Bragg-soliton formation and pulse compression in a one-dimensional periodic structure

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We present a method for efficiently exciting a Bragg soliton with a spectral content located mostly within the bandgap of a one-dimensional periodic structure. The method is based on a new interaction between Bragg solitons and on a high intensity enhancement, caused owing to the reduced propagation velocity inside periodic structures. Our method can also be used for efficient compression of optical pulses. We have theoretically demonstrated pulse compression with a compression ratio of 2800—over two orders of magnitude higher than previously reported. The results open new possibilities for experimental demonstration of Bragg soliton propagation and for pulse compression in one-dimensional periodic structures.

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I. INTRODUCTION

Bragg (or gap) solitons are solitary waves that propagate inside nonlinear periodic dielectric media owing to the interaction between Kerr effect and Bragg reflection [1,2]. An interesting class of Bragg solitons is in-gap solitons, which are solitons with a spectral content located mostly within the bandgap, or the forbidden band, of the periodic structure. The excitation of in-gap solitons is a challenging task that limits their experimental demonstration. When an incident pulse is launched into the periodic structure, it may be reflected before an in-gap soliton is formed [3,4]. In-gap solitons can be excited if high intensity incident pulses with a specific profile are used [3]. However, the coupling efficiency in this case is very low, especially for solitons with a low propagation velocity [3]. Another limitation is nonlinear instability, which causes the generation of multiple solitons instead of a single soliton [3,5]. The problem of launching a single in-gap soliton was solved by using a side-excitation technique [4,6]. This technique cannot, however, be used for exciting in-gap solitons in one-dimensional (1D) geometries such as a fiber Bragg grating (FBG).

Owing to the stringent limitations associated with in-gap soliton formation, the propagation of a single soliton in a FBG has only recently been demonstrated [7]. In Ref. [8] nonlinear switching, based on a solitonic effect, was demonstrated in a highly nonlinear waveguide. However, the spatial length of the transmitted pulse was an order of magnitude larger than the length of the grating, and, thus, soliton propagation was not demonstrated. The use of linear narrow-band resonances in gratings may theoretically reduce the intensity required for forming an in-gap soliton [9]. However, the use of such resonances poses strict limitations on the shape of the incident pulse. Moreover, when strong resonances are used, the nonlinear effect, which is not taken into account in the design, becomes dominant and severely limits the efficiency of the coupling.

In this paper, we demonstrate a method that enables, for the first time to our knowledge, an efficient excitation of in-gap solitons in 1D periodic structures. The method is based on an interaction between Bragg solitons. This interaction is used to overcome the problem of multiple-soliton formation as it enables the transmission of only the leading soliton while the trailing solitons are backreflected. We also use the reduced light velocity in periodic structures, often referred to as slow light, to reduce the required intensity for soliton formation. Using the slow-light effect, the incident pulse can be adiabatically decelerated and compressed even before nonlinear effects become dominant. In a previous work, it was noted that the slow-light effect may play a role in forming Bragg solitons [10]. However, the magnitude of the slow-light effect was limited owing to linear and nonlinear propagation effects [11,12]. We study the basic limitations of the slow-light effect and show how to use it to substantially reduce the required intensity for soliton formation.

We note that the soliton formation described in Ref. [7] is performed using a different method than reported in this paper. In that study, a slow in-gap soliton was excited from a high-intensity incident pulse with a central wavelength within the bandgap of a FBG. Although the velocity of the excited soliton was small, there was no use of the slow-light effect to adiabatically compress the incident pulse and to reduce the required intensity for soliton formation. Furthermore the soliton interaction used in our work was not utilized in that work. Using our method, a single in-gap soliton can be excited with a higher efficiency and with a lower incident-pulse intensity compared to those used in Ref. [7]

In our scheme, the generated soliton is significantly shorter than the incident pulse. Thus, our results may also be important for obtaining compression of pulses with a weak intensity. Previous works on pulse compression in FBGs were based on a high-order soliton compression or on tailoring the dispersion along the grating by using a chirp [11,12]. By adding a chirp based compression scheme to our in-gap-soliton formation method, we demonstrate theoretically a very high pulse compression of 2800—over two orders of magnitude higher than obtained theoretically and demonstrated experimentally in previous works [11,12]. The use of the nonlinear interaction between pulses enables us to obtain a single-soliton transmission for a wide range of input intensities. With the use of current technology, our compression scheme can be used for developing inexpensive picosecond-pulse sources. While nanosecond pulses can be generated by

