

# Information Rates Subjected to State Masking

Neri Merhav and Shlomo Shamai (Shitz)

EE Dept., Technion – I.I.T., Haifa 32000, Israel

Email: [merhav,sshlo]@ee.technion.ac.il

**Abstract—**We consider the problem of rate- $R$  channel coding with causal/non-causal side information at the transmitter, under an additional requirement of minimizing the amount of information that can be learned from the channel output about the state sequence, which is defined in terms of the mutual information between the state sequence and the channel output sequence. A single-letter characterization is provided for the achievable region of pairs  $\{(R, E)\}$ . Explicit results for the Gaussian case (Costa’s dirty-paper channel) are derived in full detail.

## I. INTRODUCTION

The problem of information transfer via state-dependent channels is classical (see [13] for a partial review). One of the most interesting models is the case where the channel states are available at the transmitter either causally or non-causally. This framework has been fully characterized for i.i.d. states in famous studies by Shannon [18] and by Gel’fand and Pinsker (G-P) [8], respectively. These models, and in particular the G-P setting, have gained much interest in the last few years, mainly due to the wide scope application areas, such as watermarking, [3], [14], [16], [19], [15], multi-input-multi-output (MIMO) broadcast channels, [1], [2], network [12] and cooperative networks, [10], just to name a few applications.

One of the most interesting and well known examples is the G-P channel is the Gaussian setting where the states impact the channel additively. The surprising result by Costa [4] demonstrates that no loss in capacity is suffered no matter how strong that independent interfering state sequence is. Evidently, the many applications and the challenge here motivated much work in terms of actual coding strategies that come close to the optimum. These coding strategies (see, e.g., [25] and references therein), build on the insight of random binning which is the central mechanism in showing achievability in this problem [8], and can, in fact, be interpreted as practical binning strategies. In the Gaussian channel, nick-named “dirty-paper” [4], efficient techniques based on modern codes were recently reported as well (see [7], [20] and references therein). Source-channel coding aspects in the framework of state-dependent channel of this type are also considered [17], and the source-channel separation principle has been shown valid in various scenarios, in which the model itself is intimately related to the Wyner-Ziv (W-Z) source coding problem with side information at the decoder [24], and the G-P channel [8].

While in models addressed in [17], the source and channel states are assumed independent, this is not always the case. In some applications, the channel-state process is not inherently channel-related (like in fading), but may rather be an information-bearing signal on its own. The MIMO broadcast

channel serves as a typical example, where a state sequence for one user is just the information-carrying sequence for another, and all produced at the same transmitter who addresses both users simultaneously [1]. In fact, these are exactly the cases where the justification to the non-causality is self-evident, as the transmitter controls the state sequence. The state sequence is often modelled as i.i.d. whether it is a specific codeword of a good codebook operating on a memoryless channel, which essentially mimics an i.i.d., or it is i.i.d., and it represents raw data, as say a systematic part of the information [17]. Furthermore, the state sequence may model also analogue information which is conveyed over the same channel with an overlaid digital part. This sort of applications gave rise to an interesting problem addressed in [21], [22], where the role of the transmitter is two-fold: to transmit independent reliable information on the one hand, and to boost the quality of the state estimator at the receiver, which adopts a prescribed distortion measure, on the other. A coding scheme has been suggested in [22], which combines W-Z coding, based on the side information about the state available at the receiver side, and G-P coding which is used to convey the independent reliable rate, as well as the W-Z coded information. In the Gaussian case, it has been verified that this achievable tradeoff is in fact optimal [23]. In this specific case, a simple technique where the transmitter optimally power-shares between pure information transmission via the Costa strategy and simple state amplification achieves the optimal tradeoff.

In this paper, we focus on another aspect of the problem. The state sequence is referred to as undesired information that leaks to the receiver. It indeed could model a leakage in the system of, say, secret analogue (sampled) information, or could stand for a codeword, mimicking an i.i.d. distribution, which is not intended to that receiver and is therefore to be concealed from the receiver side. Thus, the goal of the transmitter now is to try and mask this undesired information as much as possible on the one hand, and to transmit reliable independent data rate on the other. The amount of information that the receiver retrieves about the state sequence is measured by the blockwise mutual information (or, equivalently, by the equivocation<sup>1</sup>), as is customary in measuring the security of the cipher systems, in the literature of the Shannon theory. This measure guarantees that even if there is coding involved, only a small value of the associated mutual information limits the

<sup>1</sup>As the entropy of the state process is fixed, the minimization of the mutual information between the state vector and the channel output vector is equivalent to the maximization of the conditional entropy of the state vector given the channel output vector, which is the equivocation.

reliable information that the non-intended receiver can retrieve about the state sequence. It also guarantees that the distortion achieved in any attempt to estimate the state sequence would never be smaller than the distortion-rate function of the state source, computed at the rate given by this (normalized) mutual information.<sup>2</sup>

We characterize the tradeoff between the reliably transmitted rate and the masking ability of the state information, and that is in both the G-P and Shannon models, namely, where the state sequence is either available non-causally or causally respectively. We characterize, explicitly and completely, the tradeoff for the additive Gaussian example, and notice that also in this setting, an element of state cancellation (de-amplification) is optimal. In some cases, excess reliable rate can be transmitted at no cost to the masking ability.

Finally, it should be pointed out that our setting is related to that of [6] in the sense that in both problems, the encoder has two inputs, one of which should be conveyed to the decoder while the other is to be kept confidential (intended to another user in the case of [6]). The main difference, however, is that here, the sender has no control over the confidential message, and has no freedom to encode it, nor does the sender care to guarantee reliable decoding of the secret part at any other destination that may exist.

