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Noise induced in optical fibers by double Rayleigh scattering of a laser with a $1/f^{\nu}$ frequency noise

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We study, theoretically and experimentally, intensity noise induced by double Rayleigh scattering in long optical fibers. The results of the theoretical model are compared to experimental results performed with a high-coherence-length laser with a frequency noise spectrum that is dominated by $1/f^{\nu}$ noise. Excellent quantitative agreement between theoretical and experimental RF spectra were obtained for frequencies as low as 10 Hz and for fiber lengths between 4 and 45 km. Strong low-frequency intensity noise that is induced by $1/f^{\nu}$ frequency noise of the laser may limit the performance of interferometric fiber optic sensors that require high-coherence-length lasers. The intensity noise due to double Rayleigh backscattering can be suppressed by reducing the coherence length of the laser. Therefore, the intensity noise has a complex and nonmonotonic dependence on the $1/f^{\nu}$ frequency noise amplitude of the laser. Stimulated Brillouin scattering will add a significant noise for input powers greater than about 7 mW for a 30 km length fiber.

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Rayleigh backscattering in optical fibers is caused by scattering from refractive index changes formed by material density fluctuations that are "frozen" into the fused silica during the manufacture of the fiber [1]. A forward propagating pump wave that is injected into a fiber is backscattered along the fiber and generates a backward propagating wave. The scattered wave can be backscattered again and form a wave that propagates in the same direction as the pump wave. This multiple scattering process is called double Rayleigh scattering (DRS). The DRS wave has a very weak intensity; however, it can interfere with the strong pump wave and cause a significant noise at the photo-detector (PD) by conversion of the laser frequency noise into intensity noise [2,3].

In previous works, optical intensity noise that is induced by DRS has been studied for lasers with only white frequency noise [2,3] and coherence length that is significantly shorter than the fiber length. The spectrum of the intensity noise at the fiber output had a Lorentzian lineshape with a bandwidth that is on the order of few MHz. The effect of conversion of a flicker frequency noise into intensity noise through an optical interference that is formed by a pair of reflections was studied in [4].

Interferometric fiber sensors require high-coherence-length lasers [5]. In such systems, frequency noise of the laser may determine the sensitivity of the system due to unbalanced interferometer arms. To increase the sensor sensitivity, lasers with linewidths that are on the order of several kHz are used [5]. However, in such sensors, the intensity noise that is caused by conversion of the laser frequency noise into intensity noise due to DRS may limit the system performance [6]. This noise may be suppressed to some extent by modulating the laser frequency [2,6]. However, the theoretical suppression is limited [2], while the frequency modulation increases the complexity of the systems and may also limit the sensitivity of some applications. Transmitted noise is also important in optical frequency transfer, which requires highly coherent sources [7]. Recent measurements on transmitted noise induced by a 10 km fiber have shown elevated noise levels above laser intensity noise [8].

In this Letter, we study, theoretically and experimentally, intensity noise that is induced by DRS for a laser source with a general frequency noise. In particular, we study DRS caused by a laser that is dominated by a $1/f^{\nu}$ frequency noise. We also take into account the relative intensity noise (RIN) of the laser that was ignored in previous works [2,3]. This noise is important for short fibers and for lasers with a linewidth that is larger than about 1 MHz. Excellent quantitative agreement between theory and experiment was obtained for fibers with a length between 4 and 45 km. The agreement was obtained for a frequency region of 10 to 10^5 Hz. Therefore, we show that noise due to DRS is the dominant noise, even at frequencies as low as 10 Hz. Hence, even at very low frequencies, noise induced by DRS is stronger than noise caused by thermal and thermomechanical fluctuations of the fiber [9]. The effect of laser frequency noise on a self-homodyne detection of backscattered Rayleigh wave in optical fibers has been recently studied [10]. A good agreement between experiments and theory for a laser with a high-coherence-length was obtained at frequencies above

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200 Hz and fibers longer than 6 km. For shorter fibers, a good agreement was obtained only at frequencies above 500 Hz. Therefore, the dominant noise source at low frequencies in those experiments was not caused by Rayleigh backscattering of a high-coherence laser with a $1/f^{\nu}$ frequency noise.

