Optical Under-Sampling and Reconstruction of Several Bandwidth-Limited Signals

Alfred Feldster, Yuval P. Shapira, Moshe Horowitz, Amir Rosenthal, Shlomo Zach, and Lea Singer

***Abstract*—We demonstrate experimentally an optical system for under-sampling several bandwidth-limited signals with carrier frequencies that are not known apriori and can be located anywhere within a very broad frequency region between 0–18 GHz. The system is based on under-sampling asynchronously at three different sampling rates. The optical pulses required for the under-sampling are generated by a combination of an electrical comb generator and an electro-absorption optical modulator. To reduce loss and improve performance the implementation of the optical system is based on a wavelength division multiplexing technique. An accurate reconstruction of both the phase and the amplitude was obtained when two chirped signals each with a bandwidth of about 150 MHz were sampled.**

***Index Terms*—Multirate sampling (MRS), periodic nonuniform sampling (PNS).**

I. INTRODUCTION

ELECTRONIC warfare is the systems approach to the exploitation and control of the electromagnetic spectrum [1]. Electronic warfare systems require using broadband receivers tuned over desired portions of the spectrum in search of transmissions of interest such as radars or communication systems [2]. Signals that are identified should be sampled for subsequent analysis or for deception-jamming techniques. Although in modern military situation the available frequency spectrum is heavily used, at a given time only a small portion of the spectrum is occupied. Therefore, in electronic warfare systems it is required to sample and reconstruct multiband sparse signals. Mathematically, a sparse signal is a signal that occupies only a small portion of a given frequency region. Such a signal is composed of several narrowband signals with different carrier frequencies that are transmitted simultaneously. When the possible carrier frequencies of the signals are high compared to the bandwidths of the signals, it is not cost effective and often it is not feasible to sample at the assumed Nyquist rate F_{nyq} that is approximately equal to twice the maximum carrier frequency of the signals. It is therefore desirable to reconstruct the signals by under-sampling; that is to say, from samples taken at rates lower than the Nyquist rate.

There is a vast literature on theoretical methods to reconstruct signals from under-sampled data [5]–[11]. Most methods are based on a periodic nonuniform sampling (PNS) scheme. In a PNS scheme m low-rate cosets are chosen out of L cosets of samples obtained from time-uniformly distributed samples taken at a rate F where F is greater or equal to the assumed Nyquist rate F_{nyq} [6]. The sampling rate of each sampling channel is thus L times lower than F and the overall sampling rate is mF/L where $m/L \ll 1$. Thus, the use of under-sampling enables to reduce the sampling rate to a rate allowed by the current electronic technology. On the other hand the use of under-sampling slightly increases noise in the sampled data since noise from the entire spectrum is downconverted into a narrow frequency region.

In a previous work we have demonstrated theoretically a different theoretical scheme for reconstructing sparse multiband signals which we call multirate sampling (MRS)[12]. The scheme entails gathering samples at P different rates. The number P is small and does not depend on the characteristics of the signal. The approach is not intended to obtain the total minimum sampling rate. Rather, it is intended to reconstruct signals accurately with a very high probability at an overall sampling rate significantly lower than the assumed Nyquist rate under the constraint of a small number of sampling channels.

The reconstruction method in [12] does not rely on synchronization of different sampling channels. This significantly reduces hardware requirements. Moreover, unsynchronized sampling relaxes the stringent requirement in PNS schemes of a very small timing jitter in the sampling time of the channels. Simulations indicate that asynchronous multirate sampling reconstruction is robust both to different signal types and to relatively high noise. The algorithm was tested in order to reconstruct both the phase and the amplitude of 4 real signals that overlapped in time. The carrier frequencies of the signals were randomly chosen in the region 0–20 GHz and the amplitudes of the signals were chosen independently from a uniform distribution in the region 1–1.2. Each one of the four signals had a bandwidth of 200 MHz and hence the sum of the signal bandwidths was equal to 800 MHz. Therefore, about 4% of the available spectrum was exploited. In each simulation the signal was sampled using three sampling channels that operated at 3.8, 4.0, and 4.2 GSamples/s. A very high reconstruction probability (>98%) of both the signals amplitudes and phases was obtained in our simulations. The reconstruction probability depends on the total bandwidth of the signals and to some extent on the maximum number of signals. The result does not depend on the spectrum of the signals and on their chirp. Therefore, bandwidth limited signals with a spectrum having an arbitrary phase and amplitude can be reconstructed with a very high probability.

