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On Counting Graph Homomorphisms by Entropy Arguments

(Extended abstract)

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Abstract

This extended abstract applies properties of Shannon entropy to derive a lower bound on the number of homomorphisms from a complete bipartite graph to any bipartite graph. Further upper and lower bounds on homomorphism counts between arbitrary bipartite graphs, proofs and observations, are provided in the full version of this work, available as the arXiv preprint *Counting Graph Homomorphisms in Bipartite Settings (https://arxiv.org/abs/2508.06977)*.

1 Introduction

Combinatorial techniques serve a vital role in addressing problems in information theory and coding theory. Several examples where tools from combinatorics and graph theory are used to study fundamental problems in information theory were briefly surveyed in [1]. Many classical and modern results in information theory can be also derived through a combinatorial perspective, particularly using the method of types [2, 3]. The reverse direction, applying information-theoretic tools to obtain combinatorial results, has proven to be equally fruitful. Notably, Shannon entropy has significantly deepened the understanding of the structural and quantitative properties of combinatorial objects by enabling concise and often elegant proofs of classical results in combinatorics (see, e.g., [4, Chapter 37], [5], [6, Chapter 22], and [7–18]).

This extended abstract takes the latter direction, employing the Shannon entropy to derive a lower bound on the number of graph homomorphisms from a complete bipartite graph to an arbitrary bipartite graph. A full version of this work, which derives both upper and lower bounds on homomorphism counts between *arbitrary* bipartite graphs, together with complete proofs and additional observations and numerical results, is available as an arXiv preprint [18].

Graph homomorphisms provide a powerful framework for the study of graph mappings, revealing insights into structural properties, colorings, and symmetries. Their applications

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span multiple disciplines, including statistical physics, where they model spin systems [19], and computational complexity, where they underpin constraint satisfaction problems [20]. Research work has led to significant progress in understanding the problem of counting graph homomorphisms, a subject of both theoretical and practical relevance (see [7, 20–35]).

2 Preliminaries

In the sequel, let $\mathsf{V}(\mathsf{G})$ and $\mathsf{E}(\mathsf{G})$ denote the vertex set and edge set of a graph G, respectively. For adjacent vertices $u, v \in \mathsf{V}(\mathsf{G})$, let $e = \{u, v\} \in \mathsf{E}(\mathsf{G})$ denote the edge connecting them.

Let F and G be finite, simple, and undirected graphs. A *homomorphism* from a source graph F to a target graph G, denoted by $\mathsf{F} \to \mathsf{G}$, is a mapping $\psi\colon \mathsf{V}(\mathsf{F}) \to \mathsf{V}(\mathsf{G})$ such that every edge in F is mapped to an edge in G:

$$\{u, v\} \in \mathsf{E}(\mathsf{F}) \implies \{\psi(u), \psi(v)\} \in \mathsf{E}(\mathsf{G}). \tag{1}$$

The following establishes connections between graph homomorphisms and classical graph invariants. Let $\omega(\mathsf{G})$ and $\chi(\mathsf{G})$ denote the clique number and chromatic number of a graph G, respectively. Then, $\omega(\mathsf{G})$ is the largest $k \in \mathbb{N}$ such that there exists a homomorphism $\mathsf{K}_k \to \mathsf{G}$, and $\chi(\mathsf{G})$ is the smallest $k \in \mathbb{N}$ such that a homomorphism $\mathsf{G} \to \mathsf{K}_k$ exists. Consequently, such graph homomorphisms characterize the independence number, clique number, and chromatic number of a graph, problems known to be NP-hard [36].

Let $\mathrm{Hom}(\mathsf{F}, \mathsf{G})$ denote the set of all the homomorphisms $\mathsf{F} \to \mathsf{G}$, and define

$$\mathrm{hom}(\mathsf{F}, \mathsf{G}) \triangleq \bigl|\mathrm{Hom}(\mathsf{F}, \mathsf{G})\bigr| \tag{2}$$

as the number of such graph homomorphisms. These are called *homomorphism numbers*.

In addition to homomorphism numbers, we now introduce *homomorphism densities*, which are closely related.

