Beam-Quality and Guiding Field Effect on a High-Power TWT Operating at 35GHz

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Collaboration

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Outline

- Motivation
- Theoretical Model and Basic Assumptions
- Dynamics of the System
- Simulation Results
- High Order Modes
- Summary

Motivation

- For a given accelerating gradient (E_0) and group velocity, the product $P \times \omega^2 \cong$ const.
- Explicitly: $P \propto (E_0 \lambda)^2 / V_{gr}$
- We reduce the power by a factor of 16 if the frequency is increased by a factor of 4 !!

Definition of the Model

- **RF (TM$_{01}$):**

  $E_z(r,z,t)=E_0 I_0(\Gamma r)\cos(\omega t-kz-\psi)$

  $E_r(r,z,t)=-\gamma_{ph}E_0 I_1(\Gamma r)\sin(\omega t-kz-\psi)$

  $H_\phi(r,z,t)=-\gamma_{ph}\beta_{ph}(E_0/\eta_0)I_1(\Gamma r)\sin(\ldots)$

- **DC** (beam):

  $E_r(r<R_b)= -(en_0/2\varepsilon_0)r$

  $\eta_0 H_\phi(r<R_b)= -(v/c^2)(en_0/2\varepsilon_0)r$

- **Guiding Field** - B_0

Particles' Dynamics

- **Azimuthal motion:**

$$\frac{d}{dt}\left[r_i^2\left(\gamma_i\frac{d}{dt}\phi_i-\frac{1}{2}\omega_c\right)\right]=0 \qquad \left[\omega_c=\frac{eB_0}{m}\right]$$

$$\Rightarrow\frac{d}{dt}\phi_i=\frac{\omega_c}{2\gamma_i r_i^2}\left(r_i^2-r_{i,in}^2\right) \qquad \left[\left.\frac{d}{dt}\phi_i\right|_{r_i=r_{i,in}}=0\right]$$

Particles' Dynamics

- **Radial motion:**

$$\frac{d}{dt}\left(\gamma_i \frac{d}{dt} r_i\right) + \frac{\omega_c^2}{4\gamma_i r_i^3}\left(r_i^4 - r_{i,in}^4\right) - \frac{\omega_p^2}{2\gamma_i^2} r_i = -\frac{e}{m}\left(E_r + V_z \mu_0 H_\phi\right)_i$$

Particles' Dynamics

- **Longitudinal motion:**

$$\frac{d}{dt}\left(\gamma_i \beta_{z,i}\right) = -\frac{e}{mc}\left(E_z + V_r \mu_0 H_\phi\right)$$

$$\approx -\frac{e}{mc} E_z$$

$$\approx -\frac{e}{mc} E_0 \, I_0(\Gamma r_i) \cos\left[\omega t - k\, z_i(t) - \psi\right]$$

Field Dynamics

- **Poynting theorem:** $$\frac{d}{dz}\left\langle P(z)\right\rangle_t = -\int_{cs} da \left\langle \vec{J}\cdot\vec{E}\right\rangle$$
- **Interaction impedance:**

$$\left\langle P(z)\right\rangle_t = \frac{1}{2}\frac{E_0^2(\pi R_{\text{int}}^2)}{Z_{\text{int}}}$$

- **Neglect contribution of the radial motion to energy exchange**

$$\left[\,|\beta_{r,i}|\,\gamma_i << \beta_{z,i}\,\right]$$

Field Dynamics

- **Amplitude equation:**

$$\frac{d\bar{E}_0}{dz} = \frac{I\,Z_{\text{int}}}{\pi R_{\text{int}}^2}\left\langle I_0\left[\Gamma r_i(z)\right]e^{-j\chi_i}\right\rangle_i$$

- **Phase equation:**

$$\frac{d\chi_i}{dz} = \frac{\omega}{c}\frac{1}{\beta_{z,i}} - k$$

$$\gamma_{z,i} \equiv [1-\beta_{z,i}^2]^{-1/2} = \gamma_i\left[1+\left(\gamma\beta_r\right)_i^2+\left(r_i^{\,2}-r_{i,in}^{\,2}\right)^2\left(\frac{\omega_c}{2cr_i}\right)^2\right]^{-1/2}$$

