On the Corner Points of the Capacity Region of a Two-User Gaussian Interference Channel

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***Abstract*—This paper is focused on the two corner points of the capacity region of a two-user Gaussian interference channel (GIC). In a two-user GIC, the rate pairs where one user transmits its data at the single-user capacity (without interference), and the other at the largest rate for which reliable communication is still possible are called corner points. This paper provides new bounds on the corner points of the capacity region of a weak two-user GIC (i.e., when both cross-link gains in standard form are positive and below 1). A refinement of these bounds is considered for the case where the transmission rate of one user is within $\varepsilon > 0$ of the single-user capacity. The bounds on the corner points are asymptotically tight as the transmitted powers tend to infinity, and they are also useful for the case of moderate SNR and INR. New upper and lower bounds on the gap (denoted by Δ) between the sum-rate and the maximal achievable total rate at the two corner points are introduced. This is followed by an asymptotic analysis analogous to the study of the generalized degrees of freedom (where the SNR and INR scalings are coupled such that $\frac{\log(\mathbf{INR})}{\log(\mathbf{SNR})} = \alpha \geq 0$), leading to an asymptotic characterization of this gap which is exact for the whole range of α. The upper and lower bounds on Δ are asymptotically tight in the sense that they achieve the exact asymptotic characterization. Improved bounds on Δ are derived in the full version for finite SNR and INR, and their improved tightness is exemplified numerically. This conference paper presents in part the paper that is available at `http://arxiv.org/abs/1306.4934`, and it improves the bounds that were previously presented by the same author at Allerton 2013.**

1. Introduction

The two-user Gaussian interference channel (GIC) has been extensively treated in the literature during the last four decades (see, e.g., [5, Chapter 6] and [16]). For completeness and to set notation, the model of the two-user GIC in *standard form* is introduced shortly: this discrete-time, memoryless interference channel is characterized by the following relation between the inputs (X_1, X_2) and outputs (Y_1, Y_2):

$$Y_1 = X_1 + \sqrt{a_{12}}\, X_2 + Z_1 \quad (1)$$
$$Y_2 = \sqrt{a_{21}}\, X_1 + X_2 + Z_2 \quad (2)$$

where the interference coefficients a_{12} and a_{21} are time-invariant, the inputs and outputs are real valued, and Z_1 and Z_2 designate additive Gaussian noise samples that are independent of the inputs. Let $X_1^n \triangleq (X_{1,1}, \ldots, X_{1,n})$ and $X_2^n \triangleq (X_{2,1}, \ldots, X_{2,n})$ be the two transmitted codewords across the channel. No cooperation between the transmitters is allowed (so X_1^n, X_2^n are independent), nor between the receivers. The power constraints on the inputs are given by $\frac{1}{n}\sum_{i=1}^n \mathbb{E}[X_{1,i}^2] \leq P_1$ and $\frac{1}{n}\sum_{i=1}^n \mathbb{E}[X_{2,i}^2] \leq P_2$, for some $P_1, P_2 > 0$. The additive Gaussian noise samples of Z_1^n and Z_2^n are i.i.d. with zero mean and unit variance, and they are also independent of the inputs X_1^n and X_2^n. Furthermore, Z_1^n and Z_2^n can be assumed to be independent since the capacity region only depends on the marginal conditional *pdf*s of the interference channel (as the receivers are not cooperating). Finally, perfect synchronization between the pairs of transmitters and receivers is assumed, which implies that the capacity region is convex (time-sharing is possible).

In spite of the simplicity of this model, the exact characterization of the capacity region of a GIC is yet unknown, except for strong ([7], [14]) or very strong interference [2]. Specifically, the corner points of the capacity region have not yet been determined for GICs with weak interference; for GICs with mixed interference, only one corner point is known (see [9, Section 6.A] and [15, Section 2.C]).

The operational meaning of the study of the corner points of the capacity region for a two-user GIC is to explore the situation where one transmitter sends its information at the maximal achievable rate for a single-user (in the absence of interference), and the second transmitter maintains a data rate that enables reliable communication at the two receivers [3]. Two questions occur in this scenario:

Question 1: What is the maximal achievable rate of the second transmitter ?

Question 2: Does it enable the first receiver to reliably decode the messages of both transmitters ?