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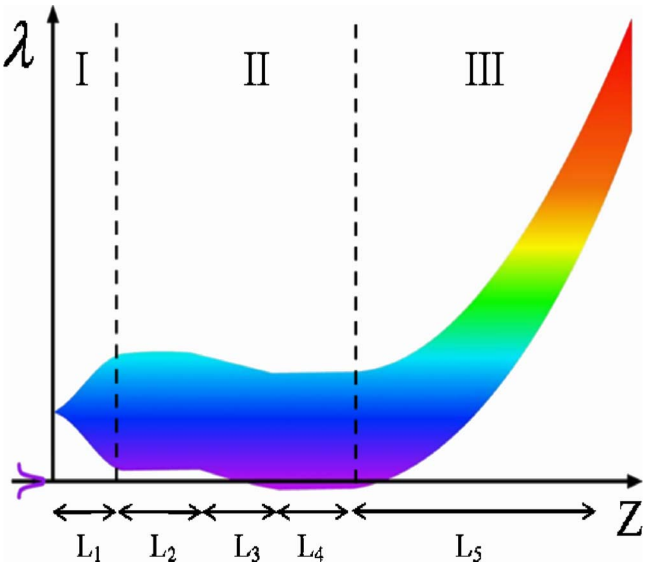

FIG. 1. (Color online) Bandgap diagram of the grating. The grating is divided into three sections. In the first section, the slow-light effect is used to enhance the pulse intensity. In the second region, a soliton interaction is used to form a single in-gap soliton. In the third region, the soliton is compressed owing to the shifting of the bandgap.

using a relatively low-cost microchip Q-switched laser, the generation of picosecond pulses requires a more complex and expensive source Ref. [12]. Using our method, picosecond pulses can be generated by efficiently compressing nanosecond Q-switched pulses.

II. ANALYSIS AND RESULTS

To demonstrate our method, we analyzed the propagation inside the grating schematically described by the bandgap diagram [13] given in Fig. 1. The colored strip in Fig. 1 represents the local bandgap, i.e., the wavelengths that are reflected from different locations of the grating when nonlinear effects are negligible [13]. The color code corresponds to the wavelengths of the bandgap. The spectrum of the incident pulse is also marked in the figure with a violet color. The grating can be divided into three main regions. In the first region, apodization is used to adiabatically decrease the pulse velocity and to enhance its intensity. In this region, most of the spectrum of the incident pulse is located outside the grating bandgap. In the second region, multiple pulses are formed owing to modulation instability. The chirp of the grating and the interaction between the pulses are used to couple a single soliton into the bandgap while the other pulses are backreflected. At the output of the second grating region, a compressed in-gap soliton is obtained. To further compress the soliton, we add a chirp to the last grating region. The soliton in this region is no longer an in-gap soliton and it is compressed owing to the adiabatic decrease in the effective dispersion along the grating.

The propagation of light in a nonlinear FBG can be described by the following coupled-mode equations [3]:

$$\frac{i}{V_g}\frac{\partial \mathcal{E}_\pm}{\partial t} \pm i\frac{\partial \mathcal{E}_\pm}{\partial z} + \delta(z)\mathcal{E}_\pm + \kappa(z)\mathcal{E}_\mp + \Gamma(|\mathcal{E}_\pm|^2 + 2|\mathcal{E}_\mp|^2)\mathcal{E}_\pm = 0, \tag{1}$$

where $\mathcal{E}_\pm$ is the envelope of the forward (+) and the backward (−) electric waves; V_g is the group velocity in absence of the grating; $\kappa(z)$ is the grating amplitude; Γ is the nonlinear coefficient; and $\delta(z)$ is the chirp parameter. The soliton solution of Eq. (1) is obtained for an infinite uniform grating and is described by two parameters: $0 \leqslant \rho \leqslant \pi$ and $|v| \leqslant 1$, where v is the velocity of the soliton, normalized by the group velocity of the medium V_g, and ρ is a free parameter [2]. The energy, E, and the spatial full-width-at-half-maximum (FWHM) of the soliton, ξ, are given by

$$E = \frac{4(1-v^2)\rho}{\Gamma(3-v^2)}, \tag{2}$$

.

$$\xi = \frac{2\sqrt{1-v^2}}{\kappa \sin(\rho)} \cosh^{-1}\sqrt{1+\cos^2\frac{\rho}{2}}. \tag{3}$$