Comparison of 35GHz & 8.75GHz

- 1D motion
- Identical beams (500kV, 200A, 3mm radius)
- Energy conservation $\Rightarrow$ same input (20kW)
- At the input $\beta_{ph} = \langle \beta_i \rangle$
- Same gain: $\mathrm{Im}(kd) \Rightarrow Z_{\mathrm{int}} \times \left(\frac{\omega}{c} d \right) = const.$
- Phase advance per cell 120^0

Comparison of 35GHz & 8.75GHz

- **Parameters:**

$$R_{\text{int}} = 6.0,\ 6.5,\ 7.0,\ 7.5[mm] \Rightarrow$$

$$Z_{\text{int}}(35GHz) = 73,\ 52,\ 37,\ 26\ [\Omega]$$

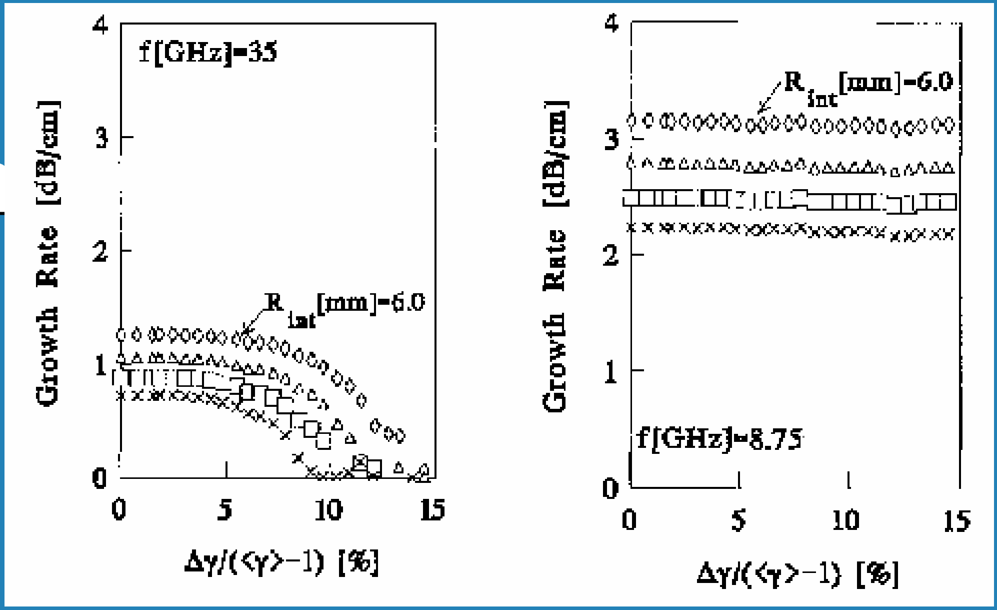
$$Z_{\text{int}}(8.75GHz) = 2700,\ 2193,\ 1812,\ 1519\ [\Omega]$$

- **At this stage it is assumed that if the phase velocity varies, the change in the interaction impedance is negligible.**