In his paper [3], Costa presented an approach suggesting that when one of the transmitters, say transmitter 1, sends its data over a two-user GIC at the maximal interference-free rate $R_1 = \frac{1}{2}\log(1 + P_1)$ bits per channel use, then the maximal rate R_2 of transmitter 2 is the rate that enables receiver 1 to decode both messages. The corner points of the capacity region are therefore related to a multiple-access channel where one of the receivers decodes correctly both messages. However, [11, pp. 1354–1355] pointed out a gap in the proof of [3, Theorem 1], though it was conjectured that the main result holds. This leads to the following conjecture:

Conjecture 1: For rate pairs (R_1, R_2) in the capacity region of a two-user GIC with positive interference coefficients a_{12}

and a_{21} and power constraints P_1 and P_2, let

$$C_1 \triangleq \frac{1}{2}\log(1+P_1), \qquad C_2 \triangleq \frac{1}{2}\log(1+P_2) \tag{3}$$

be the capacities of the single-user AWGN channels (in the absence of interference), and let

$$R_1^* \triangleq \frac{1}{2}\log\left(1+\frac{a_{21}P_1}{1+P_2}\right) \tag{4}$$

$$R_2^* \triangleq \frac{1}{2}\log\left(1+\frac{a_{12}P_2}{1+P_1}\right). \tag{5}$$

Then, the following is conjectured to hold for achieving reliable communication at both receivers:

1) If $R_2 \geq C_2 - \varepsilon$, for $\varepsilon > 0$, then $R_1 \leq R_1^* + \delta_1(\varepsilon)$ where $\delta_1(\varepsilon) \to 0$ as $\varepsilon \to 0$.
2) If $R_1 \geq C_1 - \varepsilon$, then $R_2 \leq R_2^* + \delta_2(\varepsilon)$ where $\delta_2(\varepsilon) \to 0$ as $\varepsilon \to 0$.

The discussion on Conjecture 1 is separated in [12, Section 1] into GICs with mixed, strong or one-sided interference. This is done by a restatement of some known results from [3], [4], [7], [9], [11], [13], [14] and [15].

In the following, we focus on GICs with weak interference (i.e., the channel model in (1) and (2) where $0 \leq a_{12}, a_{21} \leq 1$); for this class of GICs, the corner points of the capacity region are unknown yet. These corner points are studied in the converse part of this paper by relying on two existing outer bounds on the capacity region. The first bound is based on [6, Theorem 3] by Etkin et al., and it applies to two-user GICs with weak interference. The second bound is a specialization of the outer bound by Telatar and Tse [17] for two-user GICs (see [5, Section 6.7.2]). Although these are not the tightest existing outer bounds on the capacity region of GICs, these bounds are useful for our analysis and they lead to informative and simple closed-form expressions. Furthermore, the gap of the capacity region to each of these two outer bounds is asserted to be within one bit ([6], [17]). The interested reader is referred to various existing outer bounds on the capacity region of GICs (see, e.g., [1], [6], [8], [9], [10], [13], [15], [17]).

The structure of this paper is as follows: Conjecture 1 is considered in Section 2 for a two-user GIC with a two-sided weak interference. The excess rate for the sum-rate w.r.t. the corner points of the capacity region is considered in Section 3. A summary is provided in Section 4. The reader is referred to the full paper version in [12] that includes proofs, additional new improved bounds, discussions and further observations.

2. On the Corner Points of the Capacity Region of a Weak GIC

This section considers Conjecture 1 for a weak GIC. It is easy to verify that the points $(R_1, R_2) = (C_1, R_2^*)$ and (R_1^*, C_2) are both included in the capacity region of a weak GIC, and the corresponding receiver of the transmitter that operates at the single-user capacity can be designed to decode the messages of the two users. We proceed in the following to the converse part, which leads to the following statement:

Theorem 1: Consider a weak two-user GIC, and let C_1, C_2, R_1^* and R_2^* be as defined in (3)–(5). If $R_1 \geq C_1 - \varepsilon$ for an arbitrary $\varepsilon > 0$, then reliable communication requires that

$$R_2 \leq \min\Bigg\{R_2^* + \frac{1}{2}\log\left(1+\frac{P_2}{(1+a_{21}P_1)(1+a_{12}P_2)}\right) + 2\varepsilon, \quad \frac{1}{2}\log\left(1+\frac{P_2}{1+P_1}\right) + \left(1+\frac{1+P_1}{a_{21}P_2}\right)\varepsilon\Bigg\}. \tag{6}$$