In the first grating region, the spectrum of the incident pulse is centered outside the grating bandgap and the spectral bandwidth of the pulse is significantly smaller than the spectral width of the bandgap. Therefore, the pulse propagation in this grating region can be described by the inhomogeneous NLS equation [14]:

$$i\frac{\partial a}{\partial z} + \frac{i}{vV_g}\frac{\partial a}{\partial t} + \frac{\beta_2(z)}{2}\frac{\partial^2 a}{\partial t^2} + \bar{\Gamma}(z)|a|^2 a = -i\frac{v'}{2v}a - \alpha(z)\frac{\partial a}{\partial t} + \phi(z)a(z), \tag{4}$$

where

$$\beta_2(z) = (\kappa\gamma^3 v^3 V_g^2)^{-1}; \quad \bar{\Gamma}(z) = \frac{\Gamma(3-v^2)}{2v};$$

$$\alpha(z) = (2\kappa\gamma v^2 V_g)^{-1}\left[\frac{\kappa'}{\kappa\gamma^2} + \frac{(2+v^2)v'}{v}\right]; \tag{5}$$

the prime denotes a spatial derivative d/dz; $\gamma = 1/\sqrt{1-v^2}$; and v is the normalized group velocity:

$$v(z) = \sqrt{1 - \left(\frac{\kappa(z)}{\Omega/V_g + \delta(z)}\right)^2}, \tag{6}$$

where Ω is the temporal frequency of the fields $\mathcal{E}_\pm$. The field $a(t,z)$ is directly connected to the fields $\mathcal{E}_\pm$ [14], and its intensity is equal to $|\mathcal{E}_+|^2 + |\mathcal{E}_-|^2$. The function $\phi(z)$, given in Ref. [14], causes a phase shift in the solution and it can be eliminated by using the transformation $a(z) \rightarrow a(z)\exp[i\int_0^z \phi(\zeta)d\zeta]$. The first term in the right-hand side of Eq. (4) shows that an enhancement of the field intensity is obtained when $v'(z) < 0$.

The maximum intensity enhancement obtained owing to the slow-light effect is limited by several phenomena given in Eq. (4): Kerr nonlinearity, dispersion, and pulse deformation. In addition, effects that are neglected in Eq. (4), such as high-order dispersion terms and the reflection from the grating, may also limit the intensity enhancement. When all of the limiting effects are small, Eq. (4) indicates that the intensity of the pulse is enhanced by a factor $1/v_0$, where v_0 is the pulse velocity at the end of the first grating region, $v_0 \equiv v(z=L_1)$.

The distortive effect of the second term on the right hand side of Eq. (4) can be calculated by ignoring dispersion and nonlinear effects in Eq. (4). Assuming that the distortion is small and that $v_0^2 \ll 1$, we obtain

$$a(z=L_1,t) \simeq v_0^{-1/2}\left(1 + i\frac{1}{2v_0^2\kappa V_g}\frac{\partial}{\partial t}\right)a(z=0,t-\Delta t), \quad (7)$$

where Δt is the time delay due to the propagation in the grating and $\kappa=\kappa(z=L_1)$. Equation (7) shows that the distortion effect is negligible when $v_0 \gg (\kappa V_g T_0)^{-1/2}$, where T_0 is the temporal FWHM of the incident pulse. Defining a parameter $\mu \equiv v_0^2 \kappa V_g T_0$, we require that $\mu \gg 1$. For hyperbolic-secant pulses and $\mu=8$, the maximum contribution of the second term in Eq. (7) is less than 6% of the maximum amplitude of $a(z=0,t)$. The condition $\mu \gg 1$ can also be obtained by requiring that the effect of higher-order dispersion terms, ignored in Eq. (4), is significantly smaller than the effect of second-order dispersion. We note that the amplification due to the slow-light effect is not very sensitive to the frequency of the incident pulse. For example, by increasing the central frequency of an incident pulse with a hyperbolic-secant profile by the FWHM of the pulse spectrum, the amplification is reduced by only 18%.