The bandwidth of electronic systems is limited. Therefore, in electronic systems that are used to sample signals with a carrier frequency that can be located in a large frequency region, the spectrum is divided into small frequency bands. Each band

Manuscript received April 17, 2008; revised August 17, 2008. Current version published April 17, 2009.

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Digital Object Identifier 10.1109/JLT.2008.2005911

is down-converted and sampled by a different electronic subsystem. As a result, the size, the weight and the complexity of such electronic systems limit their applications. Alternatively, the spectrum can be scanned in time. At a given time, only a small portion of the spectrum is down-converted and sampled. Such a technique maximizes the dynamic range of the system. However, it does not allow a continuous sampling of the entire spectrum.

The bandwidth of analog optical systems is extremely high in compared to that available in electronic systems. Therefore, optical systems are capable of very high performance under-sampling [3]. The carrier frequency of the input signal can be very high on the order of 20 GHz and the dynamic range of the signals can be as high as 70 dB. The size, weight, and power consumption of optical systems make them ideal for under-sampling. The under-sampling operation is based on shifting the entire spectrum to a low frequency region called the baseband [3], [13]. The down-converted signal is then sampled using an electronic analog-to-digital (A/D) converter with a bandwidth that is significantly narrower than the carrier frequency of the signal. A previous work [3] has demonstrated the under-sampling and reconstruction of a single narrowband signal. However, the system required apriori knowledge of the carrier frequency of the signal and this knowledge was used to determine the sampling rate.

In this paper we demonstrate experimentally an optical system for sampling sparse signals with carrier frequencies that are unknown apriori and are located in a broadband frequency region of 0–18 GHz. The system entails under-sampling the signals asynchronously at three different rates. The optical pulses required for under-sampling are generated by a combination of an electrical comb generator and an electro-absorption (EA) modulator. Such an optical pulsed source is simple, small, and consumes low power. Moreover, the source is robust to changes in environmental conditions. The combination of the comb generator and the EA modulator that are used instead of two electro-absorption modulators that were used in [3] results in an increase in the output power of the pulse train by about 10 dB. The simultaneous sampling at different rates is performed efficiently by using methods based on a wavelength-division multiplexing (WDM) technique that is used in optical communication systems. Such a technique avoids loss caused by splitting of the signal between the sampling channels. This technique also enables use of the same electro-optic modulator for modulating all the sampling channels. Consequently, the frequency dependence of the modulator response curve has the same effect on all sampling channels. We have demonstrated experimentally an accurate reconstruction of both the phase and the amplitude of two chirped signals, generated simultaneously, each with a bandwidth of about 150 MHz. The carrier frequencies of the signals were not known apriori but were assumed to be within a frequency region of 0–18 GHz. The reconstruction did not depend on the chirp, the spectrum amplitudes, and the carrier frequencies of the signals. The spurious free dynamic range of the system was 94 dB-Hz$^{2/3}$.

II. Experimental Setup

A. System Description

The experimental setup of the system is shown in Fig. 1. The spectrum of the signals was down-converted to a low frequency region, called baseband, by modulating the amplitude of a train of short optical pulses by the electrical signal [3].