## II. NOTATION AND PROBLEM FORMULATION

Throughout this paper, scalar RVs will be denoted by capital letters, their sample values will be denoted by the respective lower case letters, and their alphabets will be denoted by the respective calligraphic letters. A similar convention will apply to random vectors and their sample values, which will be denoted with same symbols superscripted by the dimension. Thus, for example,  $X^n$  will denote a random  $n$ -vector  $(X_1, \dots, X_n)$ , and  $x^n = (x_1, \dots, x_n)$  is a specific vector value in  $\mathcal{X}^n$ , the  $n$ -th Cartesian power of  $\mathcal{X}$ . The notations  $x_i^j$  and  $X_i^j$ , where  $i$  and  $j$  are integers and  $i \leq j$ , will designate segments  $(x_i, \dots, x_j)$  and  $(X_i, \dots, X_j)$ , respectively, where for  $i = 1$ , the subscript will be omitted (as above). For  $i > j$ ,  $x_i^j$  (or  $X_i^j$ ) will be understood as the null string. Sequences without specifying indices are denoted by  $\{\cdot\}$ . Sources and channels will be denoted generically by the letter  $P$  or  $Q$ . Information theoretic quantities like entropies, and mutual informations will be denoted following the usual conventions of the information theory literature, e.g.,  $H(X^n)$ ,  $I(S^n; Y^n)$ , etc. Differential entropy will be denoted by  $h$ , e.g.,  $h(S^n)$ .

Consider the Gel'fand-Pinsker (G-P) discrete memoryless channel (DMC) G-P channel

$$P(y^n|x^n, s^n) = \prod_{i=1}^n P(y_i|x_i, s_i),$$

<sup>2</sup>The problem of directly maximizing the distortion in estimating  $S^n$  from  $Y^n$ , rather than minimizing  $I(S^n; Y^n)$ , subject to the communication constraint, has resisted our best efforts thus far (except for the Gaussian-quadratic case). By the same token, the problem of maximizing, rather than minimizing  $I(S^n; Y^n)$  (in the spirit of [23]), remains open.

where  $\{x_i\}$  are the transmitted symbols, taking on values in a finite alphabet  $\mathcal{X}$ ,  $\{s_i\}$  are the corresponding channel states, taking values in a finite state set  $\mathcal{S}$ , and drawn from a discrete memoryless source (DMS),

$$Q(s^n) = \prod_{i=1}^n Q(s_i),$$

and  $\{y_i\}$  are the corresponding channel outputs, taking on values in a finite output alphabet  $\mathcal{Y}$ . The channel input signal is subjected to a limitation

$$\frac{1}{n} \sum_{i=1}^n \mathbf{E}\{\phi(X_i)\} \leq \Gamma, \quad (1)$$

where  $\phi: \mathcal{X} \rightarrow \mathbb{R}^+$  is the transmission cost function and  $\Gamma > 0$  is a given constant. Let  $w \in \mathcal{W} = \{0, 1, \dots, 2^{nR} - 1\}$  denote (the index of) an  $nR$ -bit digital message,  $R$  being the coding rate, to be conveyed via the channel. The random variable  $W$  that designates the message is uniformly distributed across  $\mathcal{W}$  independently of  $S^n$ . We assume that the encoder (also referred to as the transmitter) is non-causally aware of the state sequence  $s^n$ , and it transmits an input vector  $x^n$ , which is a (possibly stochastic) function of  $w$  and  $s^n$ . A rate- $R$  encoder for  $n$ -blocks is therefore characterized by a conditional probability distribution  $P(x^n|s^n, w)$ , which maintains the channel input limitation (1), w.r.t. the randomness of  $S^n$  and  $W$  as well as the possible randomness of the transmitter itself. The corresponding decoder maps the channel output  $y^n$  to  $\hat{w} \in \mathcal{W}$ , and the probability of error  $P_e$  is defined as  $\Pr\{\hat{W} \neq W\}$ .

We are interested in the interplay between reliable coding at rate  $R$ , which we would like to keep as large as possible, and (normalized) mutual information,  $I(S^n; Y^n)/n$ , which we would like to make as small as possible. For a given  $\Gamma > 0$ , a pair  $(R, E)$  is called *achievable* if for every  $\epsilon > 0$  and sufficiently large  $n$ , there exist a rate- $R$  encoder-decoder for  $n$ -blocks such the following conditions are simultaneously satisfied:

- 1)  $\frac{1}{n} \sum_{i=1}^n \mathbf{E}\{\phi(X_i)\} \leq \Gamma$
- 2)  $P_e \leq \epsilon$
- 3)  $\frac{1}{n} I(S^n; Y^n) \leq E + \epsilon$ .

The achievable region  $\mathcal{A}$  is the set of all achievable pairs  $\{(R, E)\}$ .

Our main goal is to provide a single-letter characterization of  $\mathcal{A}$  as well as some insights on good coding schemes. We also show how our coding theorem should be modified to the case where the transmitter has causal, rather than non-causal, access to the side information. As mentioned in the Introduction, given an arbitrary distortion function  $d: \mathcal{S} \times \mathcal{S} \rightarrow \mathbb{R}$ , the distortion in estimating  $S^n$  by  $\hat{S}^n = (\hat{S}_1, \dots, \hat{S}_n)$  at the receiver, is lower bounded by

$$\frac{1}{n} \sum_{i=1}^n \mathbf{E}\{d(S_i, \hat{S}_i)\} \geq D_S(E), \quad (2)$$

where  $D_S(\cdot)$  is the distortion-rate function of  $S$ , and  $E$  is the level of the normalized mutual information  $\frac{1}{n} I(S^n; Y^n)$ . As

we shall see, in the Gaussian–quadratic case, this yields a tight result (up to possible time–sharing) regarding the distortion.

### III. THE ZERO–RATE CASE

For the sake of simplicity of the exposition, we begin with the zero–rate case, i.e.,  $R = 0$ , and then our only goal is to minimize  $I(S^n; Y^n)/n$  subject to (1).

Let  $\mathcal{F}(\Gamma)$  denote the minimum of  $I(S^n; Y^n)/n$  over all channels  $\{P(x^n|s^n)\}$  that satisfy (1). Define also the single–letter function  $F(\Gamma) = \min I(S; Y)$ , where the minimum is over all  $\{P(x|s)\}$  s.t.  $E\phi(X) \leq \Gamma$ . Our first theorem is the following:

*Theorem 1:* The minimum normalized mutual information,  $I(S^n; Y^n)/n$ , subject to the power constraint (1) is given by  $\mathcal{F}(\Gamma) = F(\Gamma)$ .

*Proof.* As for the direct part, apply the DMC

$$P^*(x^n|s^n) = \prod_{i=1}^n P^*(x_i|s_i),$$

where the single–letter channel  $P^*(x|s)$  achieves  $F(\Gamma)$ . Since the induced channel  $P(y^n|s^n)$  will be a DMC as well, and  $I(S_i, Y_i) = F(\Gamma)$  for all  $i$ , then so will be  $I(S^n; Y^n)/n = \frac{1}{n} \sum_{i=1}^n I(S_i; Y_i)$ .