The slow-light effect causes the incident pulse to be compressed when propagating through the apodized region of the grating, $z<L_1$. In the region $L_1<z<L_1+L_2$, the incident pulse will propagate as a Bragg soliton if the intensity and duration of the compressed pulse fulfill the soliton condition [14]. Assuming $v_0^2 \ll 1$, the peak power of the incident pulse that is required for soliton formation can be obtained by using Eqs. (4) and (5):

$$\mathcal{E}_{0,\max}^2 = \frac{2.06}{\sqrt{\mu}}\frac{1}{\Gamma\sqrt{(V_g T_0)^3 \kappa}}. \quad (8)$$

The length of the apodization L_1 should fulfill two requirements: it should be long enough to minimize the reflection from the grating [10,14], but short enough so that the effects of second-order dispersion and Kerr nonlinearity will not significantly affect the pulse. We assume in our calculations that the apodized section has a quarter-period sine profile. We find that if the apodization length fulfills the condition $L_1<0.15\mu V_g T_0$, the effect of both dispersion and Kerr nonlinearity is small. When $L_1<0.15\mu V_g T_0$ and only the second-order dispersion affects the pulse propagation, we obtain that the broadening of the root-mean-square pulse duration in the apodization region is less than 5%. When $L_1<0.15\mu V_g T_0$ and only Kerr nonlinearity affects the pulse propagation, the accumulated phase across the apodization length is less than 0.24π for the intensity given in Eq. (8).

The apodization length should be long enough to minimize reflections from the grating. We use the expression given in Refs. [10,14] to obtain an upper bound on the reflectivity R at the central frequency of the incident pulse, which can be calculated using Eq. (6). In our grating, we obtain that the reflectivity fulfills $R<1.6(\kappa v_0^2 L_1)^{-2}$. Thus, if the apodization length fulfills the condition $L_1=8V_g T_0/\mu$ the reflection is lower than 2.5%. In practice, we have found that even apodization lengths shorter than $8V_g T_0/\mu$ have a reflection lower than 2.5%. For $\mu \geq 8$, the two requirements on the apodization length can both be met. Hence, our estimations show that when $\mu \geq 8$ the slow-light effect can be used to enhance the pulse intensity by $1/v_0$ when the maximum pulse intensity is sufficient to generate a single Bragg soliton.

The grating profile used in the next examples had an apodized region with a quarter-period sine profile and a length of $L_1=2$ cm that began at $z=0$. For $z>2$ cm, the coupling coefficient was equal to $\kappa=9000$ m^{-1}. The lengths of the grating parts marked in Fig. 1 are $L_2=5$ cm, $L_3=4$ cm, $L_4=5$ cm, and $L_5=21$ cm. The chirp parameter did not change in the sections with the lengths L_1, L_2, and L_4 and had a linear profile for the section with the length L_3. The value of the chirp parameter $\delta(z)$ was equal to 0 at the beginning of the second grating region and was equal to -40.5 m^{-1} at its end. Owing to the chirp, about 90% of the incident pulse energy overlapped with the grating bandgap at the end of the second grating region. The chirp in the third grating region was

$$\delta(z) = -40.5 + 1.35 \times 10^6 (z-0.16)^2 \text{ m}^{-1}. \quad (9)$$

We also assumed a group velocity, $V_g=2\times10^8$ m/sec and a nonlinear coefficient $\Gamma=5$ km^{-1} W^{-1}, as used in Ref. [12].

In order to simulate the propagation of a pulse in the grating described by Fig. 1, Eq. (1) was solved by using the method in Ref. [15]. The input pulse had a hyperbolic-secant profile with a temporal FWHM of $T_0=640$ ps. The peak intensity and the central frequency of the input pulse were chosen to obtain $v_0=1/12$: $\mathcal{E}_{0,\max}^2=34$ W and $\Omega/V_g=9031$ m^{-1}, respectively. The reflectivity from the apodized region was approximately 2.5%. Figure 2 shows the propagation of the incident pulse in the grating for $z<L_1+L_2$. The figure shows that the incident pulse is spatially compressed at the first grating region ($z<L_1$) and eventually forms a soliton. The result of the simulation shows that the pulse is decelerated to a 1/12 of its original velocity and, hence, its peak intensity is enhanced by a factor of 12, in agreement with Eq. (4).

Owing to the chirp in the second grating region, the soliton cannot penetrate the grating and it is, therefore, reflected. In order to form an in-gap soliton, we used a significantly higher intensity, $\mathcal{E}_{0,\max}^2=340$ W. The use of a higher intensity caused the intensity enhancement due to the slow-light effect to decrease to about 9.1 compared to 12, obtained in the linear regime. The incident and transmitted pulses at the entrance and at the output of the whole grating are shown in Figs. 3(a) and 3(b), respectively. The transmitted pulse had a FWHM of 0.25 ps with a peak power of 160 kW, which corresponds to a compression ratio of approximately 2800. For comparison, in the compression scheme given in Ref. [12] the incident pulse had an incident power of 1400 W and the compression ratio was about 12.