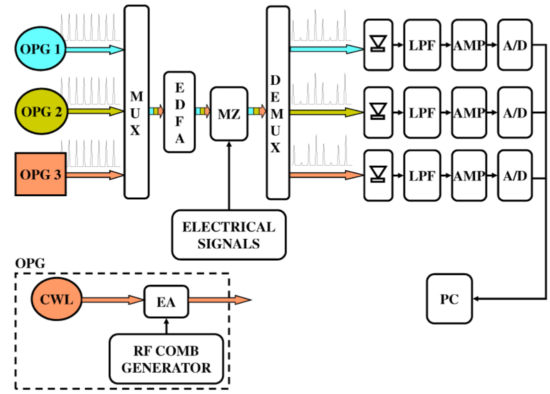

Fig. 1. Schematic description of the system used to under-sample electrical signals at three different sampling rates. Optical pulses are generated using three optical pulse generator (OPG) units consisting of a CW laser (CWL), a comb generator and an electro-absorption modulator (EA). By combining the three trains of optical pulses at different optical frequencies using a multiplexer (MUX) the input signal is sampled simultaneously at three different rates. After modulating the optical pulse trains by the electrical signal using a $LiNbO_3$ modulator (MZ), the three modulated optical pulse trains are separated using an optical demultiplexer (DEMUX), detected using optical detectors, and sampled using a bandwidth limited analog-to-digital electronic converters (A/D). AMP is an electrical amplifier, EDFA is an erbium-doped optical fiber amplifier, LPF is an electrical low pass filter, and PC is a computer used for the signal processing.

Since the reconstruction algorithm requires sampling the signal at three different rates, the down-conversion of the signal was performed separately using three optical pulsed sources that operate at different rates. The wavelengths of the optical pulsed sources were chosen to be different: 1535.04 nm, 1536.61 nm, and 1544.53 nm. Hence, the three optical pulse sources could be added by using an optical multiplexer (MUX) and then be modulated by a single $LiNbO_3$ modulator (MZ). The specified electrical full-width at half-maximum (FWHM) bandwidth of the modulator was greater than 30 GHz. The use of a single modulator reduces the sensitivity of the system to the frequency response function of the modulator and simplifies the system design. Since the modulator response is sensitive to its input polarization we used polarization maintaining (PM) fibers in all of the optical components that were connected before the input end of the modulator. In our experiments we lacked suitable optical multiplexer with PM fibers. Instead, we used two PM couplers to multiplex the three optical signals. This added an additional loss of about 6 dB that can be avoided using a PM multiplexer. The optical losses due to the electro-absorbtion modulator, the electro-optical modulators, the multiplexer, and the de-multiplexer, were about 10, 7, 6, and 1.5 dB, respectively. To compensate for these losses we used an erbium doped fiber amplifier (EDFA) with a 30 dB gain. After passing through the modulator, the optical wave passed through a demultiplexer that splits the optical signal into three pulse trains of different rates with an amplitude modulated according to the electrical signal. Each of the three modulated pulse trains was detected by an optical detector connected to a transimpedance amplifier. The input optical power at the entrance of the detectors was about −4 dBm. The output of the transimpedance amplifiers was then passed through low-pass filters with a cutoff frequency of 2.2 GHz, am-

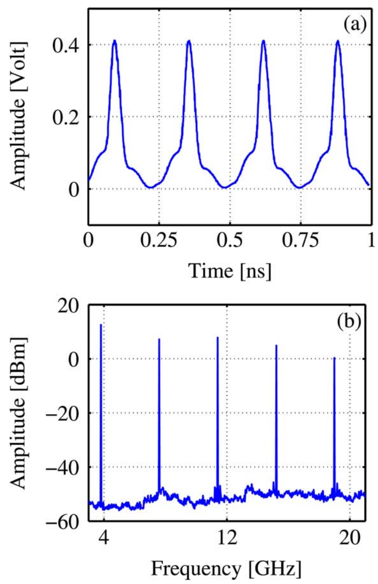

Fig. 2. Electrical pulses at the output of the comb generator measured by (a) sampling oscilloscope and (b) electrical spectrum analyzer. The repetition rate of the pulses equals 3.8 GHz.

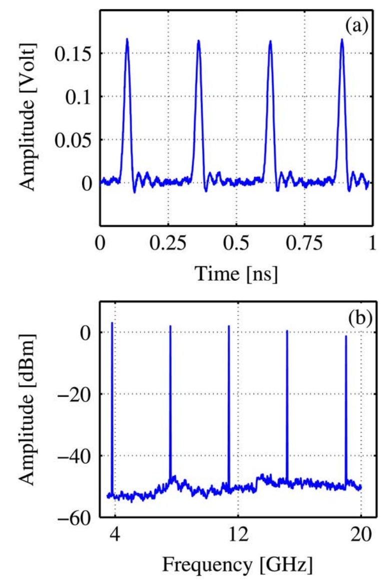

Fig. 3. Optical pulses at the output of the EA modulator, detected by an optical detector, and measured by (a) sampling oscilloscope and (b) electrical spectrum analyzer. The repetition rate of the pulses equals 3.8 GHz.

plified once again by electrical amplifiers with a gain of 20 dB, and sampled by three electronic A/D converters. Each of the A/D converters sampled at a rate of 4 Gsamples/s with 5 effective bits. In each sampling the number of sampled points in each channel was equal to 32 767.