Similarly, if $R_2 \geq C_2 - \varepsilon$, then

$$R_1 \leq \min\Bigg\{R_1^* + \frac{1}{2}\log\left(1+\frac{P_1}{(1+a_{21}P_1)(1+a_{12}P_2)}\right) + 2\varepsilon, \quad \frac{1}{2}\log\left(1+\frac{P_1}{1+P_2}\right) + \left(1+\frac{1+P_2}{a_{12}P_1}\right)\varepsilon\Bigg\}. \tag{7}$$

Consequently, the corner points of the capacity region are (R_1, C_2) and (C_1, R_2) where

$$R_1^* \leq R_1 \leq \min\left\{R_1^* + \frac{1}{2}\log\left(1+\frac{P_1}{(1+a_{21}P_1)(1+a_{12}P_2)}\right), \frac{1}{2}\log\left(1+\frac{P_1}{1+P_2}\right)\right\} \tag{8}$$

$$R_2^* \leq R_2 \leq \min\left\{R_2^* + \frac{1}{2}\log\left(1+\frac{P_2}{(1+a_{21}P_1)(1+a_{12}P_2)}\right), \frac{1}{2}\log\left(1+\frac{P_2}{1+P_1}\right)\right\}. \tag{9}$$

In the limit where P_1 and P_2 tend to infinity, which makes it an interference-limited channel,

1) Conjecture 1 holds, and it gives an asymptotically tight bound.
2) The rate pairs (C_1, R_2^*) and (R_1^*, C_2) form the corner points of the capacity region.
3) The answer to Question 2 is affirmative.

The proof of Theorem 1 relies on the two outer bounds on the capacity region that are given in [6, Theorem 3] and [8, Theorem 2]. The reader is referred to [12, Section 2] for a proof.

The following remark explains the advantage of the bound presented in Theorem 1 over the bound that was previously presented by the author at Allerton 2013. As we will see, this improvement has a significant effect on the tightness of the asymptotic results in Section 3 that are based on the analysis in [12, Section 3].

Remark 1: Consider a weak symmetric GIC where $P_1 = P_2 = P$ and $a_{12} = a_{21} = a \in (0,1)$. The corner points of the capacity region of this two-user interference channel are given by (C, R_c) and (R_c, C) where $C = \frac{1}{2}\log(1+P)$ is the capacity of a single-user AWGN channel with input power constraint P, and an additive Gaussian noise with zero mean and unit variance. Theorem 1 gives that

$$R_c \leq \min\left\{\frac{1}{2}\log\left(1+\frac{aP}{1+P}\right) + \frac{1}{2}\log\left(1+\frac{P}{(1+aP)^2}\right), \frac{1}{2}\log\left(1+\frac{P}{1+P}\right)\right\}. \tag{10}$$

In the following, we compare the two terms inside the minimization in (10) where the first term follows from the ETW bound in [6, Theorem 3], and the second term follows from Kramer's bound in [8, Theorem 2]. Straightforward algebra reveals that, for $a \in (0,1)$, the first term gives a better bound on R_{c} if and only if

$$P > \frac{2a^2 - a + 1 + \sqrt{5a^2 - 2a + 1}}{2a^2(1-a)}. \tag{11}$$

Hence, for an arbitrary cross-link gain $a \in (0,1)$ of a symmetric and weak two-user GIC, there exists a threshold for the SNR where above it, the ETW bound provides a better upper bound on the corner points; on the other hand, for values of SNR below this threshold, Kramer's bound provides a better bound on the corner points. The threshold for the SNR (P) tends to infinity if $a \to 0$ or $a \to 1$; this implies that in these two cases, Kramer's bound is better for all values of P. This is further discussed in the following:

1) If $a \to 0$ then, for every $P > 0$, the first term on the right-hand side of (10) tends to the capacity C; this forms a trivial upper bound on the value R_{c} of the corner point. On the other hand, the second term on the right-hand side of (10) gives the upper bound of $\frac{1}{2}\log\left(1 + \frac{P}{1+P}\right)$ which is smaller than C for all values of P. Note that the second term in (10) implies that, for a symmetric GIC, $R_{\text{c}} \leq \frac{1}{2}$ bit per channel use for all values of P. In fact, for a given P, the advantage of the second term in the extreme case where $a \to 0$ served as the initial motivation for incorporating it in Theorem 1.
2) If $a \to 1$ then, for every $P > 0$, the first term tends to $\frac{1}{2}\log\left(1 + \frac{P}{1+P}\right) + \frac{1}{2}\log\left(1 + \frac{P}{(1+P)^2}\right)$ which is larger than the second term. Hence, also in this case, the second term gives a better bound for all P.

