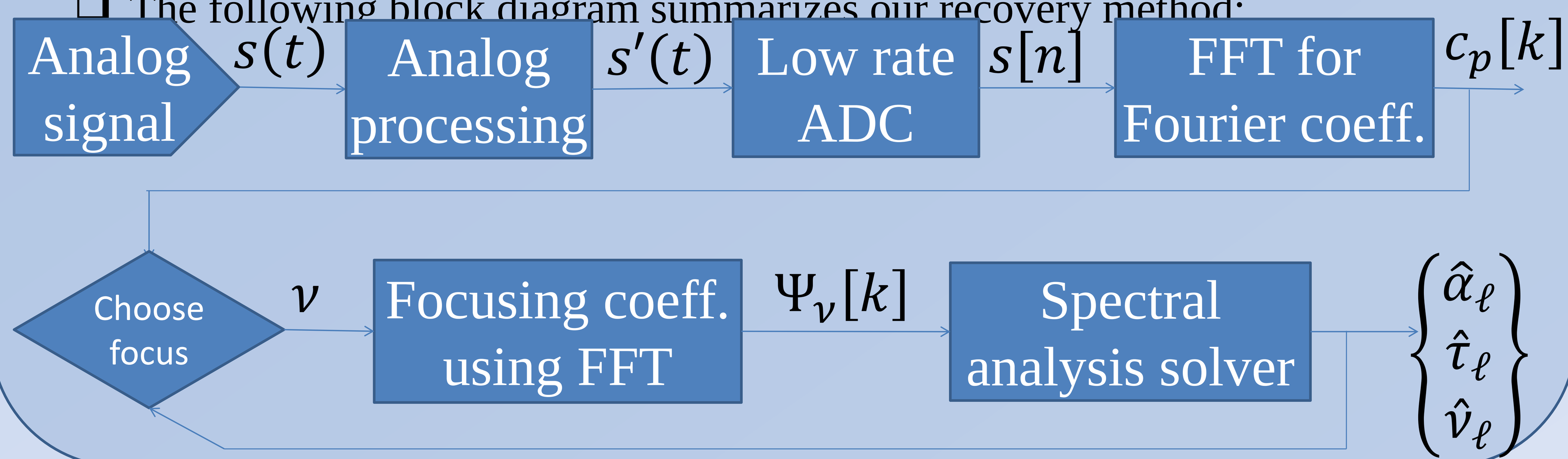
- Coherent integration of echoes from different pulses creates a single superimposed pulse. SNR scaling is linear with P

Optimal SNR performance, equivalent to a matched filter

- Instead of trying to detect targets in the delay-Doppler plane, Doppler focusing creates Doppler slices in which targets are detected using delay only

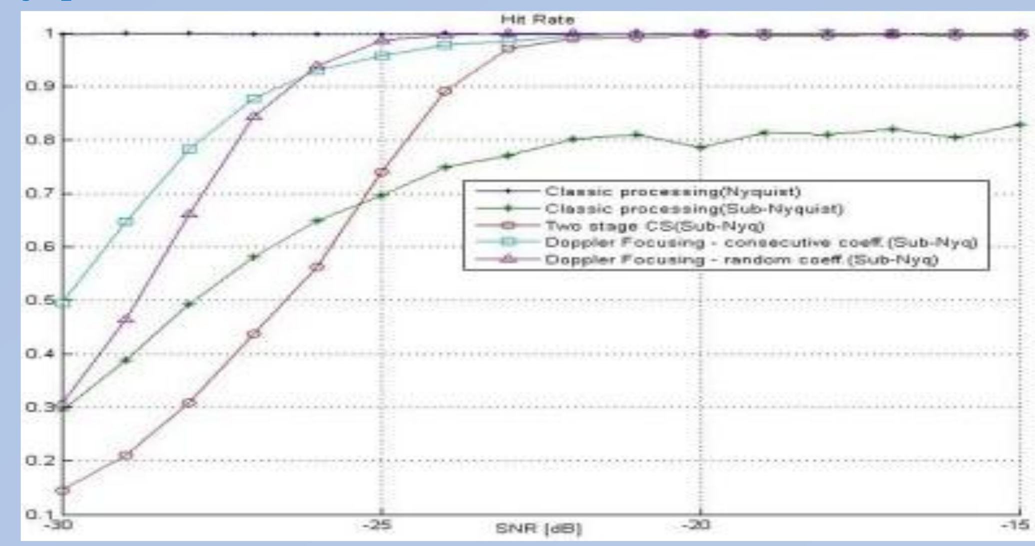


- **A hard 2D estimation problem is efficiently reduced into several easier 1D problems**
- The following block diagram summarizes our recovery method:



Simulation Results

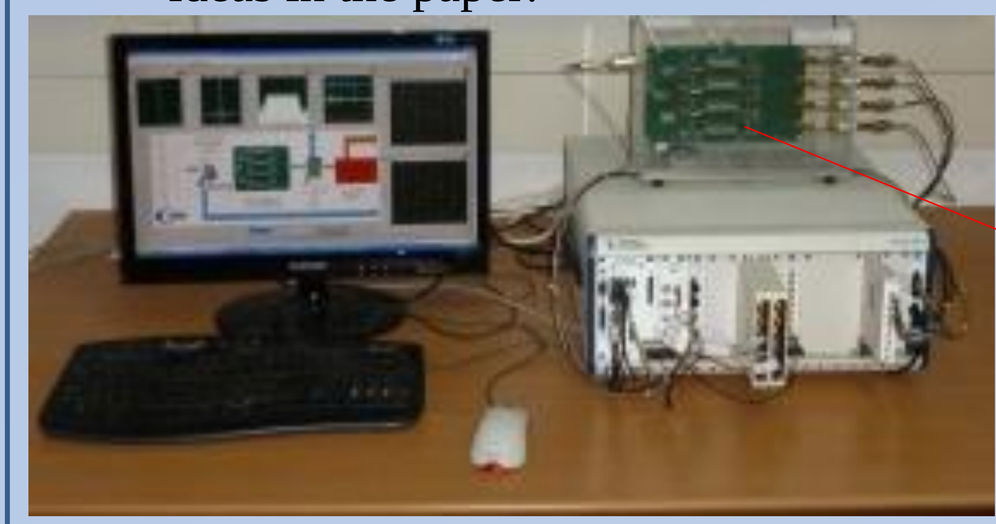
- Measuring performance using "hits" and RMS error
- A "hit" is a delay-Doppler estimate in the interior of an ellipse around the true target position
- At one tenth the Nyquist Rate and at -25bB SNR, Doppler focusing achieves performance equivalent to matched filter processing sampling at the Nyquist rate



- When we concentrate the signal's entire energy contents in the sampled frequencies, Doppler focusing based recovery outperforms matched filtering at Nyquist rate

Radar Experiment

- A Xampling-based hardware prototype board which implements the ideas in the paper:



- Samples a radar signal which classically requires a 30MHz sample rate at 1MHz, and performs recovery using Doppler focusing

- Was first demonstrated at NI Week, August 2012 (with a different recovery method without Doppler, see E. Baransky et al, "A Sub-Nyquist Radar Prototype: Hardware and Algorithms"), and a full version was demonstrated at RadarCon 2013

{omerba@tx,yonina@ee}.technion.ac.il