Definition 2.1 (Homomorphism densities)**.** Let F and G be graphs. Let $v(\mathsf{F}) \triangleq |\,\mathsf{V}(\mathsf{F})|$ and $v(\mathsf{G}) \triangleq |\,\mathsf{V}(\mathsf{G})|$. The F-*homomorphism density in* G (or simply F-density in G) is the probability that a uniformly random mapping from $\mathsf{V}(\mathsf{F})$ to $\mathsf{V}(\mathsf{G})$ induces a graph homomorphism from F to G, i.e., it is given by

$$t(\mathsf{F}, \mathsf{G}) \triangleq \frac{\mathrm{hom}(\mathsf{F}, \mathsf{G})}{v(\mathsf{G})^{\,v(\mathsf{F})}}. \tag{3}$$

By Definition 2.1, we have $t(\mathsf{K}_1, \mathsf{G}) = 1$, and

$$t(\mathsf{K}_2, \mathsf{G}) = \frac{2\,e(\mathsf{G})}{v(\mathsf{G})^2}, \tag{4}$$

where $e(\mathsf{G}) \triangleq |\mathsf{E}(\mathsf{G})|$.

Before going into technical details, we highlight why counting graph homomorphisms is an important problem. In extremal graph theory and in the study of graph limits, homomorphism counts and homomorphism densities are basic building blocks; this viewpoint goes back to work of Lovász and collaborators [25]. In computer science, they appear naturally in the context of constraint satisfaction problems, where deciding whether a homomorphism exists, or counting how many there are, is a central computational task. And in statistical physics, partition functions of spin models can be expressed as weighted homomorphism counts, so methods and insights from this area translate directly. For background, there are excellent references by Lovász [25], by Borgs–Chayes–Lovász–Sós–Vesztergombi [22], and more recently by Yufei Zhao's textbook [29], which connect these perspectives very nicely.

3 Lower bound

In the following, we rely on properties of the Shannon entropy to derive a lower bound on the number of homomorphisms from the complete bipartite graph to any bipartite graph, and examine its tightness by comparing it to the specialized lower bound that holds by the satisfiability of Sidorenko's conjecture in the examined setting. Familiarity with Shannon entropy and its basic properties is assumed, following standard notation (see, e.g., [37, Chapter 3]).

Proposition 3.1 (Number of graph homomorphisms)**.** Let G be bipartite with partite sizes n_1, n_2 and with an edge density

$$\delta \triangleq \frac{|\,\mathsf{E}(\mathsf{G})|}{n_1 n_2} \in [0,1].$$

Then, for all $s, t \in \mathbb{N}$,

$$\begin{aligned} \hom(\mathsf{K}_{s,t}, \mathsf{G}) &\geq \delta^{st} \bigl(n_1^s n_2^t + n_1^t n_2^s\bigr) && (5) \\ &= \delta^{st} \hom(\mathsf{K}_{s,t}, \mathsf{K}_{n_1,n_2}) && (6) \end{aligned}$$

Proof. Let $\mathcal{U}$ and $\mathcal{V}$ denote the partite vertex sets of the simple bipartite graph G, where $|\mathcal{U}| = n_1$ and $|\mathcal{V}| = n_2$. Let (U, V) be a random vector taking values in $\mathcal{U} \times \mathcal{V}$, and suppose that $\{U, V\}$ is distributed uniformly at random on the edges of G. Then, the joint entropy of (U, V) is given by

$$\mathrm{H}(U, V) = \log \bigl|\mathsf{E}(\mathsf{G})\bigr| = \log(\delta n_1 n_2). \tag{7}$$

The random vector (U, V) can be sampled by first sampling the value $U = u$ from the marginal probability mass function (PMF) of U, denoted by P_U, and then sampling V from the conditional PMF $\mathsf{P}_{V|U}(\cdot|u)$. Construct a random vector $(U_1, \ldots, U_s, V_1, \ldots, V_t)$ as follows:

- Let $V_1, \ldots, V_t$ be conditionally independent and identically distributed (i.i.d.) given U, having the conditional PMF

$$\mathsf{P}_{V_1,\ldots,V_t|U}(v_1, \ldots, v_t|u) = \prod_{j=1}^{t} \mathsf{P}_{V|U}(v_j|u), \quad \forall\, u \in \mathcal{U}, \;\; (v_1, \ldots, v_t) \in \mathcal{V}^t. \tag{8}$$

- Let $U_1, \ldots, U_s$ be conditionally i.i.d. given $(V_1, \ldots, V_t)$, having the conditional PMF

$$\begin{aligned} &\mathsf{P}_{U_1,\ldots,U_s|V_1,\ldots,V_t}(u_1, \ldots, u_s|v_1, \ldots, v_t) \\ &\quad = \prod_{i=1}^{s} \mathsf{P}_{U_i|V_1,\ldots,V_t}(u_i|v_1, \ldots, v_t), \quad \forall\, (u_1, \ldots, u_s) \in \mathcal{U}^s, \;\; (v_1, \ldots, v_t) \in \mathcal{V}^t, \end{aligned} \tag{9}$$

where the conditional PMFs on the right-hand side of (9) are given by

$$\begin{aligned}
&\mathsf{P}_{U_i|V_1,\ldots,V_t}(u|v_1,\ldots,v_t) \\
&= \frac{\mathsf{P}_U(u) \prod_{j=1}^{t} \mathsf{P}_{V|U}(v_j|u)}{\sum_{u' \in \mathcal{U}} \left\{ \mathsf{P}_U(u') \prod_{j=1}^{t} \mathsf{P}_{V|U}(v_j|u') \right\}}, \quad \forall\, u \in \mathcal{U}, \;\; (v_1,\ldots,v_t) \in \mathcal{V}^t, \;\; i \in [s]. \qquad (10)
\end{aligned}$$

By the construction of the random vector $(U_1,\ldots,U_s,V_1,\ldots,V_t)$ in (8)–(10), the following holds (see [18]):

1) $U_1,\ldots,U_s$ are identically distributed random variables, and $U_i \sim U$ (i.e., $\mathsf{P}_{U_i} = \mathsf{P}_U$) for all $i \in [s]$.

2) For all $i \in [s]$ and $j \in [t]$, $(U_i, V_j) \sim (U, V)$, and $(U_i, V_1, \ldots, V_t) \sim (U, V_1, \ldots, V_t)$.

Recall that, by assumption, the bipartite graph G has no isolated vertices, thus making the above construction feasible.

The joint entropy of the random subvector $(U_1, V_1, \ldots, V_t)$ then satisfies

$$\begin{aligned}
\mathrm{H}(U_1, V_1, \ldots, V_t) &= \mathrm{H}(U_1) + \sum_{j=1}^{t} \mathrm{H}(V_j | U_1) && (11) \\
&= \mathrm{H}(U) + t\, \mathrm{H}(V|U) && (12) \\
&= t\, \mathrm{H}(U,V) - (t-1)\, \mathrm{H}(U) && (13) \\
&= t \log(\delta n_1 n_2) - (t-1)\, \mathrm{H}(U) && (14) \\
&\geq t \log(\delta n_1 n_2) - (t-1) \log n_1 && (15) \\
&= \log(\delta^t n_1 n_2^t), && (16)
\end{aligned}$$

where (11) holds by the chain rule of the Shannon entropy, since (by construction) $V_1,\ldots,V_t$ are conditionally independent given U (see (8)) and since $(U_1, V_1, \ldots, V_t) \sim (U, V_1, \ldots, V_t)$; (12) relies on the property $(U_i, V_j) \sim (U, V)$; (13) holds by a second application of the chain rule; (14) holds by (7), and finally (15) follows from the uniform bound, which states that if X is a discrete random variable supported on a finite set $\mathcal{S}$, then $\mathrm{H}(X) \leq \log|\mathcal{S}|$. In this case, $\mathrm{H}(U) \leq \log|\mathcal{U}| = \log n_1$. Consequently, the joint entropy of the random vector $(U_1,\ldots,U_s,V_1,\ldots,V_t)$ satisfies