Each pulsed source was implemented by a combination of an EA modulator and a comb generator. The comb generator is based on using a step-recovery diode (SRD) and is used to generate short electrical pulses from a sinusoidal electrical wave. Fig. 2 shows the electrical pulse train at the output of the comb generator. The train is measured by a sampling oscilloscope and by an RF spectrum analyzer. The measurement shows the existence of strong ripples between the pulses. Moreover, the difference between the harmonics at 3.8 and 19 GHz is 12.3 dB. In [3] it has been shown that the bandwidth of the system is limited by the bandwidth of the RF spectrum of the optical pulses. Consequently, the pulse train at the output of the comb generator cannot meet the requirement of sampling signals whose carrier frequency may be as high as 18 GHz. To rectify this, we connected the output of the comb generator to the electrical input port of an EA modulator with an electrical bandwidth of more than 30 GHz. The optical input to the modulator was a continuous-wave (CW) laser with a power of 11.5 dBm. The nonlinearity of the modulator enabled shortening of the optical pulse duration as well as a reduction in the intensity of ripples between pulses. Such improvement in the pulses is essential for high performance and high bandwidth sampling. Fig. 3 shows the optical pulses at the output of the EA modulator after conversion to an electric signal by an optical detector and measured by an RF spectrum analyzer and a sampling oscilloscope. The pulse duration obtained at the output of the electro-absorption modulator was about 26 ps. The specified bandwidth of the sampling oscilloscope with the detector was greater than 50 GHz. The FWHM of the impulse response of our sampling oscilloscope and the detector was less than 9 ps. The RF spectrum of the optical pulses, shown in Fig. 3(b) shows that the difference between the RF harmonics at 3.8 GHz and the harmonics at 19 GHz was only 4.3 dB. The average power of each pulse train was about -11 dBm. The repetition rate of the optical pulses can be controlled easily by choosing different frequencies of the sinusoidal waves at the input of the comb generators. For a given comb generator, the input electrical sinusoidal wave could be changed by up to about $\pm 2.5\%$ with respected to the specified frequency of the device. By choosing three comb generators with different specified frequencies we could generate the three different pulse trains as required to the sampling. Using a comb generator and an EA modulator instead of two EA modulators as were used in [3] enables a reduction in the loss in the pulsed source by about 10 dB. We note that the three optical pulse generators in our system were uncorrelated in time.

B. Principle of Operation

The under-sampling is performed in two steps. In the first step the entire signal spectrum is down-converted to a low frequency region called baseband by modulating the amplitude of an optical pulse train by the electrical signal. In the second step the down-converted optical signal is transformed into an electronic signal that is then sampled by a bandwidth-limited electronic A/D converter. The bandwidth of the electronic A/D converter is significantly narrower than the maximum carrier frequency of the signals. The optical under-sampling is performed at three different rates. The repetition rate of the optical pulses in each of the sampling channels is chosen to be different and should be lower than the sampling rate of the A/D converters to avoid additional aliasing. On the other hand, the sampling rate of each channel is chosen to be approximately equal to the maximum sampling rate allowed by cost and technology. Under-sampling at high rates has a fundamental advantage when applied to signals contaminated by noise. The spectrum evaluated at a baseband frequency f_b in a channel that samples at a rate F is the sum of the spectrum of the original signal at all frequencies $f_b + mF$ that are located in the overall system operational bandwidth, where m is an integer [12]. Thus, the larger the value of F, the fewer terms contribute to this sum. As a result, sampling at a higher rate lowers the noise contribution and hence increases the signal-to-noise ratio (SNR) in the baseband. Moreover, when the sampling rate of each channel is increased, our reconstruction method, described in [12], enables to decrease the number of sampling channels. The sampling rates that were used in our experiments were 3.8, 3.9, and 4 GHz.