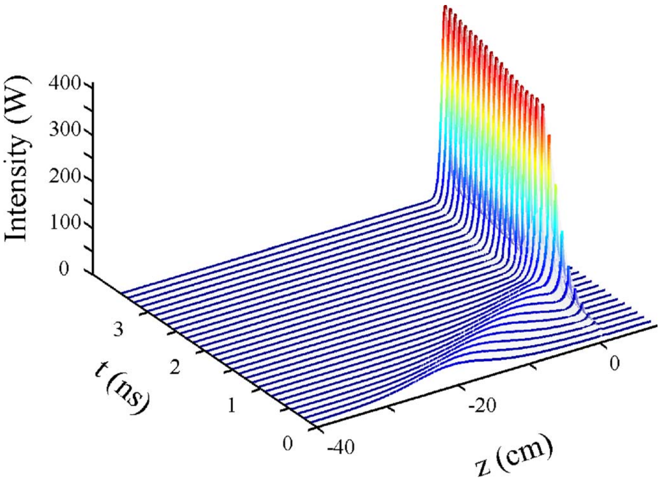

FIG. 2. (Color online) Formation of a soliton at the grating region $0<z<L_1+L_2$. The pulse is spatially compressed by a factor of 12 owing to the slow-light effect and evolves into a Bragg soliton.

When an in-gap soliton is excited, nonlinear instability often causes the generation of multiple pulses [3,5]. Figure 3(c) shows that three pulses were generated at the input of the second grating region. However, the linear chirp and the nonlinear interaction between the pulses in the second grating region were used to overcome the nonlinear instability. Figure 3(d) shows that only the leading pulse penetrates the grating bandgap whereas the trailing pulses are backreflected. We have numerically validated that the transmitted pulse at the output of the second grating region is an in-gap soliton. The energy of the in-gap soliton is equal to 50% of the input pulse intensity. In contrast, when a hyperbolic-secant pulse with a peak power of about 1700 W is directly launched into a uniform grating, as performed in Ref. [3], the transmitted soliton carries only about 4% of the incident pulse energy, assuming that the grating has the same parameters as in the end of the second grating region.

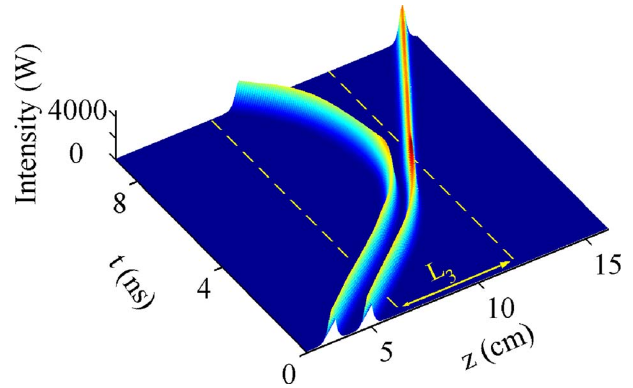

FIG. 4. (Color online) Propagation of two identical solitons in the second region of the grating. The interaction between the solitons causes only the leading soliton to be transmitted while the trailing soliton is backreflected.

The interaction between the pulses in the second region of the grating can be understood by considering the interaction between two identical solitons, demonstrated in Fig. 4. The soliton parameters are $\rho=0.05$ and $v=0.1$. When the two solitons propagate through the chirped region ($7<z<12$ cm) they are decelerated [16]. Since the leading soliton starts decelerating before the second soliton, the distance between the two solitons decreases and their interaction strength increases. The figure shows that the interaction causes the leading soliton to be transmitted while the trailing soliton is backreflected. This result is not sensitive to the phase of the input pulses and it is similar to that obtained in Fig. 3.

We have validated that the in-gap soliton formation is not sensitive to changes in the bandgap of the grating and to changes in the intensity of the incident pulse. For example, we obtained that a single soliton was transmitted for incident pulse intensities in the range of 230 to 500 W. The in-gap soliton formation was also not sensitive to the input pulse