Example 1: The condition in (11) is consistent with [9, Figs. 10 and 11], as explained in the following:

1) According to [9, Fig. 10], for $P = 7$ and $a = 0.2$, Kramer's outer bound gives a better upper bound on the corner point than the ETW bound. For $a = 0.2$, the complementary of the condition in (11) implies that Kramer's bound is indeed better in this respect for $P < 27.725$.
2) According to [9, Fig. 11], for $P = 100$ and $a = 0.1$, the ETW is nearly as tight as Kramer's bound in providing an upper bound on the corner point. For $a = 0.1$, the complementary of the condition in (11) implies that Kramer's outer bound gives a better upper bound on the corner point than the ETW bound if $P < 102.33$; hence, for $P = 100$, there is only a slight advantage to Kramer's bound over the ETW bound that is not visible in [9, Fig. 11]: Kramer's bound gives an upper bound on R_{c} that is equal to 0.4964 bits per channel use, and the ETW bound gives an upper bound of 0.5026 bits per channel use.

3. The Excess Rate for the Sum-Rate w.r.t. the Corner Points of the Capacity Region

The sum-rate of a mixed, strong or one-sided GIC is attained at a corner point of its capacity region. This is in contrast to a (two-sided) weak GIC whose sum-rate is not attained at a corner point of its capacity region. It is therefore of interest to examine the excess rate for the sum-rate w.r.t. these corner points by measuring the gap between the sum-rate (C_{sum}) and the maximal total rate ($R_1 + R_2$) at the corner points of the capacity region:

$$\Delta \triangleq C_{\text{sum}} - \max\bigl\{R_1 + R_2 \colon (R_1, R_2) \text{ is a corner point}\bigr\}. \tag{12}$$

The parameter Δ measures the excess rate for the sum-rate w.r.t. the case where one transmitter operates at its single-user capacity, and the other reduces its rate to the point where reliable communication is achievable. We have $\Delta = 0$ for mixed, strong and one-sided GICs. This section derives bounds on Δ for weak GICs, and it also provides an asymptotic analysis analogous to the study of the generalized degrees of freedom (where the SNR and INR scalings are coupled such that $\frac{\log(\text{INR})}{\log(\text{SNR})} = \alpha \geq 0$). This leads to an asymptotic characterization of this gap which is demonstrated to be exact for the whole range of α. The upper and lower bounds on Δ are shown to be asymptotically tight in the sense that they achieve the exact asymptotic characterization. Improvements of the bounds on Δ are derived in [12] for finite SNR and INR, and these bounds are exemplified numerically in [12].

A. An Analogous Measure to the Generalized Degrees of Freedom and its Implications

Consider a two-user symmetric GIC whose interference coefficient a scales like $P^{\alpha-1}$ for some fixed value of $\alpha \geq 0$. For this GIC, the GDOF is defined as the asymptotic limit of the normalized sum-rate $\frac{C_{\text{sum}}(P, P^{\alpha-1})}{\log P}$ when $P \to \infty$. This GDOF refers to the case where the SNR (P) tends to infinity, and the interference to noise ratio ($\text{INR} = aP$) scales such that $\frac{\log(\text{INR})}{\log(\text{SNR})} = \alpha$ is kept fixed for a non-negative α. The GDOF for a two-user symmetric GIC (without feedback) gets the following closed-form expression:

$$d^\star(\alpha) \triangleq \lim_{P \to \infty} \frac{C_{\text{sum}}(P, P^{\alpha-1})}{\log P} \tag{13}$$

$$= \begin{cases} 1 - \alpha, & \text{if } 0 \leq \alpha < \frac{1}{2} \\ \alpha, & \text{if } \frac{1}{2} \leq \alpha < \frac{2}{3} \\ 1 - \frac{\alpha}{2}, & \text{if } \frac{2}{3} \leq \alpha < 1 \\ \frac{\alpha}{2}, & \text{if } 1 \leq \alpha < 2 \\ 1, & \text{if } \alpha \geq 2 \end{cases}. \tag{14}$$

