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Sub-Nyquist Radar

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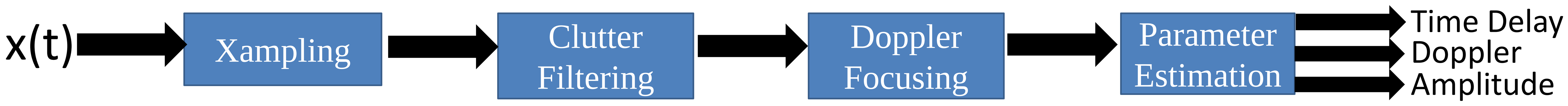
Motivation and Goals

- High resolution radar requires high bandwidth signals
- Wideband signals need a complex analog front end receiver design which consumes high power
- Digital processing of wideband signals requires large memory and large computational power
- We present a sub-Nyquist sampling and recovery method implemented in hardware which reduces the rate by 30 fold
- This approach provides both simple recovery and robustness to noise by performing beamforming on the low rate samples
- Clutter rejection is also performed on the sub-Nyquist samples by adapting standard methods to our setting

Sub-Nyquist Radar Algorithm



- Xampling– A process of sampling a signal at a low rate in such a way that preserves the required information
- Clutter Filtering – Adaptation of standard clutter algorithms to fit our low rate samples
- Doppler Focusing - A method of digitally beamforming the low rate samples which is both numerically efficient and robust to noise
- Estimation – A modified OMP, matched to our samples, produces target locations and Doppler frequencies

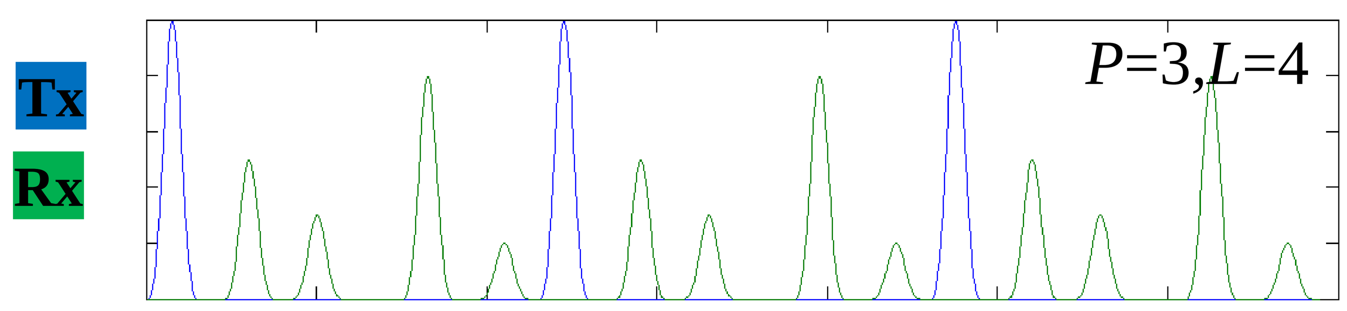
Input Signal Model

- L targets, each defined by 3 degrees of freedom: amplitude α_ℓ, delay τ_ℓ, and Doppler frequency ν_ℓ
- After transmitting P equispaced high-bandwidth pulses $h(t)$, the received signal*:

$$x(t) = \sum_{p=0}^{P-1}\sum_{l=0}^{L-1} \alpha_l h(t-\tau_l - p\tau) e^{-j\nu_l p\tau}$$

(* some assumptions on target dynamics are needed for this model)