We denote the input wave spectrum at the RF input of the modulator by $S(f)$. The signal is sampled using an optical pulse train with a temporal pulse shape $p(t)$ and a repetition rate F. The current spectrum $i(f)$ of the optical detector is proportional to [3]

$$i(f) \propto F \sum_{n=-\infty}^{\infty} S(f - nF)P(nF) \tag{1}$$

where $P(f)$ is the Fourier transform of a single optical pulse $p(t)$ and n is an integer number. The spectrum at the output of each sampling channel contains replicas of the original spectrum $S(f)$ shifted by an integer multiple of the repetition rate F. The signal spectrum is therefore down-shifted to a baseband where it can be sampled using a conventional electronic A/D converter. Because the sampling rate of each sampling channel is different, the corresponding spectrum in the baseband of each channel is different.

A signal processing algorithm utilizes the information from the three sampling channels to reconstruct the original signal [12]. The algorithm can correctly reconstruct both the phase and the amplitude of the signals in almost all cases; even in the cases when aliasing deteriorates the spectrum in parts of the basebands. The algorithm is based on extracting a spectrum that minimizes the error between the three down-converted spectra of the reconstructed signal and the spectra measured in the three sampling channels. The signal can be reconstructed accurately when each of the frequencies of the signal spectrum is unaliased at least in one of the sampling channels. Our simulations indicate that the multirate sampling scheme is robust both to different signal types and to relatively high noise.

C. Experimental Results

In our experiment we have demonstrated the sampling and the reconstruction of two chirped signals that are generated simultaneously. Fig. 4 shows the spectrum of the input RF wave as measured using an RF spectrum analyzer. The wave consisted of two chirped pulses with approximate square time profiles. The waves were generated using two independent fast voltage-controlled-oscillators (VCOs). The input voltage to the VCOs was linearly changed in time. However, because of the nonlinear dependence and the saturation of the VCO frequency as a function of the input control voltage, the chirp of the pulses did not change linearly in time in parts of the pulses. The first signal had a central frequency of 6.87 GHz and a bandwidth of 150 MHz. The second signal had a carrier frequency of 9.13 GHz and a bandwidth of 133 MHz. The average RF power of the superposed signals was approximately

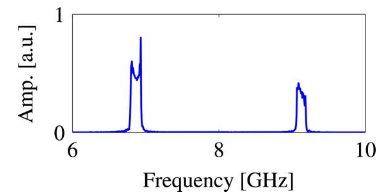

Fig. 4. Spectrum of the electrical signal at the input of the MZ modulator as measured by an RF spectrum analyzer. The wave contains two chirped signals centered around 6.87 and 9.13 GHz with a bandwidth of 150 and 135 MHz, respectively.

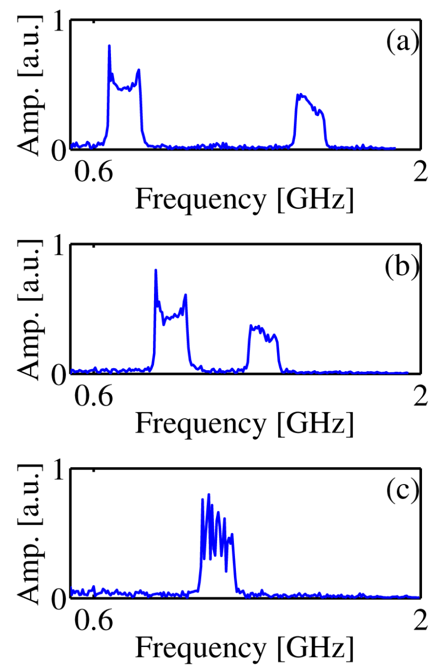

Fig. 5. Baseband spectrum after sampling using a pulse train with a repetition rate of (a) 3.8, (b) 3.9, and (c) 4.0 GHz. Each spectrum is the amplitude of the Fourier transform of the corresponding sampled data.

-14 dBm. The two signals were generated simultaneously and therefore we have studied the worst case scenario where the two signals almost completely overlap on time. The duration of the combined pulse was 1.35 μs. The repetition rate of the pulses was 2 kHz. We note that our simulation results indicate that the sampling system can be used to efficiently sample and reconstruct more than 4 chirped signals that are transmitted simultaneously [12]. However, our source that generated the input signals could only generate simultaneously two chirped signals.