Turning now to the converse part, we first observe that  $F(\Gamma)$  is convex. This is very easy to see in the very same manner as the classical informational rate–distortion is shown to be convex [5]: The mutual information  $I(S^n; Y^n)$  is a convex functional of  $\{P(y^n|s^n)\}$ , which is a linear functional of  $\{P(x^n|s^n)\}$ , which in turn is subjected to the power constraint, which is linear. Thus,

$$\begin{aligned} I(S^n; Y^n) &\geq \sum_{i=1}^n I(S_i; Y_i) \\ &\geq \sum_{i=1}^n F(\mathbf{E}\phi(X_i)) \\ &\geq nF\left(\frac{1}{n} \sum_{i=1}^n \mathbf{E}\phi(X_i)\right) \\ &\geq nF(\Gamma), \end{aligned} \quad (3)$$

where the first inequality is by the memorylessness of  $S^n$ , the second is by definition of  $F$ , the third by convexity, and the fourth by monotonicity. This completes the proof of Theorem 1.

It is interesting to observe that in the zero–rate case considered here, the optimum transmitter works in a single–letter (scalar) fashion, i.e., no long blocks are needed. This means that the solutions to the causal and non-causal problems coincide in the zero–rate case. It also means that the solution is strictly optimum and not only asymptotically so. As we shall see, this will no longer be true for positive rates.

We next study the example of the “dirty–paper” channel in some detail. It should be noted that the derivation in this

example is a generalization of [9, Corollary 3] from the case  $\sigma_z^2 = 0$  to the case  $\sigma_z^2 > 0$ .<sup>3</sup>

*Example.* Consider the channel

$$Y = X + S + Z, \quad (4)$$

where  $S$  and  $Z$  are zero–mean, (not necessarily Gaussian) RV’s with variances  $\sigma_s^2$  and  $\sigma_z^2$ , respectively, where  $Z$  is independent of  $(X, S)$ . We would like to characterize the optimum conditional distribution  $P^*(x|s)$ . Since

$$I(S; Y) = h(S) - h(S|Y), \quad (5)$$

and  $h(S)$  is given, minimization of  $I(S; Y)$  is equivalent to maximization of  $h(S|Y)$ , provided that the differential entropies are finite. Now, for a given  $\sigma_x^2 \leq \Gamma$ , and  $\rho = \mathbf{E}(XS)/(\sigma_x\sigma_s)$ , we have:

$$\begin{aligned} h(S|Y) &= h(S - \mathbf{E}(S|Y)|Y) \\ &\leq h(S - \mathbf{E}(S|Y)) \\ &\leq \frac{1}{2} \log [2\pi e \cdot \mathbf{E}(S - \mathbf{E}(S|Y))^2] \\ &\leq \frac{1}{2} \log [2\pi e \cdot \min_a \mathbf{E}(S - aY)^2] \\ &= \frac{1}{2} \log [2\pi e(\sigma_s^2 - \sigma_s^2)] \end{aligned} \quad (6)$$

with

$$\sigma_s^2 = \frac{(\sigma_s^2 + \rho\sigma_x\sigma_s)^2}{\sigma_s^2 + 2\rho\sigma_x\sigma_s + \sigma_x^2 + \sigma_z^2}, \quad (7)$$

which is the variance of the optimum linear estimator of  $S$  based on  $Y$ . The last inequality is due to the fact that the MSE of the optimum linear estimator of  $S$  is never smaller than the MSE of the optimum estimator, which is the conditional mean. As is easily seen, all inequalities become equalities if in addition to the above–mentioned assumptions,  $(X, S, Z)$  are jointly Gaussian, which will be our assumption henceforth through this example. It remains then to minimize  $\sigma_s^2$  w.r.t.  $(\sigma_x^2, \rho)$  over the rectangle  $[0, \Gamma] \times [-1, 1]$ . First observe that whenever  $\Gamma \geq \sigma_s^2$ , the solution is trivially  $X = -S$ . Assume then that  $\Gamma < \sigma_s^2$ . First, let us rewrite  $\sigma_s^2$  as follows:

$$\begin{aligned} \sigma_s^2 &= \frac{(\sigma_s^2 + \rho\sigma_x\sigma_s)^2}{2(\sigma_s^2 + \rho\sigma_x\sigma_s) + \sigma_x^2 + \sigma_z^2 - \sigma_s^2} \\ &= \frac{t^2}{2t + \alpha} \end{aligned} \quad (8)$$

where

$$t = \sigma_s^2 + \rho\sigma_x\sigma_s \quad (9)$$

and

$$\alpha = \sigma_x^2 + \sigma_z^2 - \sigma_s^2. \quad (10)$$

and where for a given  $\sigma_x^2$ , we have the freedom to minimize  $\sigma_s^2$  over  $t$  in the interval  $[\sigma_s(\sigma_s - \sigma_x), \sigma_s(\sigma_s + \sigma_x)]$ . It is easy to see that within this interval, the denominator is never negative, namely,  $t > -\alpha/2$ . First, observe that  $\sigma_x^2 = \Gamma$  is

<sup>3</sup>The final version of [9] has also appeared in *IEEE Trans. Inform. Theory* (vol. 49, no. 4, pp. 951–963, April 2003), but without Corollary 3.