$$\begin{aligned}
\mathrm{H}(U_1,\ldots,U_s,V_1,\ldots,V_t) &= \mathrm{H}(V_1,\ldots,V_t) + \sum_{i=1}^{s} \mathrm{H}(U_i | V_1,\ldots,V_t) && (17) \\
&= \mathrm{H}(V_1,\ldots,V_t) + s\, \mathrm{H}(U_1 | V_1,\ldots,V_t) && (18) \\
&= s\big[\mathrm{H}(V_1,\ldots,V_t) + \mathrm{H}(U_1|V_1,\ldots,V_t)\big] - (s-1)\, \mathrm{H}(V_1,\ldots,V_t) && (19) \\
&= s\, \mathrm{H}(U_1, V_1,\ldots,V_t) - (s-1)\, \mathrm{H}(V_1,\ldots,V_t) && (20) \\
&\geq s \log(\delta^t n_1 n_2^t) - (s-1)\, \mathrm{H}(V_1,\ldots,V_t) && (21) \\
&\geq s \log(\delta^t n_1 n_2^t) - (s-1) \log(n_2^t) && (22) \\
&= \log(\delta^{st} n_1^s n_2^t), && (23)
\end{aligned}$$

where (17) holds by the chain rule and since (by construction) the random variables $U_1, \ldots, U_s$ are conditionally independent given $V_1, \ldots, V_t$ (see (9)); (18) holds since, by construction, all the U_i's ($i \in [s]$) are identically conditionally distributed given $(V_1, \ldots, V_t)$ (see (10)); (19) is simple algebra; (20) holds by another use of the chain rule; (21) holds by (16), and finally (22) holds by the uniform bound which implies that $\mathrm{H}(V_1, \ldots, V_t) \leq \log(|\mathcal{V}|^t) = \log(n_2^t)$.

Each vector $(U_1, \ldots, U_s, V_1, \ldots, V_t)$ can be mapped to a homomorphism from $\mathsf{K}_{s,t}$ to G via an injective mapping, where each vertex in the partite set of size s in $\mathsf{K}_{s,t}$ is mapped to a vertex from the partite set of size n_1 in G, and each vertex in the partite set of size t in $\mathsf{K}_{s,t}$ is mapped to a vertex in the second partite set of size n_2 in G. Denote the subset of such homomorphisms by $\mathcal{H}_1 \subseteq \mathrm{Hom}(\mathsf{K}_{s,t}, \mathsf{G})$. To define this mapping explicitly, label the vertices of the complete bipartite graph $\mathsf{K}_{s,t}$ by the elements of $[s+t]$, assigning the labels $1, \ldots, s$ to the vertices in the partite set of size s, and the labels $s+1, \ldots, s+t$ to those in the second partite set of size t. For every $i \in [s]$, map vertex $i \in \mathsf{V}(\mathsf{K}_{s,t})$ to vertex $U_i \in \mathsf{V}(\mathsf{G})$, and for every $j \in [t]$, map vertex $s+j \in \mathsf{V}(\mathsf{K}_{s,t})$ to vertex $V_j \in \mathsf{V}(\mathsf{G})$. Under this mapping, each edge $\{i, s+j\} \in \mathsf{E}(\mathsf{K}_{s,t})$ is mapped to the edge $\{U_i, V_j\} \in \mathsf{E}(\mathsf{G})$, thereby defining a homomorphism $\mathsf{K}_{s,t} \to \mathsf{G}$ in $\mathcal{H}_1$, since $\{U_i, V_j\} \in \mathsf{E}(\mathsf{G})$ holds by construction (see (10)). Recall that in (10), $\{U, V\}$ is uniformly distributed over the edges of the graph G, where $U \in \mathcal{U}$ and $V \in \mathcal{V}$ (by construction), P_U denotes the marginal PMF of U, and $\mathsf{P}_{V|U}$ denotes the conditional PMF of V given U. The suggested mapping is injective since it maps distinct such vectors to distinct homomorphisms in $\mathcal{H}_1 \subseteq \mathrm{Hom}(\mathsf{K}_{s,t}, \mathsf{G})$. By (2) and the uniform bound, it then follows that

$$\mathrm{H}(U_1, \ldots, U_s, V_1, \ldots, V_t) \leq \log |\mathcal{H}_1|. \tag{24}$$