Mathematically, since the signals in our experiments are real functions each real signal is composed of two bands. One band with a spectrum $S(f)$ is located in the positive frequency region $(f > 0)$, and another band with a spectrum $S^*(-f)$ is located in the negative frequency region [12]. Therefore, mathematically, the signal spectrum in our system is composed of four frequency bands. We note that due to sampling each two of the four down-converted bands can overlap.

Fig. 5 shows the spectra at the output of the three sampling channels. The spectra were calculated by performing a discrete Fourier transform on the digital samples from the outputs of the

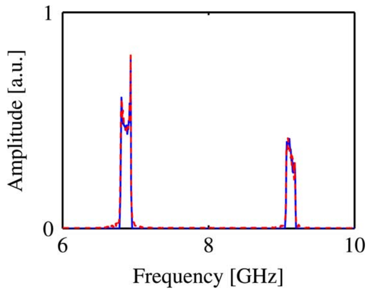

Fig. 6. Amplitude of the reconstructed signal spectrum (blue solid line) compared to the spectrum of the input electrical signal that was measured using a spectrum analyzer (red-dashed line).

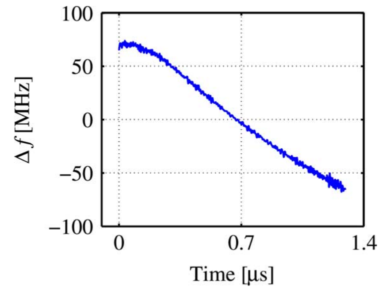

Fig. 7. Change in the instantaneous frequency $\Delta f = (1)/(2\pi)(d\phi)/(dt) - f_0$ of the reconstructed signal that is centered around a frequency $f_0 = 6.87$ GHz. $\phi(t)$ is the phase of the reconstructed signal in the time domain.

three A/D converters. We note that after down-converting the signal using a pulse train with a rate of F the spectrum becomes periodic with a periodicity of F. Moreover, since the signal is real the down-converted spectrum obeys: $i(f) = i^*(-f)$. Therefore, when sampling at a rate of F, all the information about the down-converted signal is contained in the frequency region $[0, F/2]$. Fig. 5(c) shows that down-converting the signal at a rate of 4 GHz resulted in an interference in the spectrum at a frequency around 1.1 GHz due to aliasing effect. The aliasing was between the signal components around −6.87 GHz and 9.13 GHz.

Since our reconstruction method does not require synchronization of the sampling channels we did not have to adjust precisely the sampling rates. Moreover, our reconstruction method is highly robust since it does not require inverting ill-pose matrices, as in multicoset sampling schemes [9]. Therefore, a nonuniformity of about 10% between the signal amplitudes in different sampling channels did not significantly affect the reconstruction. Hence, we did not need to calibrate the amplitudes between the different channels after building the system. The runtime of our reconstruction algorithm using a standard personal computer (PC) and the Matlab software was approximately 0.2 s. The runtime can be significantly decreased by about three orders of magnitude using a software implemented in digital-signal-processor (DSP).

The algorithm for reconstructing the signal from the three sampled down-converted signals is described in detail in [12]. Fig. 6 shows the amplitude of the reconstructed spectrum. For the comparison, the original spectrum as measured by the RF spectrum analyzer was added to the figure. Fig. 6 shows an excellent quantitative agreement between the reconstructed spectrum amplitude and the spectrum amplitude measured using the analog RF spectrum analyzer. An accurate reconstruction was obtained despite aliasing at the channel corresponding to a sampling rate of 4 GHz. Since the measurement of the RF spectrum analyzer is based on a slow frequency scan, measuring the whole spectrum requires many RF pulses. Our system requires only a single RF pulse for measuring the entire spectrum and hence it can be used in real-time applications. Our system also allows a measurement of the phase of the input electrical signal. This important information can not be obtained using the RF spectrum analyzer. Fig. 7 shows the instantaneous frequency i.e., the time derivative $d\phi(t)/dt$ of the time-domain phase of the reconstructed signal centered around the frequency $f_0 = 6.87$ GHz.