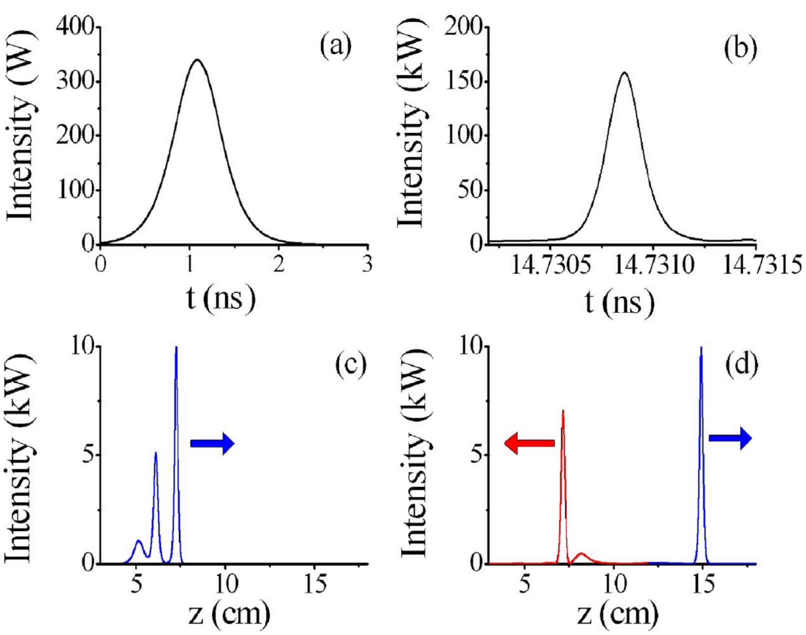

FIG. 3. (Color online) Pulses at the (a) input and (b) output of the grating and (c) at the beginning and (d) the end of the second region of the grating. Multiple pulse are obtained at the beginning of the second grating region. Only the leading pulse is transmitted while the trailing pulses are backreflected owing to the nonlinear interaction between the pulses. The arrows indicate the movement direction of the pulses.

shape; we obtained the same qualitative results for an input Gaussian pulse.

The spatial FWHM of the transmitted gap soliton at the output of the second region of the grating is about 2.1 mm, which corresponds to a compression factor of about 61. In the third grating region, a quadratic chirp is used to further compress the transmitted soliton. The soliton in this region is accelerated. When the soliton is adiabatically compressed, and the pulse energy is approximately conserved, Eqs. (2) and (3) show that when v approaches one, the soliton spatial width ξ decreases and the parameter ρ increases. Since stable soliton propagation is possible only when the parameter ρ is smaller than approximately $\pi/2$ [17], the maximum achievable velocity v is bounded and limits the maximum pulse compression. Although the compression scheme is most effective when the final velocity v is close to one, to the best of our knowledge, pulse compression has not been previously studied in this case. Previous works were based on using the inhomogeneous NLS equation to analyze the pulse compression. However, the NLS equation can only describe Bragg solitons that fulfill $\rho \ll 1$ [18]. Therefore, the amount of compression that was obtained by using the inhomogeneous NLS equation was limited to about 3–10 [11]. In our example, we found that the velocity of the soliton before exiting the grating was $v=0.986$, which corresponds to $\rho=0.75$. The spatial compression of the pulse in the third grating region was equal to 47, giving a total compression factor of about 2800.

III. CONCLUSIONS

In conclusion, we have demonstrated a method for an efficient in-gap soliton formation in FBGs. Our method is based on using a soliton interaction, which enables us to obtain a single-soliton transmission when multiple solitons are formed. We have also used the slow-light effect to substantially reduce the required intensity for the soliton formation. While the slow-light effect in FBGs has been studied in previous works [10–12], we have studied, for the first time to our knowledge, the fundamental limitations of this effect. Our analysis enabled us to significantly increase the magnitude of the slow-light effect beyond what was obtained in previous works.

When using our method for in-gap soliton formation, the spatial width of the formed soliton is significantly shorter than the width of the incident pulse. Thus, our soliton-formation method can also be used for pulse compression. By adding a chirp-based compression scheme to our in-gap-soliton formation method, we have theoretically demonstrated a very high pulse compression of 2800—over two orders of magnitude higher than obtained theoretically and demonstrated experimentally in previous works [11,12]. The incident pulse had a peak power of only 340 W, which is significantly lower than the power used in previous works on pulse compression in FBGs.

The compression scheme described in this paper may be applied to generate high-intensity picosecond pulses. Using our scheme, nanosecond pulses, generated by Q-switched lasers, can be efficiently compressed into picosecond pulses. Therefore, using our method for compressing pulses generated by Q-switched lasers may be an alternative to mode-locked lasers used for generating picosecond pulses. The repetition rate of pulses generated by Q-switched lasers is significantly lower than that of pulses generated by modelocked laser. Therefore, Q-switched lasers can be easily amplified in order to obtain very high optical-pulse intensities.

ACKNOWLEDGMENTS

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