We compare the RF noise due to DRS for lasers with different parameters of white and $1/f^{\nu}$ frequency noise. We show, theoretically, that in the case where the laser coherence length becomes shorter than the fiber length, the intensity noise is reduced. The coherence length of the laser can be decreased by increasing the amplitude of the $1/f^{\nu}$ frequency noise of the source.

We also show that for optical power of 1.7 mW that was used in our experiments, noise induced by the stimulated Brillouin scattering (SBS) [11] is negligible. It is expected that this noise source will become important for sufficiently high power. For example, for optical power of 7 mW, the intensity noise due to SBS for a 30 km length fiber becomes equal to noise induced by DRS at a frequency of 1 kHz.

We model the laser source by a quasi-monochromatic wave:

$$E_{\rm in}(t) = E_0[1 + m(t)]\exp[j\varphi(t) + j\omega_0 t], \tag{1}$$

where $m(t)$ represents the relative amplitude noise of the laser with an autocorrelation function $r_m(\tau) = \langle m(t)m^*(t-\tau)\rangle$ and zero mean, $\varphi(t)$ denotes the laser phase noise, and ω_0 is the carrier frequency. The laser relative intensity noise $S_{\rm RIN}(\omega)$ is equal to the Fourier transform of the autocorrelation function $\langle |E_{\rm in}(t)|^2|E_{\rm in}(t-\tau)|^2\rangle/\langle|E_{\rm in}(t)|^2\rangle^2$, where $\langle\rangle$ denotes a time-average. The effect of the polarization state changes during the propagation in a random birefringent fiber can be added by calculating the degree of polarization of the double backscattered field by using the properties of a Mueller matrix [3,12]. Hence, we use a scalar field in the following derivations, and we add the polarization effect by multiplying the calculated spectrum by a constant factor of 5/9.

We model Rayleigh backscattering of an electrical field by a coefficient $\kappa(z)$ that describes the scattering per unit of length that is modeled as a complex white Gaussian zero-mean process with an ensemble average $\langle\kappa(z_1)\kappa^*(z_2)\rangle \approx \sigma_\kappa^2\delta(z_1 - z_2)$ [13]. Back reflections from locations z_1 and z_2 along the fiber are integrated, and the double Rayleigh backscattered electrical field at the fiber output is given by [2,3]

$$E_{\rm DRS}(t) = \int_0^L {\rm d}z_1 \int_0^{z_1} {\rm d}z_2 E_{\rm in}[t - nL/c - 2n/c(z_1 - z_2)] \times \exp(-\alpha L/2)\exp[-\alpha(z_1 - z_2)]\kappa(z_1)\kappa(z_2), \tag{2}$$

where L is the fiber length, n is a fiber refractive index, c is a speed of light in the vacuum, and α is a power loss coefficient per fiber length. The total electrical field at the PD is

$$E(t) = E_s(t) + E_{\rm DRS}(t), \tag{3}$$

where $E_s(t) = E_{\rm in}(t - nL/c)\exp(-\alpha L/2)$ is the electrical field of the source at the fiber output.

The optical intensity at the output of the fiber equals $I(t) = (1/2)\epsilon_0 nc|E(t)|^2$, where ϵ_0 is the vacuum permittivity. To calculate the normalized autocorrelation function of the optical intensity at the PD input, $r(\tau) = \langle I(t)I(t-\tau)\rangle/\langle I(t)\rangle^2$, we use Eqs. (2) and (3) and neglect terms that do not depend on time and terms that result in zero value due to statistical properties of the backscattering coefficient κ [3]. We also keep terms that are on the order of $L^2\sigma_\kappa^4$ and neglect noise components that are on the order of $L^4\sigma_\kappa^8$. We then obtain

$$\begin{aligned} r(\tau) = \{&\langle|E_s(t)|^2|E_s(t-\tau)|^2\rangle \\ &+ 2\,{\rm Re}\langle E^*_{\rm DRS}(t)E_s(t)E^*_s(t-\tau)E_{\rm DRS}(t-\tau)\rangle \\ &+ \langle|E_s(t)|^2|E_{\rm DRS}(t-\tau)|^2\rangle \\ &+ \langle|E_s(t-\tau)|^2|E_{\rm DRS}(t)|^2\rangle\}/[|E_0|^4\exp(-2\alpha L)]. \end{aligned} \tag{4}$$