Our sampling method enables to reconstruct sparse signals with an arbitrary spectrum. Our theoretical analysis [12] shows that in order to obtain a very high reconstruction probability ($>98\%$) the sum of the bandwidths of the signals in our system should not exceed about 800 MHz assuming that the maximum number of signals is four. We do not take into account any assumption on the signal chirp or on the spectrum amplitude. Indeed, Fig. 7 shows that the signal chirp that was measured does not have a simple time-dependent function. The chirp saturates at the beginning of pulse due to the response saturation of the VCOs used to generate the chirped RF pulses. The instantaneous frequency change during the pulse duration is equal to about 142 MHz. This result is in good quantitative agreement with the measured overall signal bandwidth of about 150 MHz. An accurate reconstruction of two signals was obtained in all of our experiments when the carrier frequencies of the two signals were chosen randomly from the frequency region 5–11 GHz. This frequency region was imposed by the bandwidth of our RF sources that were used to generate the signals, and not by the limitations of the sampling system. We note that in our reconstruction method we identify signals that are about 2 dB above the noise floor of our system. Since our reconstruction algorithm recognizes accurately the frequency bands of all the signals, the parts of the spectrum where the signal is not detected are set to zero. It is theoretically impossible to reconstruct the noise between the signals since the total sampling rate is lower than the Nyquist rate [8].

Since the carrier frequencies of the signals are not known apriori the reconstruction algorithm has to calculate first the frequency band of each signal. Then, the frequencies in each sampled spectrum that are unaliased are calculated. The reconstruction is performed according to all the sampled data that is not aliased. In case that the reconstruction algorithm identifies that a spectrum frequency is unaliased in more than one sampled spectrum, the reconstruction at this frequency is performed by averaging the corresponding data from all of the sampling channels that are unaliased. The averaging improves the SNR of the reconstruction. Moreover, the reconstruction algorithm does not require that a complete signal will be unaliased in one of the sampling channels. Different parts of the signal spectrum can be reconstructed from different sampling channels. Therefore, although most of the spectrum in the 4 GHz sampling channel is aliased, the reconstruction algorithm uses part of the data in this channel that is not aliased (about 20 MHz around a frequency of 1.2 GHz).

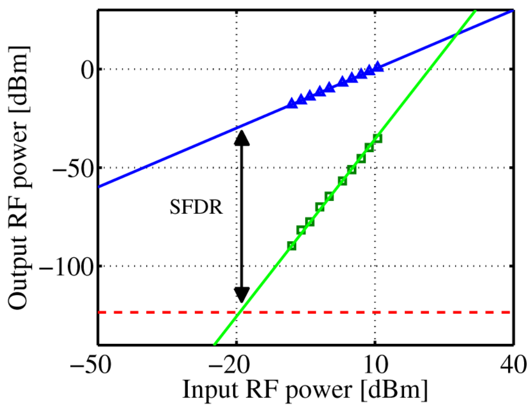

Fig. 8. Measurement of the spurious free dynamic range (SFDR) of the sampling channel obtained using a pulse train with a repetition rate of 3.9 GHz. The triangles show the measured input power of each of two CW RF signals at frequencies 6.197 and 6.198 GHz. The squares show the corresponding measured power of the third order products at the output of the optical detector (see Fig. 1). The red dashed line shows the average noise floor per Hz, as measured by the spectrum analyzer. The SFDR, shown in the figure, was equal to 94 dB-Hz$^{2/3}$.

Fig. 8 summarizes the measurement of the spurious free dynamic range of the system. In this measurement the electro-optical modulator was driven by two CW RF signals of the same power at adjacent frequencies of 6.197 GHz and 6.198 GHz. The power of the two CW electrical signals was gradually changed between -8 dBm and 10.6 dBm. For every value of the input power, the power of the third order products at frequencies 6.196 and 6.199 GHz was measured at the output of each of the three photo-detectors. The SFDR is defined as the difference in dB between the signals power and the power of the third order products, when the third order products are equal to the noise floor of the system. The result is normalized assuming that the spectral bandwidth of the system is equal to one Hz. The measurement of the noise power density was performed using the spectrum analyzer. The noise floor was measured five times for a resolution bandwidth that varied between 1 and 100 kHz. In each measurement the noise floor was normalized to a spectral window of 1 Hz. The average noise floor that was obtained was equal to -124 dB/Hz (the red dashed line in Fig. 8). The obtained spurious free dynamic range of each of the channels due to third-order distortion was about 94 dB-Hz$^{2/3}$. The bias voltage to the electro-optic modulator in our system was adjusted to minimize the second-order distortion at a frequency $6.198 + 6.197 = 12.395$ GHz. The SFDR_2 that is caused by the second-order distortion was equal to about 98 dB-Hz$^{1/2}$. Therefore, the spurious free dynamic range of our system was limited by the third-order distortion.