For large P, let us consider in an analogous way the asymptotic scaling of the normalized excess rate for the sum-rate w.r.t. the corner points of the capacity region. To this end, we study the asymptotic limit of the ratio $\frac{\Delta(P, P^{\alpha-1})}{\log P}$ for a fixed $\alpha \geq 0$ when P tends to infinity. Similarly to (13), the denominator of this ratio is equal to the asymptotic sum-rate of two parallel AWGN channels with no interference. However, in the latter expression, the excess rate for the sum-rate w.r.t. the corner points is replacing the sum-rate that appears in the numerator on the right-hand side of (13). Correspondingly, for an arbitrary $\alpha \geq 0$, let us define

$$\delta(\alpha) \triangleq \lim_{P\to\infty} \frac{\Delta(P, P^{\alpha-1})}{\log P}. \tag{15}$$

provided that this limit exists. In the following, we demonstrate the existence of this limit and provide a closed-form expression for δ.

Theorem 2: The limit in (15) exists for every $\alpha \geq 0$, and the function δ admits the following closed-form expression:

$$\delta(\alpha) = \begin{cases} \left|\frac{1}{2} - \alpha\right|, & \text{if } 0 \leq \alpha < \frac{2}{3} \\ \frac{1-\alpha}{2}, & \text{if } \frac{2}{3} \leq \alpha < 1 \\ 0, & \text{if } \alpha \geq 1 \end{cases}. \tag{16}$$

The proof of Theorem 2 is provided in [12].

Equations (14) and (16) imply that, for an interference level $\alpha \in [0, 1]$, the difference between the GDOF (denoted here by $d(\alpha)$) and $\delta(\alpha)$ is half a bit per channel use.

The following are some implications of Theorem 2:

- The GDOF is known to be a non-monotonic function of α over the interval [0,1] (see [6, pp. 5542–5543] and (14)). From (16), it also follows that δ is a non-monotonic function over this interval. For $P > 1$, the cross-link gain $a = P^{\alpha-1}$ forms a monotonic increasing function of $\alpha \in [0, 1]$, and it is a one-to-one mapping from the interval $[0, 1]$ to itself. This implies that, for large P, the excess rate for the sum-rate w.r.t. the corner points (denoted by $\Delta(P, a)$) is a non-monotonic function of a over the interval [0,1]. This observation is supported by numerical results in [12, Section 4].
- For a weak and symmetric two-user GIC, the excess rate for the sum-rate w.r.t. the corner points is the difference between the sum-rate of the capacity region and the total rate at any of the two corner points of the capacity region. According to Theorem 1, for large P, the total rate at a corner point is an increasing function of $a \in (0, 1]$. Although it is known that, for large P, the sum-rate of the capacity region is not monotonic decreasing in a, a priori, there was a possibility that by subtracting from it a monotonic increasing function in a, the difference (that is equal to the excess rate Δ) would be monotonic decreasing in a. However, it is shown not to be the case. The fact that, for large P, the excess rate $\Delta(P, a)$ is not a monotonic decreasing function of a is a stronger property than the non-monotonicity of the sum-rate.
- For large P, the excess rate $\Delta(P, a)$ jumps drastically when the cross-link gain just varies slightly from $\frac{1}{\sqrt{P}}$ to $\frac{1}{\sqrt[3]{P}}$. This observation follows from (16), due to the fact that δ obtains its maximum and minimum, respectively, at $\alpha = \frac{1}{2}$ and $\alpha = \frac{2}{3}$ (this refers, respectively, to $a = \frac{1}{\sqrt{P}}$ and $\frac{1}{\sqrt[3]{P}}$). This observation is supported by numerical results in [12, Section 4].

Various additional implications and numerical results that support all these implications are considered in [12].

B. New Simple Bounds on Δ for finite SNR and INR

The following theorem introduces new bounds on the excess rate Δ for finite SNR and INR, and these bounds are demonstrated to be asymptotically tight in the sense of achieving the asymptotic result in Theorem 2.