always the optimum choice – this choice both maximizes  $\alpha$  and broadens the range of allowable values of  $t$  as much as possible. Let us set then  $\sigma_x^2 = \Gamma$ . Thus, our problem is to minimize the function  $f(t) = t^2/(2t + \alpha)$  in the range  $t \in [\sigma_s(\sigma_s - \sqrt{\Gamma}), \sigma_s(\sigma_s + \sqrt{\Gamma})]$ . At this point, we have to distinguish between two cases:  $\alpha \geq 0$  and  $\alpha < 0$  (corresponding to  $\sigma_s^2 \leq \Gamma + \sigma_z^2$  and  $\sigma_s^2 > \Gamma + \sigma_z^2$ , respectively). Assume first that  $\alpha \geq 0$ . In this case, the global minimum of the function  $f(t) = t^2/(2t + \alpha)$  (in the range  $t > -\alpha/2$ ) is at  $t = 0$ , and this function is monotonically increasing for  $t \geq 0$ . Moreover, since we are assuming that  $\Gamma < \sigma_s^2$ , then the above defined allowable interval of  $t$  is within  $\mathbb{R}^+$ , and so, the optimum  $t$  is the left edgepoint of this interval, namely,  $t^* = \sigma_s(\sigma_s - \sqrt{\Gamma})$ , which corresponds to  $\rho^* = -1$ , which in turn means that the optimum transmission is  $X = -\sqrt{\frac{\Gamma}{\sigma_s^2}} \cdot S$ . Next, consider the case  $\alpha < 0$ . In this case, for  $t > -\alpha/2$ , the derivative of  $f(t)$  vanishes at  $t = -\alpha$ , and  $f(t)$  is monotonically increasing for  $t \geq -\alpha$ . In this case, there are two possibilities: If  $\sigma_s < \sqrt{\Gamma} + \sigma_z^2/\sqrt{\Gamma}$  (which is still possible when  $\alpha < 0$ ), then again, the minimum of  $f(t)$  within the allowable interval is obtained at the left edgepoint,  $t^* = \sigma_s(\sigma_s - \sqrt{\Gamma})$ , and so, once again,  $X = -\sqrt{\frac{\Gamma}{\sigma_s^2}} \cdot S$ . If, on the other hand,  $\sigma_s \geq \sqrt{\Gamma} + \sigma_z^2/\sqrt{\Gamma}$ , then the derivative of  $f(t)$  vanishes within the allowable interval, and so the minimum lies at  $t = -\alpha = \sigma_s^2 - \Gamma - \sigma_z^2$ , which corresponds to

$$\rho^* = -\frac{\Gamma + \sigma_z^2}{\sqrt{\Gamma}\sigma_s},$$

and the corresponding transmission is

$$X = -S \cdot \left( \frac{\Gamma}{\sigma_s^2} + \frac{\sigma_z^2}{\sigma_s^2} \right) + V,$$

where  $V$  is a zero-mean Gaussian RV whose variance is determined so that  $\mathbf{E}(X^2) = \Gamma$ .

To summarize then, the solution divides into three cases, according to the intensity of the interference,  $S$ :

- *Weak interference*: If  $\sigma_s^2/\Gamma \leq 1$ , then take  $X = -S$ , and then  $F(\Gamma) = 0$ .
- *Moderate interference*: If

$$1 < \frac{\sigma_s^2}{\Gamma} \leq \left( 1 + \frac{\sigma_z^2}{\Gamma} \right)^2, \quad (11)$$

then

$$X = -\sqrt{\frac{\Gamma}{\sigma_s^2}} \cdot S, \quad (12)$$

and then

$$F(\Gamma) = \frac{1}{2} \log \left[ 1 + \left( \frac{\sigma_s}{\sigma_z} - \frac{\sqrt{\Gamma}}{\sigma_z} \right)^2 \right]. \quad (13)$$

- *Strong interference*: If

$$\frac{\sigma_s^2}{\Gamma} > \left( 1 + \frac{\sigma_z^2}{\Gamma} \right)^2, \quad (14)$$

then

$$X = -S \cdot \left( \frac{\Gamma}{\sigma_s^2} + \frac{\sigma_z^2}{\sigma_s^2} \right) + V, \quad (15)$$

where  $V$  is a zero-mean Gaussian RV, independent of  $S$ , with variance

$$\sigma_v^2 \triangleq \Gamma \left[ 1 - \frac{\Gamma}{\sigma_s^2} \left( 1 + \frac{\sigma_z^2}{\Gamma} \right) \right]^2 \quad (16)$$

and in this case,

$$F(\Gamma) = \frac{1}{2} \log \left( 1 + \frac{A}{B} \right), \quad (17)$$

where

$$A = \sigma_s^2 \left[ 1 - \frac{\Gamma}{\sigma_s^2} \left( 1 + \frac{\sigma_z^2}{\Gamma} \right) \right]^2 \quad (18)$$

and

$$B = \sigma_v^2 + \sigma_z^2 = \Gamma \left[ 1 - \frac{\Gamma}{\sigma_s^2} \left( 1 + \frac{\sigma_z^2}{\Gamma} \right) \right]^2 + \sigma_z^2. \quad (19)$$

#### IV. THE POSITIVE RATE CASE

Turning now to the more general positive rate case, our main result is the following:

*Theorem 2*: The achievable region  $\mathcal{A}$  is the set of pairs  $\{(R, E)\}$  for which there exist random variables  $U$  and  $X$  that satisfy the following conditions at the same time:

- 1)  $U \rightarrow (X, S) \rightarrow Y$  is a Markov chain, i.e., the joint probability distribution of  $(U, X, S, Y)$  is given by  $P(u, x, s, y) = Q(s)P(u|s)P(x|u, s)P(y|x, s)$ .
- 2)  $\mathbf{E}\phi(X) \leq \Gamma$ .
- 3)  $R \leq I(U; Y) - I(U; S)$ .
- 4)  $E \geq I(S; U, Y)$ .

*Proof*. We begin with the converse part. The channel-coding part is exactly as in [8], except that here, we present it slightly differently in order to establish the fact that the same random variable  $U$  that meets the rate requirement, also meets the mutual information (or, equivalently, equivocation) requirement and the power constraint. For  $i = 1, \dots, n$ , let  $U_i = (W, Y^{i-1}, S_{i+1}^n)$ . Define a RV  $T$ , uniformly distributed over  $\{1, 2, \dots, n\}$  (independently of the other RV's), and let  $U \triangleq (U_T, T)$  (where  $U_T$  is  $U_i$  for  $i = T$ ). We also define the RV's  $Y = Y_T$ ,  $S = S_T$ , and

$$\delta(\epsilon) = \epsilon \log \epsilon - (1 - \epsilon) \log(1 - \epsilon) + \epsilon R,$$

for  $\epsilon \in [0, 1]$ . Now,

$$\begin{aligned} R - \delta(\epsilon) &\leq R - \delta(P_e) \\ &\leq \frac{1}{n} \sum_{i=1}^n [I(U_i; Y_i) - I(U_i; S_i)] \\ &= I(U_T; Y_T|T) - I(U_T; S_T|T) \\ &= I(U_T, T; Y_T) - I(T; Y_T) - \\ &\quad I(U_T, T; S_T) + I(T; S_T) \\ &\leq I(U_T, T; Y_T) - I(U_T, T; S_T) + I(T; S_T) \\ &= I(U_T, T; Y_T) - I(U_T, T; S_T) \\ &= I(U; Y) - I(U; S), \end{aligned} \quad (20)$$