- This is an FRI model as $x(t)$ is completely defined by $3L$ parameters



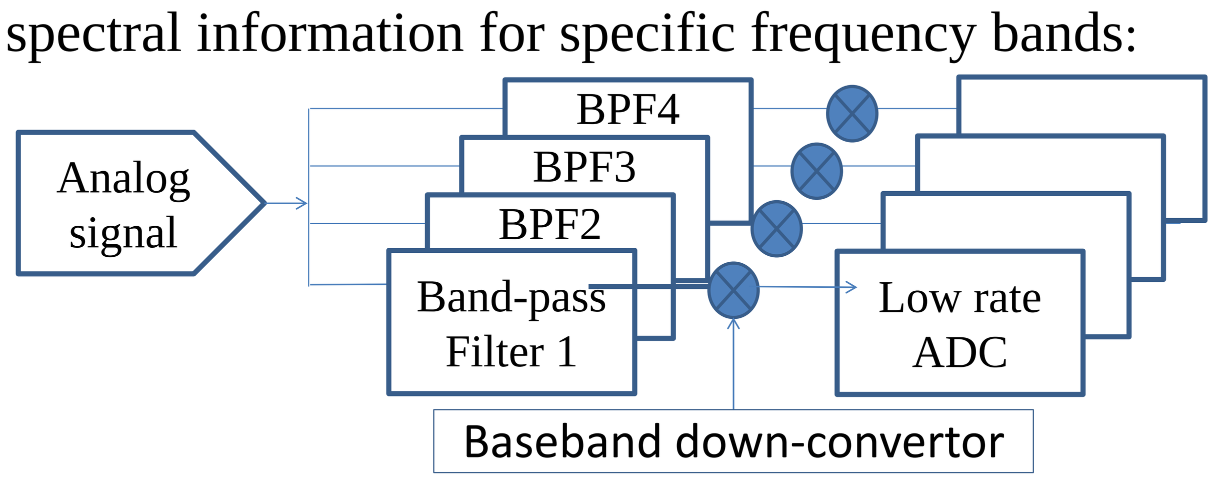
- The signal's Fourier coefficients contain the required parameters:

$$\boxed{c_p[k] = \frac{1}{\tau}\int_0^\tau x_p(t) e^{-j2\pi kt/\tau} dt = \frac{1}{\tau} H(2\pi/\tau) \sum_{\ell=0}^{L-1} \alpha_\ell e^{-j2\pi k\tau_\ell/\tau} e^{-j\nu_\ell p\tau}}$$

- Standard radar methods sample and process at the Nyquist rate

Xampling Scheme – Acquiring Fourier Coefficients

- The signal's parameters are embodied in its Fourier coefficients
- Multichannel analog processing and low rate sampling scheme are used to extract spectral information for specific frequency bands:



- Calculating Fourier coefficients is performed digitally after sampling

Clutter Filtering

- The target signal is contaminated with clutter + thermal noise:

$$y(t) = x(t) + c(t) + n(t)$$

- Assume the clutter interference is modelled as "colored" noise - a WSS random process whose spectrum is Gaussian:

$$S_C(f) = P_C \cdot \frac{1}{\sqrt{2\pi}\sigma_c} \cdot exp\left[\frac{(f-f_c)^2}{2\sigma_c^2}\right]$$

- Clutter + Thermal Noise autocorrelation matrix:

$$M(m,n) = (P_c/P_N) e^{-2(\pi\sigma_c T\cdot(m-n))^2} e^{-j\cdot 2\pi(m-n)f_c} + \delta_{m,n}$$

- Filtering is performed by using the whitening matrix $M^{-\frac{1}{2}}$ to whiten the interference and proceeding with Doppler focusing.

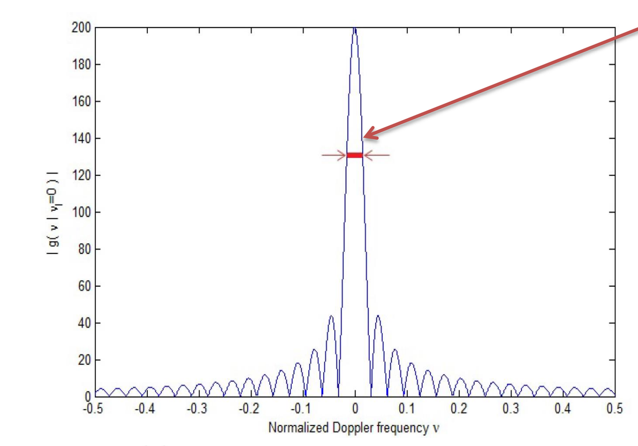
Doppler Focusing

- Transforms a simultaneous Delay-Doppler estimation problem into a set of delay-only problems with specific Doppler frequency
- Focusing on Doppler frequency ν for sampled Fourier coefficients:

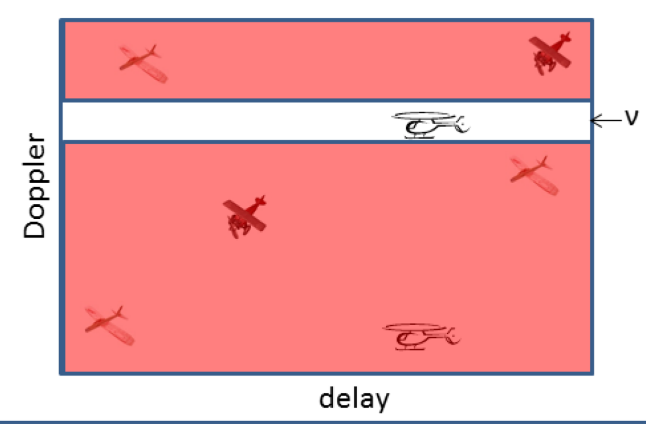
$$\boxed{\Psi_\nu[k] = \sum_{p=0}^{P-1} \tilde{C}_p[k] e^{j\nu p\tau} = \frac{1}{\tau} H(2\pi k/\tau) \underbrace{\sum_{\ell=0}^{L-1} \alpha_\ell e^{-j2\pi k\tau_\ell/\tau}}_{\text{Spectral analysis problem}} \underbrace{\sum_{p=0}^{P-1} e^{j(\nu-\nu_\ell)p\tau}}_{\text{Focusing term}}}$$

Focusing term

$$\sum_{p=0}^{P-1} e^{j(\nu-\nu_\ell)p\tau} \cong \begin{cases} P, & |\nu-\nu_\ell| < \pi/P\tau \\ 0, & otherwise \end{cases}$$

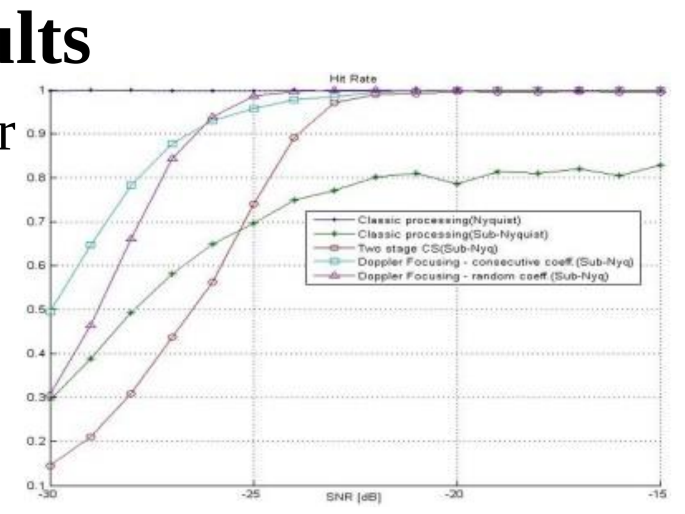


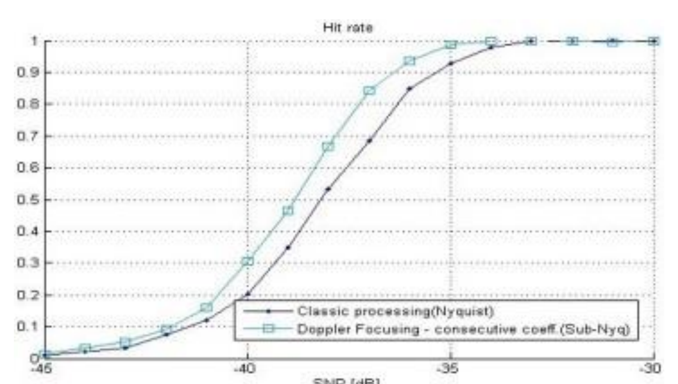
- Coherent integration of echoes from different pulses creates a single superimposed pulse. SNR scaling is linear with P
- Instead of detecting targets in the delay- Doppler plane, Doppler focusing creates slices in which targets are detected using delay only
- **A hard 2D estimation problem is efficiently reduced into several easier 1D problems.**



Simulation Results

- Measure performance by"hits" and RMS error
- A "hit" is a Delay-Doppler estimate in the interior of an ellipse around the true target one tenth the Nyquist Rate and at -25bB SNR, Doppler focusing achieves performance equivalent to matched filter processing sampling at the Nyquist rate
- When we concentrate the signal's energy contents in the sampled frequencies, Doppler focusing outperforms matched filtering at Nyquist rate





- Under SNR of -16dB and 100 pulses used:
- Without clutter filtering, only 3 out of 5 targets are detected
- Using clutter filtering algorithm, all 5 targets are detected.
- **The performance of sub-Nyquist algorithm is equivalent to classic Nyquist rate processin**