Combining (23) and (24) yields

$$|\mathcal{H}_1| \geq \delta^{st} n_1^s n_2^t. \tag{25}$$

Likewise, denote by $\mathcal{H}_2$ the subset of homomorphisms $\mathsf{K}_{s,t} \to \mathsf{G}$, where each vertex in the partite set of size s in $\mathsf{K}_{s,t}$ is mapped to a vertex in the partite set of size n_2 in G, and each vertex in the partite set of size t in $\mathsf{K}_{s,t}$ is mapped to a vertex in the other partite set of size n_1 in G. Similarly to (25), we get

$$|\mathcal{H}_2| \geq \delta^{st} n_1^t n_2^s. \tag{26}$$

Since the subsets $\mathcal{H}_1$ and $\mathcal{H}_2$ form a partition of the set $\mathrm{Hom}(\mathsf{K}_{s,t}, \mathsf{G})$ (as G is a nonempty bipartite graph), it follows from (25) and (26) that

$$\mathrm{hom}(\mathsf{K}_{s,t}, \mathsf{G}) = |\mathcal{H}_1| + |\mathcal{H}_2| \geq \delta^{st} (n_1^s n_2^t + n_1^t n_2^s), \tag{27}$$

which proves the leftmost inequality in (5). □

An equivalent form of the leftmost inequality in (5) is next obtained, using the identity

$$\mathrm{hom}(\mathsf{K}_{s,t}, \mathsf{K}_{n_1,n_2}) = n_1^s n_2^t + n_1^t n_2^s. \tag{28}$$

Corollary 3.1. Let G be a simple bipartite graph with partite sets of sizes n_1 and n_2, no isolated vertices, and $\delta n_1 n_2$ edges for some $\delta \in (0, 1]$. Then, for all $s, t \in \mathbb{N}$,

$$\mathrm{hom}(\mathsf{K}_{s,t}, \mathsf{G}) \geq \delta^{st} \, \mathrm{hom}(\mathsf{K}_{s,t}, \mathsf{K}_{n_1,n_2}). \tag{29}$$

In particular, if $\mathsf{G} = \mathsf{K}_{n_1,n_2}$, then inequality (29) holds with equality (as $\delta = 1$).

Definition 3.1 (Sidorenko graph)**.** A graph H is said to be Sidorenko if it has the property that for every graph G

$$t(\mathsf{H},\mathsf{G}) \geq t(\mathsf{K}_2,\mathsf{G})^{\,e(\mathsf{H})}, \tag{30}$$

where $e(\mathsf{H}) \triangleq |\mathsf{E}(\mathsf{H})|$, or, by (3) and (4),

$$\frac{\hom(\mathsf{H},\mathsf{G})}{v(\mathsf{G})^{\,v(\mathsf{H})}} \geq \left(\frac{2\,e(\mathsf{G})}{v(\mathsf{G})^2}\right)^{e(\mathsf{H})}. \tag{31}$$

Therefore, a graph H is said to be Sidorenko if the probability that a random uniform mapping from its vertex set $\mathsf{V}(\mathsf{H})$ to the vertex set of any graph G forms a homomorphism is at least the product over all edges in H of the probability that the edge is mapped to an edge in G.

Sidorenko's conjecture states that every bipartite graph is Sidorenko. Although this conjecture remains an open problem in its full generality, it is known that every bipartite graph containing a vertex adjacent to all vertices in its other part is Sidorenko (see, e.g., [29, Theorem 5.5.14], originally proved in [30], and simplified in [31]). Additional classes of bipartite graphs that are Sidorenko have been established in [32].