where the first inequality is by the requirement that  $P_e \leq \epsilon$ , the second is as in [8, Proposition 3, Lemma 4], and in the second to the last equality we have used the fact that  $S = S_T$  is independent of  $T$  (due to stationarity). As for the mutual information,  $I(S^n; Y^n)$ , we have the following:

$$\begin{aligned}
I(S^n; Y^n) &= I(S^n; Y^n, W) - I(S^n; W|Y^n) \\
&\geq H(S^n) - H(S^n|Y^n, W) - H(W|Y^n) \\
&\geq \sum_{i=1}^n [H(S_i) - H(S_i|S_{i+1}^n, Y^n, W)] - n\delta(P_e) \\
&\geq \sum_{i=1}^n [H(S_i) - H(S_i|Y_i, S_{i+1}^n, Y^{i-1}, W)] \\
&\quad - n\delta(P_e) \\
&\geq \sum_{i=1}^n [H(S_i) - H(S_i|Y_i, U_i)] - n\delta(\epsilon) \\
&= n[H(S_T|T) - H(S_T|Y_T, U_T, T)] - n\delta(\epsilon) \\
&= n[H(S) - H(S|Y, U) - \delta(\epsilon)] \\
&= n[I(S; Y, U) - \delta(\epsilon)], \tag{21}
\end{aligned}$$

where the second inequality is by Fano's inequality, and where we have used again the fact that  $S_T$  is independent of  $T$ . The channel input constraint is maintained by definition of  $X_T$ . Finally, note that due to the stationarity of the memoryless channel  $P(y|x, s)$ , the Markov relation  $U \rightarrow (X, S) \rightarrow Y$  is maintained (and is not violated by the presence of the RV  $T$ ).

Regarding the direct part, consider the ordinary construction of the G-P code using binning. Reliable decoding is proved exactly as in [8]. The power constraint is maintained by joint typicality considerations. As for the mutual information between  $S^n$  and  $Y^n$ , first, we have the following:

$$\begin{aligned}
I(S^n; Y^n) &\leq I(S^n; U^n, Y^n) \\
&= I(S^n; U^n) + I(S^n; Y^n|U^n) \\
&= I(S^n; U^n) + H(Y^n|U^n) - H(Y^n|S^n, U^n) \tag{22}
\end{aligned}$$

The first term is bounded as follows:

$$\begin{aligned}
I(S^n; U^n) &\leq I(S^n; W, U^n) \\
&= I(S^n; U^n|W) \\
&= H(U^n|W) - H(U^n|S^n; W) \\
&\leq H(U^n|W) \\
&\leq n[I(U; S) + \epsilon], \tag{23}
\end{aligned}$$

where the equality is due to the independence between  $S^n$  and  $W$ , and the last inequality is due to the fact that the size of each bin is less than  $2^{n[I(U; S) + \epsilon]}$ . As for the second term on the right-most side of (22), we have:

$$H(Y^n|U^n) \leq \sum_{i=1}^n H(Y_i|U_i) = nH(Y|U), \tag{24}$$

where we have used the fact that the empirical distribution of each codeword  $U^n$  is according to the desired choice of

the distribution of  $U$  and that each  $Y_i$  is generated from  $U_i$  according to

$$P(y|u) = \sum_{s,x} P(s|u)P(x|u, s)P(y|x, s). \tag{25}$$

As for the third term on the right-most side of (22):

$$H(Y^n|S^n, U^n) = \sum_{i=1}^n H(Y_i|S_i, U_i) = nH(Y|S, U), \tag{26}$$

where the first equality is due to the memorylessness of the channel  $P(y|u, s)$  (which is the cascade of the memoryless channel  $P(x|u, s)$  and the memoryless channel  $P(y|x, s)$ ), and the second equality is explained similarly as before. Thus, we obtain:

$$\begin{aligned}
I(S^n; Y^n) &\leq n[I(U; S) + \epsilon] + nH(Y|U) - nH(Y|S, U) \\
&= n[I(S; Y, U) + \epsilon]. \tag{27}
\end{aligned}$$

The power constraint is maintained by joint typicality considerations. This completes the proof of Theorem 2.

A few comments are in order: First, as the auxiliary RV  $U$ , includes the time variable  $T$ , the achievable region is convex. Second, the cardinality of the alphabet of  $U$  is by two letters larger than in ordinary G-P coding because of the additional mutual information and power constraints. Finally, note that here, unlike the pure G-P coding, the channel  $P(x|u, s)$  is not necessarily deterministic: For example, in the Gaussian case with  $R = 0$  that was studied earlier,  $U$  is degenerate, but  $P(x|u, s) = P(x|s)$  is non-deterministic in the case of very strong interference.

We next revisit the Gaussian example, this time, for positive rates. One of the interesting points in this example is that it turns out that the same RV  $U$  that maximizes the information rate,  $I(U; Y) - I(U; S)$  (as in Costa's channel) turns out to minimize  $I(S; Y, U)$  and bring it to the level of  $I(S; Y)$ . In other words,  $U$  does not improve on the estimation of  $S$  once  $Y$  is observed even in the single-letter level (see also [23, footnote no. 2]).

*Example – "dirty-paper" channel revisited.* First, it should be noted, that similarly as in [4], here too, Theorem 2 extends to continuous alphabets by taking limits of  $I(U; Y) - I(U; S)$  and  $I(S; Y, U)$  over sequences of successively refined partitions of the alphabets  $\mathcal{U}$ ,  $\mathcal{S}$  and  $\mathcal{Y}$ . As before, the actual input power  $\sigma_x^2 \triangleq \mathbf{E}(X^2)$  will be assumed less than or equal to  $\Gamma$ . However, observe that in case of  $R > 0$ , the best choice of  $\sigma_x^2$  is always  $\sigma_x^2 = \Gamma$ , because the part of the power of  $X$  that may not be needed to cancel  $S$  (when  $\sigma_s^2 < \Gamma$ ), is always fully utilized to convey digital information. Thus,  $\sigma_x^2$  and  $\Gamma$  are two notations for the same entity, in this example.