Equation (4) is similar to that obtained in [2,3]. However, it includes noise that is added due to the laser RIN that was ignored in [2,3]. The Fourier transform of the first term in Eq. (4) is equal to the laser RIN, $S_{\rm RIN}(\omega)$. This direct contribution of the laser intensity noise to the output noise does not depend on the fiber length, and it is an important noise source in short fibers. The last two terms in Eq. (4) are also due to the laser intensity noise. The laser amplitude noise can be double scattered and interfere with the laser wave, and it can interfere with the DRS of the laser wave. Following the derivation below for the second term in Eq. (4), it can be easily shown that the spectrum caused by these terms is equal to the spectrum of the laser RIN passing through a linear filter due to DRS. However, the contribution of these terms to the spectrum is on the order of $L^2\sigma_\kappa^4 S_{\rm RIN}(\omega)$, while the contribution of the first term in Eq. (4) is equal to $S_{\rm RIN}(\omega)$. Hence, the last two terms in Eq. (4) can be neglected.

The second term in Eq. (4) results from the beating between the double Rayleigh backscattered wave and the source wave [2,3]:

$$\begin{aligned} &\langle E^*_{\rm DRS}(t)E_s(t)E^*_s(t-\tau)E_{\rm DRS}(t-\tau)\rangle \\ &= \exp(-2\alpha L)\sigma_\kappa^4\int_0^L {\rm d}w\int_0^w {\rm d}u\exp(-2\alpha u) \\ &\times\left\langle E_{\rm in}(t)E^*_{\rm in}(t-\tau)E^*_{\rm in}\left(t-\frac{2nu}{c}\right)E_{\rm in}\left(t-\frac{2nu}{c}-\tau\right)\right\rangle. \end{aligned} \tag{5}$$

The laser phase noise $\varphi(t)$ is related to the angular-frequency noise $\Delta\omega(t)$ via $\varphi(t) = \int_0^t {\rm d}t'\Delta\omega(t')$. We assume that $\Delta\omega(t)$ is a zero mean Gaussian random process [14] with a one-sided power spectral density $S_{\Delta\omega}(\omega)$ that includes white and $1/f^{\nu}$ frequency noise of the laser or any other general noise. Using the frequency noise spectrum $S_{\Delta\omega}(\omega)$, we calculate [10,15]

$$\begin{aligned} &\left\langle E_{\rm in}(t)E^*_{\rm in}(t-\tau)E^*_{\rm in}\left(t-\frac{2nu}{c}\right)E_{\rm in}\left(t-\frac{2nu}{c}-\tau\right)\right\rangle \\ &\approx |E_0|^4 R_{S_{\Delta\omega}(\omega)}(u,\tau)[1 + O(r_m)], \end{aligned} \tag{6}$$

where

$$R_{S_{\Delta\omega}(\omega)}(z,\tau) = \exp\left[-\frac{4}{\pi}\int_0^\infty \sin^2\left(\frac{\omega\tau}{2}\right)\sin^2\left(\frac{\omega\tau_z}{2}\right)\frac{S_{\Delta\omega}(\omega)}{\omega^2}{\rm d}\omega\right],$$

and $\tau_z = 2nz/c$ is the round-trip propagation delay over a fiber section of a length z. By substituting Eq. (6) and Eq. (5) into Eq. (4), the normalized autocorrelation function equals

$$r(\tau) = 2r_m(\tau) + 2T_P\sigma_\kappa^4\int_0^L {\rm d}w \times\int_0^w {\rm d}u\exp(-2\alpha u)R_{S_{\Delta\psi}(\omega)}(u,\tau) + O(r_m^2). \tag{7}$$

In Eq. (7), we added a factor $T_P = 5/9$ that is due to a random polarization of the low-birefringence fiber [3,12].