The maximum spurious free dynamic range of a system based on under-sampling is lower than that of systems that sample at Nyquist rate. The reason is that the noise from the entire spectrum is down-converted to baseband [12]. Since our sampling rate in each channel was around 4 GHz while the system bandwidth was about 20 GHz the noise power spectral density at baseband is expected to be about $2 * 5$ times higher than that in the original signal. However, electronic sampling at Nyquist rate of 40 Gsamples/s is currently unattainable. Moreover, it is expected that the huge bandwidth of a sampler with a sampling rate of 40 Gsamples/s will result in a very high noise in the sampling. The SFDR of our commercial spectrum analyzer was about 108 dB-Hz$^{2/3}$. However, the spectrum analyzer is based on a slow scan of the spectrum using a narrowband filter. Due to the use of a narrowband filter the noise in the measurement is very small. The SFDR obtained by our system is sufficient for most applications. Moreover, we believe that by using narrow optical filters, an optical multiplexer instead of couplers, and an improved signal processing algorithm we will be able to reduce significantly the noise floor in the system.

The bandwidth of the system was measured by scanning the frequency of a CW RF source that was connected to the system input. The power of the down-converted signal was measured as a function of input signal frequency. The 3 dB bandwidth was about 17.8 GHz. The decrease in the transmission of our electro-optic modulator between 100 and 18 GHz was less than 1 dB. The FWHM bandwidth of our optical demultiplexer was about 30 GHz and hence it should not limit significantly the system bandwidth. The system bandwidth that was measured is in agreement with the bandwidth of RF spectrum of the optical pulses -18 GHz, as indicated in Fig. 3(b). Indeed, the theory shows that the maximum system bandwidth is limited by the bandwidth of the electrical spectrum of a single optical pulse [3]. Therefore, shortening the optical pulses will increase the system bandwidth.

III. Conclusion

We have demonstrated a novel optical system for under-sampling sparse multiband signals. Such signals are composed of several bandwidth limited signals. The carrier frequencies of the signals are not known apriori and can be located anywhere within a very broad frequency region (0–18 GHz). The system is based on under-sampling asynchronously the signals at three different rates. The under-sampling is performed by down-converting all of the signals spectrum to a low-frequency region called baseband. The baseband is then sampled using electronic A/D converters with a bandwidth that is significantly narrower than twice the maximum carrier frequency of the input signals. By separating the frequency down-conversion and the analog to digital conversion operations it becomes possible to use a single frequency resolution that is common to all of the sampling channels. This facilitates a robust reconstruction algorithm that is used to detect and reconstruct the phase and the amplitude of the signals. The reconstruction method does not rely on synchronization between different sampling channels. This significantly reduces hardware requirements. The system also does not require an accurate calibration of the sampling channels. Because the entire frequency region of the signal is down-converted to baseband, aliasing may cause a deterioration in the down-converted spectrum. However, when the sum of the signals forms a sparse wave, a very high theoretical successful reconstruction percentage (98%) can be obtained using our reconstruction algorithm when the sum of the bandwidths of the signals does not exceed about 800 MHz. Our system uses standard optical components that are used in optical communication systems. The optical system is robust against changes in environmental conditions. It is a turn-key system that does not require any adjustments prior to the start of its operation. The weight, power consumption, dynamic range, and bandwidth of the optical system are significantly superior to those of electronic systems capable of meeting similar requirements. The performance of the reconstruction algorithm can be further enhanced by synchronizing the three sampling channels. With such a synchronization aliasing can often be resolved by solving a set of linear equations. The system bandwidth can be further increased by shortening the optical pulses by using an improved comb generator. The SFDR of the system can be further increased by adding optical filters, reducing the system loss, and by improving the reconstruction algorithm.

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