*Proposition 1:* Let  $Y = X + S + Z$ , where  $S$  is a zero-mean (not necessarily Gaussian) RV with variance  $\sigma_s^2$ , and  $Z \sim \mathcal{N}(0, \sigma_z^2)$  is independent of  $X$  and  $S$ , and where  $\mathbf{E}(X^2) = \sigma_x^2$ , and  $\mathbf{E}(XS) = \rho\sigma_s\sigma_x$ . Further, let  $U$  be an RV that satisfies the Markov relation  $U \rightarrow (X, S) \rightarrow Y$ . Then,

$$I(U; Y) - I(U; S) \leq \frac{1}{2} \log \left[ 1 + \frac{\sigma_x^2(1 - \rho^2)}{\sigma_z^2} \right]. \tag{28}$$

*Proof.* Let  $\tilde{X} = X - aS$ , where  $aS$  stands for the best linear estimator of  $X$  given  $S$ , that is,  $a = \rho\sigma_x/\sigma_s$ . Thus,  $Y$  can be represented as

$$Y = \tilde{X} + (1+a)S + Z, \quad (29)$$

where  $\tilde{X}$  is uncorrelated with  $S$ , and  $\mathbf{E}(\tilde{X}^2) = \sigma_x^2(1 - \rho^2)$ . Now,

$$\begin{aligned} & I(U; Y) - I(U; S) \\ & \leq I(U; Y, S) - I(U; S) \\ & = I(U; Y|S) \\ & \leq I(X, S; Y|S) \\ & = I(\tilde{X}, S; Y|S) \\ & = I(\tilde{X}; \tilde{X} + Z|S) \\ & = h(\tilde{X} + Z|S) - h(\tilde{X} + Z|S, \tilde{X}) \\ & \leq h(\tilde{X} + Z) - h(\tilde{X} + Z|\tilde{X}) \\ & \leq \frac{1}{2} \log [2\pi e (\sigma_x^2(1 - \rho^2) + \sigma_z^2)] - \frac{1}{2} \log(2\pi e \sigma_z^2) \\ & = \frac{1}{2} \log \left[ 1 + \frac{\sigma_x^2(1 - \rho^2)}{\sigma_z^2} \right], \end{aligned} \quad (30)$$

where the second inequality is due to the Markov relation  $U \rightarrow (X, S) \rightarrow Y$  and the data processing theorem, the following equality is due to the fact that the transformation from  $(X, S)$  to  $(\tilde{X}, S)$  is one-to-one, and the following inequality is due to the fact that conditioning cannot increase entropy and the fact that  $\tilde{X} + Z$  is independent of  $S$  given  $\tilde{X}$  (since  $Z$  is independent of both  $\tilde{X}$  and  $S$ ). This completes the proof.

*Proposition 2:* Let  $Y = X + S + Z$ , where  $S \sim \mathcal{N}(0, \sigma_s^2)$  and  $Z$  is a zero-mean (not necessarily Gaussian) RV with variance  $\sigma_z^2$ , independent of  $(X, S)$ . Further, let  $U$  be an additional RV jointly distributed with  $(X, S, Z)$ . Then,

$$I(S; Y, U) \geq \frac{1}{2} \log \frac{\sigma_s^2}{\sigma_s^2 - \sigma_s^2}, \quad (31)$$

where  $\sigma_s^2$  is as in eq. (7).

*Proof.*  $I(S; Y, U) \geq I(S; Y)$  and the rest is like in the Gaussian example for  $R = 0$ , taking into account that for  $S \sim \mathcal{N}(0, \sigma_s^2)$ ,  $h(S) = \frac{1}{2} \log(2\pi e \sigma_s^2)$ .

*Corollary 1:* Let

$$R < \frac{1}{2} \log \left( 1 + \frac{\sigma_x^2}{\sigma_z^2} \right), \quad (32)$$

and

$$\varrho(R) = \sqrt{1 - (2^{2R} - 1) \frac{\sigma_z^2}{\sigma_x^2}}, \quad (33)$$

and let  $E(\varrho)$ ,  $\varrho \geq 0$ , denote the minimum of

$$\frac{1}{2} \log \frac{\sigma_s^2}{\sigma_s^2 - \sigma_s^2}$$

over  $\rho \in [-\varrho, +\varrho]$ , where  $\sigma_s^2$  as in eq. (7). Then, for the channel  $Y = X + S + Z$ , where  $S \sim \mathcal{N}(0, \sigma_s^2)$  and  $Z \sim \mathcal{N}(0, \sigma_z^2)$  is independent of  $(X, S)$ , and for a given  $R$  as

in (32), the minimum achievable per-symbol masking mutual information is lower bounded by

$$\frac{1}{n} I(S^n; Y^n) \geq E(\varrho(R)).$$

*Comment:* Referring to the discussion after eq. (8), the interval of  $t$  where optimum is sought, shrinks to  $[\sigma_s(\sigma_s - \varrho\sigma_x), \sigma_s(\sigma_s + \varrho\sigma_x)]$ .

*Proposition 3:* Let  $Y = X + S + Z$  where  $Z \sim \mathcal{N}(0, \sigma_z^2)$  is independent of  $S \sim \mathcal{N}(0, \sigma_s^2)$  and of  $X$ . Then,  $E(\varrho(R))$  is achievable.

*Proof.* Given  $R$ , let  $\rho$  be the achiever of  $E(\varrho(R))$ . Now, apply dirty-paper coding to the (modified) Costa channel

$$Y = \tilde{X} + (1+a)S + Z,$$

where  $\tilde{X}$  is, as was shown above, Gaussian and independent of  $S$ , and where  $U = \tilde{X} + c(1+a)S$ , with

$$c = \frac{\sigma_x^2(1 - \rho^2)}{\sigma_x^2(1 - \rho^2) + \sigma_z^2}.$$

This means that

$$U = \tilde{X} + c(1+a)S = X - aS + c(1+a)S = X + bS,$$

where

$$b = c(1+a) - a = \frac{\sigma_x^2(1 - \rho^2) - \rho\sigma_z^2\sigma_x/\sigma_s}{\sigma_x^2(1 - \rho^2) + \sigma_z^2}.$$

Since the power of  $\tilde{X}$  is  $\sigma_x^2(1 - \rho^2)$ , any coding rate up to

$$\frac{1}{2} \log \left[ 1 + \frac{\sigma_x^2(1 - \rho^2)}{\sigma_z^2} \right]$$

is achievable as in [4].