In our experiments, we measured the normalized power spectral density of the detected optical signal at the fiber output and compared the results to a numerical calculation of Eq. (7). A continuous wave laser passes through a variable optical attenuator (VOA) that controls the power that is injected into a long optical fiber with a length L. Optical isolators (OI) are placed at the fiber entrance and at the fiber end to reduce back reflections from connectors. The optical wave is detected by a PD and measured with an RF spectrum analyzer (SA). The laser source was a RIO "Orion" laser [16] with a specified linewidth smaller than 1 kHz, as defined by the laser white frequency noise. Several fiber sections ("Corning" SMF-28) with lengths 4, 20, 20.5 km, and 25 km were fusion spliced to obtain fiber lengths of 4, 8, 28.5, and 45.5 km. The attenuation of the SMF-28 fibers that was measured by using optical time-domain reflectometer (OTDR) was about 0.2 dB/km. The optical power at the entrance of the fiber was less than 1.7 mW. We have measured the laser frequency noise by using a Mach–Zehnder interferometer as described in [17]. The spectrum of the measured noise, $S_{\Delta\omega}(\omega)$, was empirically fitted by $S_{\Delta\omega}(\omega) = 2(2\pi)^2 \times S_0 + 2(2\pi)^{\nu+2} \times k/\omega^{\nu}$ [(rad/s)^2/Hz], where $S_0 = 225$ Hz^2/Hz, $\nu = 1.15$, and $k = 10^{7.17}$ Hz$^{\nu+2}$/Hz. The spectrum is composed of a white noise and $1/f^{1.15}$ noise. The RIN of the laser was directly measured by using a PD, and it was empirically fitted by $S_{\mathrm{RIN}}(\omega) = 10^{-16} + 2\pi \times 10^{-12}/\omega$ [1/Hz]. The empirical fits of the frequency and amplitude noise of the laser were used in the theoretical calculation. The Rayleigh backscatter coefficient σ_κ^4 that we used in the theoretical calculations was equal to 3.7×10^{-15} m^{-2}. Such a coefficient gives a power reflection of −82 dB at a wavelength of 1550 nm for 1 ns pulse width [10].

Figure 1 shows a comparison between the one-sided normalized power spectral density of the detected signal at the fiber output (solid blue curves) and the spectrum that is calculated by numerically solving Eq. (7) (dashed red curves). Figure 1 shows that excellent quantitative agreement between theory and experiments was obtained for the whole frequency region of 10–10^5 Hz and for fiber lengths between 4 and 45 km. The maximum differences between the theoretical and experimental results shown in Fig. 1 are less than about 1.5 dB. The largest difference of 1.5 dB is obtained for the 28.5 km length fiber. Part of this small difference is caused by splice losses. The 28.5 km fiber contained two splices, each with a measured power attenuation of 0.2 dB. The 45.5 km length fiber contained a single splice, and the difference was about 1 dB. Other causes for the small difference between theory and experiments are changes of the random birefringence between different fibers.

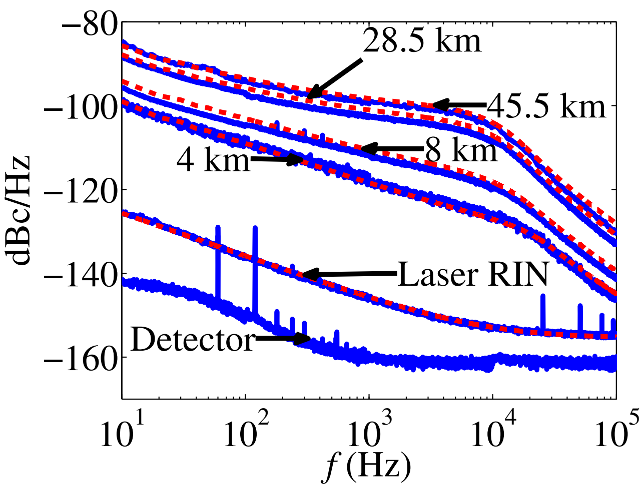

Fig. 1. Comparison between measured normalized RF spectrum of the detected signal at the fiber output (solid blue curves) and the theoretical power spectral density calculated by solving Eq. (7) (dashed red curves) for fiber lengths of 4, 8, 28.5, and 45.5 km. Also shown in the figure are the RIN of the laser and the noise spectrum of the detector.