Theorem 3: Consider a two-user symmetric GIC with weak interference in standard form where $P_1 = P_2 = P$ and $a_{12} = a_{21} = a \in (0, 1]$. Then,

$$\begin{aligned}\Delta(P, a) \leq \frac{1}{2} \Bigg[&\min\left\{ \log(1 + P) + \log\left(1 + \frac{P}{1 + aP}\right), \right. \\ &\left. 2\log\left(1 + aP + \frac{P}{1 + aP}\right) \right\} \\ &- \log\big(1 + (1 + a)P\big) \Bigg]\end{aligned}$$

and, if $P \geq 2.551$,

$$\begin{aligned}\Delta(P, a) \geq \frac{1}{2} \Bigg[&\min\left\{ \log\big(1 + (a + 1)P\big) + \log\left(1 + \frac{P}{1 + aP}\right), \right. \\ &\left. 2\log\left(1 + aP + \frac{P}{1 + aP}\right) \right\} \\ &- \min\left\{ \log\big(1 + (a + 1)P\big) + \log\left(1 + \frac{P}{(1 + aP)^2}\right), \right. \\ &\left. \log(1 + 2P) \right\} \Bigg] - 1.\end{aligned}$$

Furthermore, the upper and lower bounds on $\Delta(P, P^{\alpha-1})$ are asymptotically tight, as we let P tend to infinity, in the sense of achieving the asymptotic limit $\delta(\alpha)$ for an arbitrary $\alpha \geq 0$.

The proof of this theorem is provided in [12, Section 3].

Corollary 1: Consider a two-user symmetric GIC with weak interference in standard form where $P_1 = P_2 = P$ and $a_{12} = a_{21} = a \in (0, 1]$. Then,

$$\frac{1}{2}\log\left(1 + \frac{1}{a}\right) - 1 \leq \lim_{P\to\infty} \Delta(P, a) \leq \frac{1}{2}\log\left(\frac{1}{a}\right)$$

where the base of the log is 2.

- This provides the correct scaling of $\Delta(P, a)$ for large P and fixed cross-link gain a.
- The gap between the upper and lower bounds is at most 1 bit, and the bounds are tight at $a = 1$ (both bounds are zero). Indeed, for $a = 1$, the capacity region is the polyhedron that is obtained by intersecting the capacity regions of two Gaussian multiple-access channels; hence, $\Delta(P, 1) = 0$ for $P > 0$.

Consider the capacity region of a weak and symmetric two-user GIC, and the bounds on the excess rate for the sum-rate w.r.t. the corner points in Theorem 3. In this case, the transmission rate of one of the users is assumed to be equal to the single-user capacity of the respective AWGN channel. Consider now the case where the transmission rate of this user is reduced by no more than $\varepsilon > 0$, so it is within ε of the single-user capacity. Then, from Theorem 1, it follows that the upper bound on the transmission rate of the other user cannot increase by more than

$$f(\varepsilon) \triangleq \max \left\{ 2\varepsilon, \, \left(1 + \frac{1+P}{aP}\right) \varepsilon \right\}.$$

Consequently, the lower bound on the excess rate for the sum-rate in Theorem 3 is reduced by no more than $f(\varepsilon)$. Furthermore, the upper bound on this excess rate cannot increase by more than ε (note that if the first user reduces its transmission rate by no more than ε, then the other user can stay at the same transmission rate; overall, the total transmission rate it decreased by no more than ε, and consequently the excess rate for the sum-rate cannot increase by more than ε). Revisiting the analysis in this sub-section by introducing a positive $\varepsilon \triangleq \varepsilon(P)$ to the calculations, before taking the limit of P to infinity, leads to the conclusion that the corresponding characterization of δ in (16) stays unaffected as long as

$$\lim_{P \to \infty} \frac{\varepsilon(P)}{\log P} = 0$$

which then implies that

$$\lim_{P \to \infty} \frac{f\big(\varepsilon(P)\big)}{\log P} = 0$$

when the value of the cross-link gain a is fixed. For example, this happens to be the case if ε scales like $(\log P)^\beta$ for an arbitrary $\beta \in (0, 1)$ (so, in the limit where $P \to \infty$, we have $\varepsilon(P) \to \infty$ but $\frac{\varepsilon(P)}{\log P} \to 0$).

4. Summary

This paper considers the corner points of the capacity region of a two-user Gaussian interference channel (GIC). The operational meaning of the corner points is a study of the situation where one user sends its information at the single-user capacity (in the absence of interference), and the other user transmits its data at the largest rate for which reliable communication is possible at the two non-cooperating receivers. The approach used in this work for the study of the corner points relies on some existing outer bounds on the capacity region of a two-user GIC.