Discussion 3.1 (Comparison to Sidorenko's lower bound)**.** Every complete bipartite graph is known to be Sidorenko (see [29, Theorem 5.5.12]). Specializing (31) to a complete bipartite graph $\mathsf{H} = \mathsf{K}_{s,t}$, where $s, t \in \mathbb{N}$, yields inequality (31) with $v(\mathsf{H}) = s + t$ and $e(\mathsf{H}) = st$. Let us now further specialize it to the case where G is a simple bipartite graph with partite sets of sizes n_1 and n_2, has no isolated vertices, and contains $\delta n_1 n_2$ edges for some $\delta \in (0, 1]$. In this specialized setting, (31) gives

$$\hom(\mathsf{K}_{s,t},\mathsf{G}) \geq (2\delta)^{st}(n_1+n_2)^{s+t-2st}(n_1n_2)^{st} \triangleq \mathrm{LB}_1. \tag{32}$$

This lower bound on $\hom(\mathsf{K}_{s,t},\mathsf{G})$ is compared to the bound $\mathrm{LB}_2 \triangleq \delta^{st}\left(n_1^s n_2^t + n_1^t n_2^s\right)$, which appears as the leftmost inequality in (5). To compare these two lower bounds, which are symmetric in n_1 and n_2 and also in s and t, we examine the ratio $\frac{\mathrm{LB}_2}{\mathrm{LB}_1}$. Without loss of generality, assume that $p \geq q$, and let $r \triangleq \frac{\max\{n_1,n_2\}}{\min\{n_1,n_2\}} \geq 1$. By straightforward algebra, we get

$$\begin{aligned}\frac{\mathrm{LB}_2}{\mathrm{LB}_1} &= 2^{-s}\left(\frac{(1+r)^2}{2r}\right)^{s(t-1)}\,(1+r)^{s-t}\,\left(1+r^{-(s-t)}\right) &(33)\\ &\geq 2^{st-(s+t)}\,\left(1+r^{-|s-t|}\right). &(34)\end{aligned}$$

By the symmetry of the right-hand side of (34) in s and t, the earlier assumption that $s \geq t$ can be dropped. Consequently, the following cases hold:

(1) If $s = t$, then it follows from (34) that $\mathrm{LB}_2 \geq 2^{(s-1)^2}\,\mathrm{LB}_1$, and in particular, $\mathrm{LB}_2 \geq \mathrm{LB}_1$.

(2) Else, if $s > 1$ and $t = 1$ (i.e., $\mathsf{K}_{s,t}$ is a star graph), then by Jensen's inequality

$$\frac{\mathrm{LB}_2}{\mathrm{LB}_1} = \frac{\frac{1}{2}\left(n_1^{1-s}+n_2^{1-s}\right)}{\left(\frac{n_1+n_2}{2}\right)^{1-s}} \geq 1, \tag{35}$$

so $\mathrm{LB}_2 \geq \mathrm{LB}_1$. Due to symmetry in s and t, it also holds if $s = 1$ and $t > 1$.

(3) Otherwise (i.e., if $s, t \geq 2$ and $s \neq t$), we get from (34) that $\mathrm{LB}_2 > 2^{st-(s+t)}\,\mathrm{LB}_1 \geq 2\,\mathrm{LB}_1$.

To conclude, the lower bound on $\hom(\mathsf{K}_{s,t},\mathsf{G})$ in (5) compares favorably to Sidorenko's lower bound given in (32).

4 Concluding Remarks

We highlight several results from the full paper version on arXiv [18], which investigates homomorphism counts from arbitrary bipartite source graphs to bipartite target graphs. In this extended abstract, we focus only on the second and fourth items, choosing to present two results with proofs rather than listing all findings without any proof. The full version [18] also contains observations and numerical results that are omitted here due to space limitations.

- Combinatorial and two entropy-based lower bounds are derived for complete bipartite source graphs.
- The first entropy-based bound was also introduced here, which depends only on the sizes of the partite sets in the source and target graphs, along with the edge density of the target graph.
- The second entropy-based lower bound further incorporates the degree profiles within the target's partite sets, yielding a strengthening of the first.
- Both entropy-based bounds improve upon the inequality that is implied by Sidorenko's conjecture for complete bipartite graph sources.
- These lower bounds, combined with new auxiliary results, yield general bounds on homomorphism counts between arbitrary bipartite graphs.
- A known reverse Sidorenko inequality by Sah, Sawhney, Stoner, and Zhao [35] is used to derive corresponding upper bounds.
- Numerical comparisons with exact counts in tractable cases support the effectiveness of the proposed computable bounds.

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