Therefore, noise induced by DRS is the dominant noise for frequencies as low as 10 Hz. Noise that is due to thermal and thermo-mechanical fluctuations of the fiber [9] did not significantly affect the results, even at low frequencies. We note that the laser RIN does not affect the results that are shown in Fig. 1. Our theoretical results, given in Eq. (4), indicate that the contribution of the laser RIN to the output RIN does not depend on the fiber length. Hence, the laser RIN is an important noise source only for short fibers with a length below about 500 m. For a fiber with a length of 500 m, the laser RIN is expected to affect the measured spectrum at frequencies that are higher than about 20 kHz. For example, at a frequency of 70 kHz, the laser RIN will add about 7 dB to the output RIN.

Figure 1 indicates that the output noise spectrum contains at low frequencies a significant noise component with $1/f^{\gamma}$ frequency dependence. The power γ depends on the fiber length L, and it decreases with increasing L. For example, $\gamma = 0.88$ for a fiber with $L = 4$ km and $\gamma = 0.4$ for a fiber with $L = 45.5$ km. Above a certain frequency that is higher than about 10 kHz, the dependence of the noise changes to $1/f^{2.3}$. This corner frequency depends on the ratio between the coherence length of the laser and the fiber length. Reflections at frequencies below the corner frequency are coherently added along the whole fiber. The $1/f^{\nu}$ frequency noise component of the laser that was used in the experiments, has an important contribution to the laser coherence length. For such a laser, the coherence length can be estimated by calculating the full width at half-maximum (FWHM) linewidth of the laser that we denote as $\Delta\nu_c$. In [18], $\Delta\nu_c$ was approximated through the integration of the frequency noise spectrum of the laser in the frequency region that starts from the inverse of the observation time. When the coherence length of the laser is shorter than the fiber length, the corner frequency approximately equals to $2\Delta\nu_c$, and it does not significantly change with the fiber length, as was obtained in [3] for a laser with only a white frequency noise. Hence, the corner frequency for fibers with length of 28.5 and 45.5 km approximately equals 10 kHz. However, when the coherence length is longer than the fiber length, the corner frequency approximately equals to $f_c = c/(2nL)$. For a 4 and 8 km fibers, f_c equals to 26 and 13 kHz, respectively. These values are in accordance with the results in Fig. 1.

We have calculated the dependence of the power spectrum at the output of a 10 km fiber as a function of the white and $1/f^{\nu}$ frequency noise parameters S_0 and k by solving Eq. (7). We compared the results obtained for the "Orion" laser to the noise induced by an EM4 laser. The frequency noise of the EM4 laser was modeled by using an empirical fit of the frequency noise spectrum measured in [19] for a 5 mW optical power: $S_{\Delta\omega}(\omega) = 2(2\pi)^2 \times S_0 + 2(2\pi)^{\nu+2} \times k/\omega^{\nu}$ [(rad/s)^2/Hz], where $S_0 = 10^5$ Hz^2/Hz, $k = 10^{10}$ Hz$^{\nu+2}$/Hz, and $\nu = 1$. We have also used an empirical fit to the measured EM4 laser RIN [19] and obtained $S_{\mathrm{RIN}}(\omega) = 10^{-14} + 2\pi \times 10^{-11}/\omega$ [1/Hz]. The RIN of the EM4 laser was used

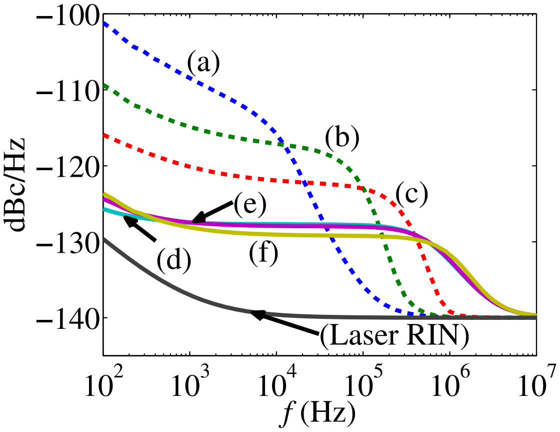

Fig. 2. Calculated power spectrum of the noise that is due to DRS at the output of a 10 km fiber obtained by solving Eq. (7). The fiber parameters correspond to the "Corning" SMF-28 fiber. The laser parameters for curve (a) correspond to the frequency noise parameters of the "Orion" laser, and the parameters for curve (f) correspond to the EM4 laser. For dashed curves (b) and (c), the $1/f^{\nu}$ frequency noise ($\nu = 1.15$) magnitude is increased by factors of 10^2 and 10^3 with respect to that used in curve (a). For solid curves (d)–(f), the $1/f^1$ frequency noise coefficient k is the same as for curves (a)–(c), respectively. The figure also shows the RIN of the EM4 laser that was used in all curves.