This work provides bounds on the corner points of the capacity region of a two-user GIC with weak interference (see Theorem 1). The excess rate for the sum-rate w.r.t. the corner points of the capacity region (denoted by Δ) is introduced and studied in this paper. An asymptotic analysis of this gap is provided in [12], analogously to the study of the GDOF (where the SNR and INR scalings are coupled), leading to Theorem 2. It is shown to be tight in the whole range of this scaling. Upper and lower bounds on Δ are introduced in Theorem 3, for finite values of SNR and INR, based on the analysis in [12]. These upper and lower bounds on Δ are demonstrated to be asymptotically tight in the sense of reproducing the exact asymptotic characterization of this gap in Theorem 2. Moreover, the bounds in Theorem 3 are further improved in [12] for finite SNR and INR, and these improvements are exemplified (this part of the work is skipped here due to space limitations). The reader is referred to [12] that includes proofs, additional improved bounds and numerical results, discussions and further observations.

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References

[1] V. S. Annapureddy and V. V. Veeravalli, "Gaussian interference networks: sum capacity in the low-interference regime and new outer bounds on the capacity region," *IEEE Trans. on Information Theory*, vol. 55, no. 7, pp. 3032–3050, July 2009.

[2] A. B. Carleial, "A case where interference does not reduce capacity," *IEEE Trans. on Information Theory*, vol. 21, no. 5, pp. 569–570, September 1975.

[3] M. H. M. Costa, "On the Gaussian interference channel," *IEEE Trans. on Information Theory*, vol. 31, no. 5, pp. 607–615, September 1985.

[4] M. H. M. Costa, "Noisebergs in Z- Gaussian interference channels," *Proceedings of the 2011 Information Theory and Applications Workshop (ITA)*, p. 1–6, San-Diego, California, USA, February 2011.

[5] A. El Gamal and Y. H. Kim, *Network Information Theory*, Cambridge university press, 2011.

[6] R. Etkin, D. Tse and H. Wang, "Gaussian interference channel capacity to within one bit" *IEEE Trans. on Information Theory*, vol. 54, no. 12, pp. 5534–5562, December 2008.

[7] T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. on Information Theory*, vol. 27, no. 1, pp. 49–60, January 1981.

[8] G. Kramer, "Outer bounds on the capacity of Gaussian interference channels," *IEEE Trans. on Information Theory*, vol. 50, no. 3, pp. 581–586, March 2004.

[9] A. S. Motahari and A. K. Khandani, "Capacity bounds for the Gaussian interference channel," *IEEE Trans. on Information Theory*, vol. 55, no. 2, pp. 620–643, February 2009.

[10] J. Nam and G. Caire, "A new outer bound on the capacity region of Gaussian interference channels," *Proceedings of the 2012 IEEE International Symposium on Information Theory*, pp. 2816–2820, MIT, Boston, MA, USA, July 2012.

[11] I. Sason, "On achievable rate regions for the Gaussian interference channels," *IEEE Trans. on Information Theory*, vol. 50, no. 6, pp. 1345–1356, June 2004.

[12] I. Sason, "On the corner points of the capacity region of a Gaussian interference channel," submitted to the *IEEE Trans. on Information Theory*, August 2013. [Online]. Available: http://arxiv.org/abs/1306.4934.

[13] H. Sato, "On degraded Gaussian two-user channels," *IEEE Trans. on Information Theory*, vol. 24, no. 5, pp. 637–640, September 1978.

[14] H. Sato, "The capacity of the Gaussian interference channel under strong interference," *IEEE Trans. on Information Theory*, vol. 27, no. 6, pp. 786–788, November 1981.

[15] X. Shang, G. Kramer and B. Chen, "A new outer bound and the noisy-interference sum-rate capacity for Gaussian interference channels," *IEEE Trans. on Information Theory*, vol. 55, no. 2, pp. 689–699, February 2009.

[16] X. Shang and B. Chen, "Two-user Gaussian interference channels: an information theoretic point of view," *Foundations and Trends in Communications and Information Theory*, vol. 10, no. 3, pp. 247–378, December 2013.

[17] E. Telatar and D. Tse, "Bounds on the capacity region of a class of interference channels," *Proceedings of the 2007 IEEE International Symposium on Information Theory*, pp. 2871–2874, Nice, France, June 2007.