Regarding the mutual information between  $S^n$  and  $Y^n$ , we now show that with this choice of  $U$ , we have  $I(S; U, Y) = I(S; Y)$ , i.e., in the presence of  $Y$ , the observation of  $U$ , defined as in Costa [4], does not improve the MSE of linear estimation of  $S$ , and so the lower bound to  $I(S; Y, U)$  is met. In other words, the above choice of  $U$  simultaneously maximizes  $I(U; Y) - I(U; S)$  and minimizes  $I(S; Y, U)$ . To show this, consider the minimum mean square error associated with optimum (linear) estimation of  $S$  given  $Y = \tilde{X} + (1+a)S + Z$  and  $U = \tilde{X} + bS$ , i.e.,  $\mathbf{E}(S - \alpha Y - \beta U)^2$ . We have to show that for the optimum coefficients  $(\alpha^*, \beta^*)$ , we have  $\beta^* = 0$ . Now, by solving the linear equations associated with  $(\alpha^*, \beta^*)$ , it is readily seen that  $\beta^*$  is given by a ratio of two expressions whose numerator is given by

$$\mathbf{E}(Y^2) \cdot \mathbf{E}(SU) - \mathbf{E}(UY) \cdot \mathbf{E}(SY). \quad (34)$$

Thus, proving that  $\beta^* = 0$  is equivalent to proving that

$$\mathbf{E}(Y^2) \cdot \mathbf{E}(SU) = \mathbf{E}(UY) \cdot \mathbf{E}(SY). \quad (35)$$

Now, the left-hand side of the last equation is given by

$$\mathbf{E}(Y^2) \cdot \mathbf{E}(SU) = [\mathbf{E}(\tilde{X}^2) + (1+a)^2\sigma_s^2 + \sigma_z^2] \cdot b\sigma_s^2 \quad (36)$$

whereas the right-hand side is given by

$$\mathbf{E}(UY) \cdot \mathbf{E}(SY) = [E(\tilde{X}^2) + b(1+a)\sigma_s^2] \cdot (1+a)\sigma_s^2. \quad (37)$$

By using the above defined expressions of  $\mathbf{E}(\tilde{X}^2)$ ,  $a$ ,  $b$ , and  $c$ , the equality between the two expressions is readily verified. This completes the proof.

Note that as long as the achiever  $\rho$  of  $E(1)$  has absolute value strictly less than unity (which is the case of strong interference, cf. the Gaussian example at rate  $R = 0$ ), then it is possible to transmit at a positive rate, without any loss in mutual information between  $Y^n$  and  $S^n$ . In other words, the random variable  $V$ , in the earlier Gaussian example pertaining to  $R = 0$ , could be used for dirty-paper coding a la Costa, at rate up to

$$R = \frac{1}{2} \log \frac{B}{\sigma_z^2},$$

where  $B$  is defined as in eq. (19). A similar comment applies to weak interference, where the remainder power, not used to cancel  $S$ , can be harnessed to convey information at any rate up to

$$R = \frac{1}{2} \log \left( 1 + \frac{\Gamma - \sigma_s^2}{\sigma_z^2} \right).$$

Another observation is that while in general, the channel  $P(x|s, u)$  might be stochastic (in contrast to the ordinary G-P problem), in the Gaussian case, it remains always deterministic when  $R > 0$  ( $U = X + bS$  is equivalent to  $X = U - bS$ ). Recall that for the case  $R = 0$ , it is not necessarily true.

#### The Quadratic Distortion in the Gaussian Case.

Let us now consider the quadratic distortion (mean square error) in the estimation of  $S^n$  at the receiver. While our secrecy criterion, throughout this paper, is the minimum mutual information between  $S^n$  and  $Y^n$ , in the Gaussian-quadratic case, this yields also a closed-form result with regard to the maximum mean-square distortion at the receiver: On the one hand, if one applies the achievability scheme described above, then eq. (2) results in:

$$\begin{aligned} & \frac{1}{n} \sum_{i=1}^n \mathbf{E}(S_i - \hat{S}_i)^2 \\ & \geq D_S(\mathbf{E}(\varrho(R))) \\ & = \max_{\rho \in [-\varrho(R), \varrho(R)]} \sigma_s^2 \cdot \exp_2 \left\{ -2 \cdot \frac{1}{2} \log \frac{\sigma_s^2}{\sigma_s^2 - \sigma_z^2} \right\} \\ & = \sigma_s^2 - \min_{\rho \in [-\varrho(R), \varrho(R)]} \sigma_z^2. \end{aligned} \quad (38)$$

On the other hand, this distortion level is achievable at the receiver by using the optimal linear estimator of  $S_i$  given  $Y_i$ ,  $i = 1, \dots, n$  on a symbol-by-symbol basis.

While  $D_S(\mathbf{E}(\varrho(R)))$  was obtained above as a tight upper and lower bound on the maximum achievable distortion, which is based on the above proposed achievability scheme, we now show an achievable upper bound on the maximum distortion that *any* encoder can enforce while maintaining reliable communication at rate  $R$ . First, let us refer to the proof of the converse part of Theorem 2, where we defined

$U_i = (W, Y^{i-1}, S_{i+1}^n)$  and have shown (cf. eq. (20)) that

$$\begin{aligned} R - \delta(\epsilon) & \leq \frac{1}{n} \sum_{i=1}^n [I(U_i; Y_i) - I(U_i; S_i)] \\ & \leq \frac{1}{2n} \sum_{i=1}^n \log \left( 1 + \frac{\sigma_x^2(1 - \rho_i^2)}{\sigma_z^2} \right), \end{aligned} \quad (39)$$

where in the second inequality we have used Proposition 1 with  $\rho_i \triangleq \mathbf{E}\{X_i S_i\} / \sigma_x \sigma_s$  for all  $i = 1, \dots, n$ . Defining  $D(R)$  as the upper concave envelope<sup>4</sup> of  $D_S(\mathbf{E}(\varrho(R)))$ , we have:

$$\begin{aligned} & \frac{1}{n} \sum_{i=1}^n \mathbf{E}\{(S_i - \hat{S}_i)^2\} \\ & \leq \frac{1}{n} \sum_{i=1}^n \left[ \sigma_s^2 - \frac{(\sigma_s^2 + \rho_i \sigma_x \sigma_s)^2}{\sigma_s^2 + 2\rho_i \sigma_x \sigma_s + \sigma_x^2 + \sigma_z^2} \right] \\ & \leq \frac{1}{n} \sum_{i=1}^n D_S(\mathbf{E}(\varrho(I(U_i; Y_i) - I(U_i; S_i)))) \\ & \leq \frac{1}{n} \sum_{i=1}^n D(I(U_i; Y_i) - I(U_i; S_i)) \\ & \leq D \left( \frac{1}{n} \sum_{i=1}^n [I(U_i; Y_i) - I(U_i; S_i)] \right) \\ & \leq D(R - \delta(P_e)), \end{aligned} \quad (40)$$

where the first inequality bounds the mean-square error of the best estimator in terms of the best linear, symbol-by-symbol estimator, the second inequality follows from Proposition 1 applied to each  $i = 1, \dots, n$  (cf. the right inequality of (39)), the third inequality is by definition of  $D(\cdot)$  as the upper concave envelope of  $D_S(\mathbf{E}(\varrho(\cdot)))$ , the fourth inequality is due to the concavity of  $D(\cdot)$ , and the last inequality is due to the non-increasing monotonicity of  $D(\cdot)$  and by (20). This bound can be achieved by an encoder that time-shares the two working points,  $(R_1, D_1)$  and  $(R_2, D_2)$ , on the curve  $D_S(\mathbf{E}(\varrho(R)))$  whose convex combination achieves  $D(R)$ . In each of these working points, we apply our achievability scheme at the appropriate rate. We now summarize this conclusion as a corollary:

*Corollary 2:* Let  $Y = X + S + Z$  where  $S \sim \mathcal{N}(0, \sigma_s^2)$  and  $Z \sim \mathcal{N}(0, \sigma_z^2)$  is independent of  $(X, S)$ . Then

$$\sup_{n \rightarrow \infty} \limsup \frac{1}{n} \sum_{i=1}^n \mathbf{E}(S_i - \mathbf{E}\{S_i | Y^n\})^2 = D(R) \quad (41)$$

where the supremum on the left-hand side is over all sequences of encoders that allow reliable communication at rate  $R$ .

Finally, returning to our achievability scheme for the Gaussian case, it is interesting to compare the coding strategy here, which is essentially based on coherent subtraction (where  $X$  is negatively correlated to  $S$ ), as opposed to [23], where the

<sup>4</sup>The function  $D_S(\mathbf{E}(\varrho(R)))$  may not be a concave function of  $R$  in general.

strategy is based on coherent addition between  $X$  and  $S$ . Also, it is not surprising that in certain cases, the encoder in our setting is stochastic, whereas in [23] it is always deterministic.

## V. CAUSAL SIDE INFORMATION

In analogy to the Shannon model of causal side information [18], the question of trading off coding rate and mutual information,  $I(S^n; Y^n)$ , is applicable also when the channel input is a *causal* (stochastic) function of  $s^n$  and the message  $w$ , i.e.,

$$P(x^n|s^n, w) = \prod_{i=1}^n P(x_i|s^i, x^{i-1}, w). \quad (42)$$

We argue that the characterization of the achievable region of pairs  $\{(R, E)\}$  is the same as before, with the additional constraint that  $U$  is independent of  $S$ , and so,  $I(U; S) = 0$  in the rate inequality, and  $I(S; Y, U) = I(S; Y|U)$  in the mutual information inequality (cf. [11]).

As for the converse part, we first note that the auxiliary RV  $U_i = (W, Y^{i-1}, S_{i+1}^n)$  is independent of  $S_i$  whenever the encoder is as in eq. (42). In other words, given  $T$ , the RV's  $U = (U_T, T)$  and  $S = S_T$  are independent, i.e.,

$$P(u, s|t) = P(u|t)P(s|t) = P(u|t)P(s),$$

where the second equality follows again from the fact that  $S$  and  $T$  are independent. Thus,

$$\begin{aligned} P(u, s) &= \frac{1}{n} \sum_{t=1}^n P(u, s|t) \\ &= \left[ \frac{1}{n} \sum_{t=1}^n P(u|t) \right] P(s) = P(u)P(s). \end{aligned} \quad (43)$$

This means that  $I(U; S) = 0$  in the rate inequality, and  $I(S; Y, U) = I(S; Y|U)$  in the mutual information inequality.

As for the direct part, let  $U$  and  $X$  be random variables, where  $U$  is independent of  $S$ , the Markov relation  $U \rightarrow (X, S) \rightarrow Y$  is met, and the power constraint

$$\sum_{x, u, s} P(s)P(u)P(x|u, s)\phi(x) \leq \Gamma,$$

the rate constraint,  $R < I(U; Y)$ , and the mutual information constraint,  $E \geq I(S; Y|U)$ , are all met. Randomly select  $2^{nR}$  independent codewords  $\{u^n(1), \dots, u^n(2^{nR})\}$  with uniform distribution within the type class corresponding to  $P_U$ . Given a message  $w$  and a state sequence  $s^n$ ,  $x^n$  is generated by the product channel (42), where

$$P(x_i|x^{i-1}, s^i, w) = P(x_i|s_i, u_i(w)). \quad (44)$$

First, observe that this induces a memoryless channel from  $U^n$  to  $Y^n$ , given by  $P(y^n|u^n) = \prod_{i=1}^n P(y_i|u_i)$ , thus,  $U^n(W)$  is communicated reliably for  $R < I(U; Y)$ , by the ordinary coding theorem for DMC's. Regarding the mutual information,  $I(S^n; Y^n)$ , consider again the inequality (22). Now,  $I(S^n, U^n) = I(S^n; U^n(W)) = 0$ , as  $S^n$  and  $W$  are

independent. The second term,  $H(Y^n|U^n)$  is upper bounded by  $\sum_{i=1}^n H(Y_i|U_i)$ , as before, and

$$H(Y^n|U^n, S^n) = \sum_{i=1}^n H(Y_i|U_i, S_i) = nH(Y|U, S),$$

since  $P(y^n|u^n, s^n)$  is a DMC and the joint statistics of  $U$  and  $S$  are according to  $P(u, s) = P(u)P(s)$  (again, due to the independence between  $S^n$  and  $W$ ).

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