in calculation of the output spectrum for all the results shown in Fig. 2. This RIN is higher than that of the "Orion" laser that was used in the experiments. Nevertheless, the RIN of the EM4 does not have a significant effect on the noise induced by the 10 km fiber.

Figure 2 shows the intensity noise spectrum at the output of a 10 km fiber that was calculated using Eq. (7). The frequency noise parameters of the dashed curve (a) correspond to the "Orion" laser such that $S_0 = 225\ \mathrm{Hz}^2/\mathrm{Hz}$, $k = 10^{7.17}\ \mathrm{Hz}^{\nu+2}/\mathrm{Hz}$, and $\nu = 1.15$. For dashed curves (b)–(c), we increased the $1/f^{\nu}$ frequency noise coefficient k that was used for curve (a) by factors of 10^2 and 10^3, respectively. The parameter of the white frequency noise that was used to calculate curves (d)–(f) corresponds to the EM4 laser such that $S_0 = 10^5\ \mathrm{Hz}^2/\mathrm{Hz}$. The magnitudes of the $1/f^1$ frequency noise coefficients k in curves (d)–(f) are the same as for curves (a)–(c), respectively. The parameters for curve (f) correspond to the EM4 laser.

The maximum propagation time of the DRS wave in a 10 km fiber is about 0.1 ms. For such an observation time, the coherence length of the "Orion" laser that corresponds to a curve (a) is determined only by the laser white frequency noise so that the coherence length is equal to about 230 km. For curve (b), the coherence length for the observation time of 0.1 ms is reduced due to flicker frequency noise to about 4.5 km. For curves (c)–(f), the coherence length is shorter than 1 km.

When the frequency noise of the laser increases, the coherence length decreases. If the coherence length becomes much shorter than the fiber length, the intensity of the noise that is due to DRS can be decreased, since waves that are reflected along the fiber are not coherently accumulated. For the results shown in curves (b)–(c), the white frequency noise is small, and the laser coherence length is mainly determined by the $1/f^{\nu}$ frequency noise of the laser. Hence, increasing the $1/f^{\nu}$ frequency noise causes a large decrease of the spectrum at low frequencies below about 10 kHz. A significant increase of the white frequency noise for the EM4 laser from $10^2\ \mathrm{Hz}^2/\mathrm{Hz}$ to $10^5\ \mathrm{Hz}^2/\mathrm{Hz}$ in curves (d)–(f) causes a large reduction of the intensity noise with respect to curve (a). For curves (d)–(f), the coherence length is mainly determined by the white frequency noise. Hence, increasing the $1/f^{\nu}$ frequency noise slightly increases the low-frequency noise. We have checked that similar qualitative behavior of the normalized noise spectrum, as shown in Fig. 2, is obtained also for a 30 km length fiber.

For sufficiently high input optical power, SBS will induce a significant intensity noise with approximately Lorentzian function with linewidth and power that depend on the incident optical power [11]. Using the computational model in [11], we have calculated the optical power at the entrance of the fiber that would induce SBS that is equal to the noise induced by DRS at a frequency of 1 kHz. These calculated incident powers for 4, 8.5, and 28.5 km fibers are equal to 20, 12, and 7 mW, respectively with corresponding FWHM of 90, 52, and 30 kHz. For a 45.5 km fiber and incident power of 1.7 mW in our experiments, SBS will cause noise with FWHM linewidth of 5 kHz and power density of −180 dBc/Hz that is much below the noise of the detector.

In conclusion, we showed, theoretically and experimentally, that noise induced by DRS of a high-coherence-length laser in long fibers is the dominant noise at low frequencies starting at 10 Hz. Increasing the laser $1/f^{\nu}$ frequency noise may decrease the induced noise due to the reduction of the coherence length of the laser.

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