Comparison of 35GHz & 8.75GHz

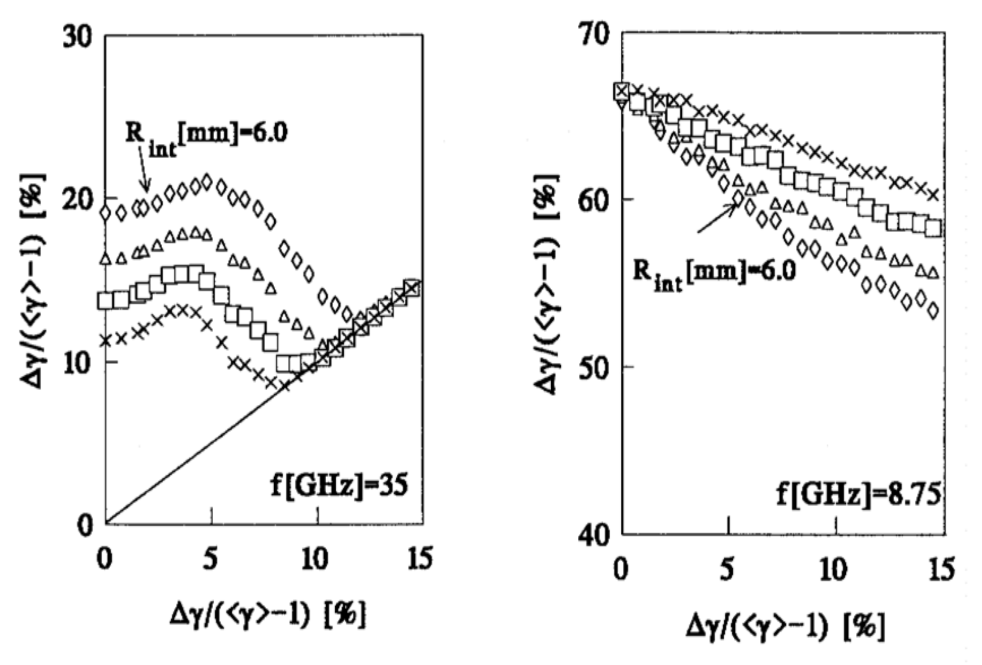
- Growth-rate at 35GHz sensitive to beam quality.
- Not critical !!

  In practice, energy sprea of 1% is achievable.



Comparison of 35GHz & 8.75GHz

- Energy spread at the output as a function of the energy spread at the input for the same parameters.
- Note the dramatic effect of high interaction impedance.

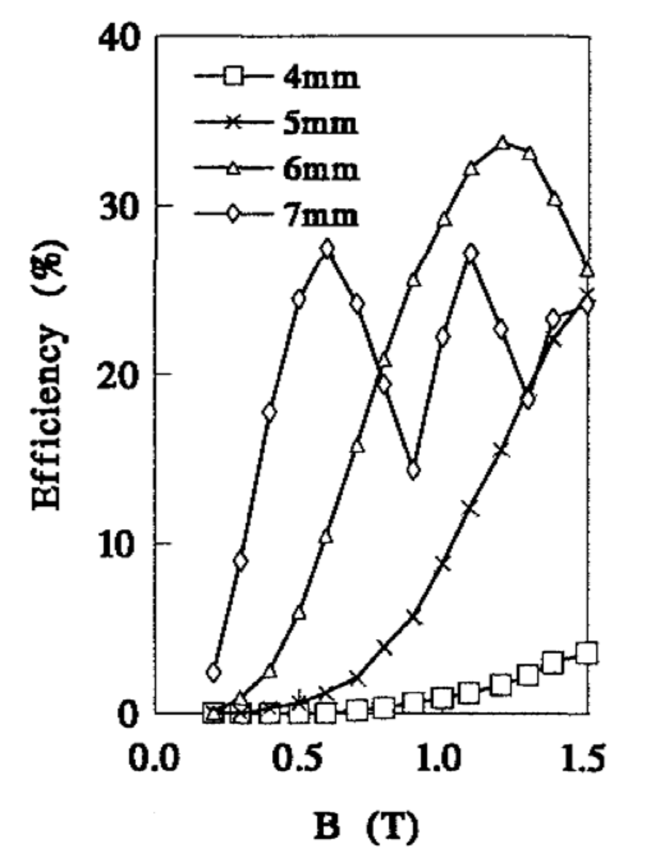


Guiding Field Effect

- Energy spread at the input 1% .
- Tapered structure:

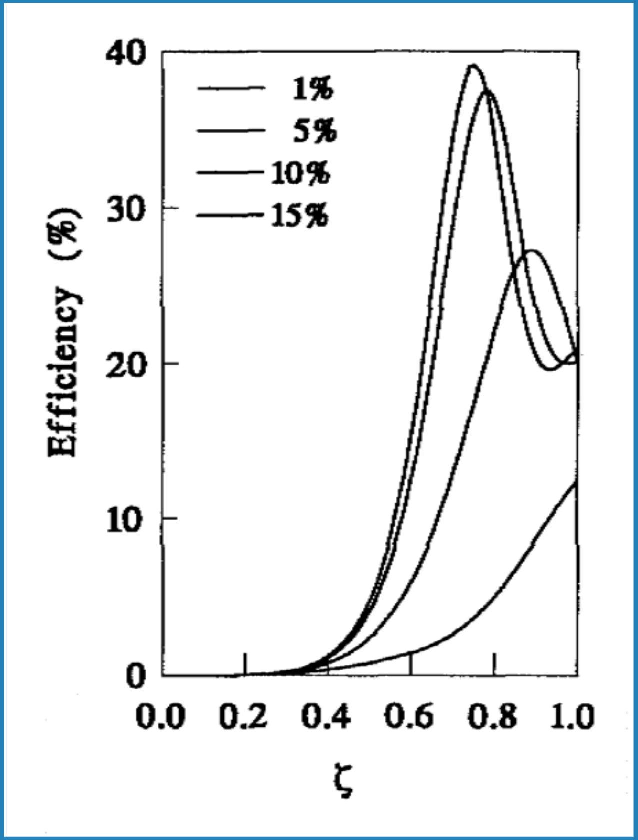
$$\frac{1}{\beta_{ph}} = \left\langle \frac{1}{\beta_i} \right\rangle$$

- Simulation terminates if: particles hit the wall or are reflected.



Beam-Quality Effect

- R_{int}=6mm
- B=0.5T



Beam-Quality Effect

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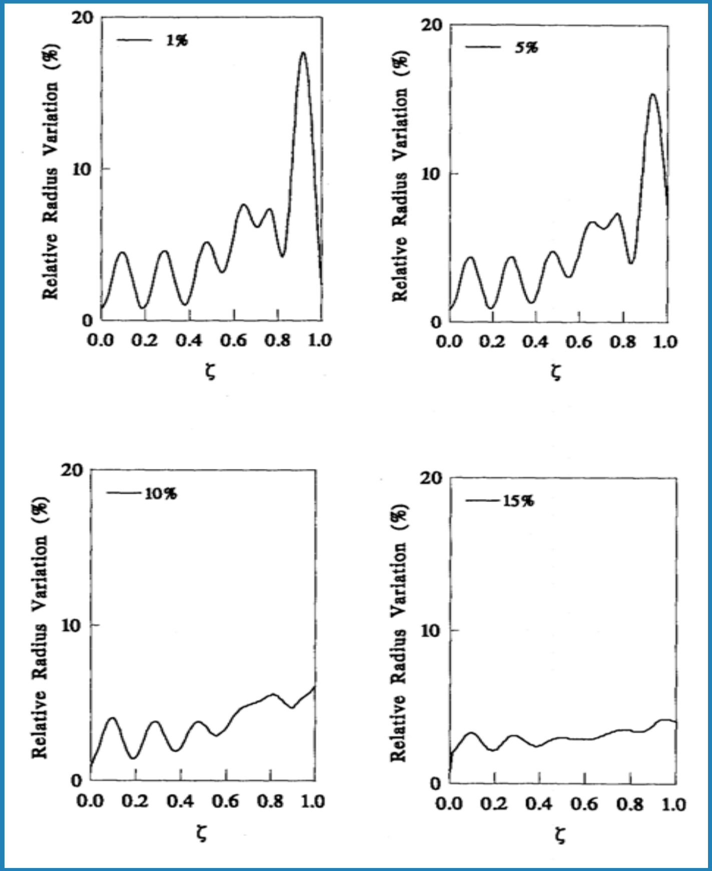


Varying Interaction Impedance

- The dependence of the interaction impedance on the phase velocity was calculated analytically at 35GHz:

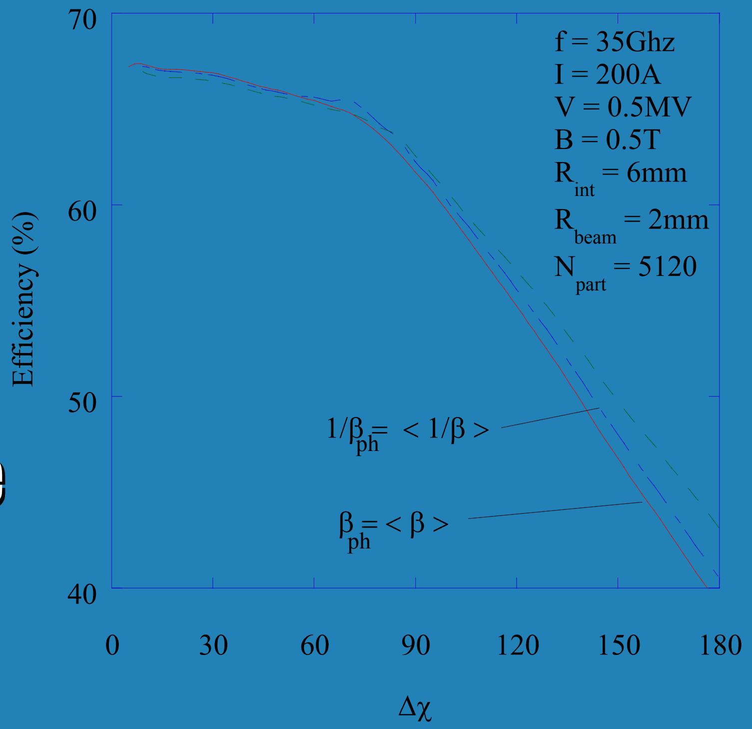
$$Z_{\text{int}}(R_{\text{int}} = 6mm) = 302\,\beta_{ph}^{8.76}\,[Ohm]$$

$$Z_{\text{int}}(R_{\text{int}} = 7mm) = 219\,\beta_{ph}^{11.28}\,[Ohm]$$

$$Z_{\text{int}}(R_{\text{int}} = 8mm) = 166\,\beta_{ph}^{14.02}\,[Ohm]$$

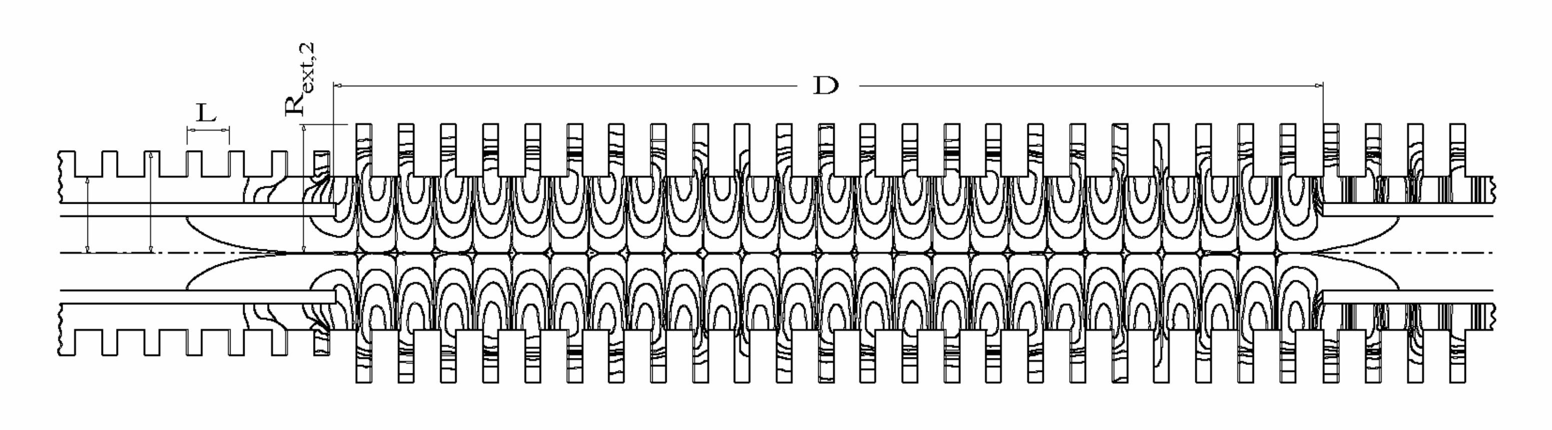
High-Efficiency Interaction

- Varying interaction impedance
- Three tapering criteria
- For a good density modulated beam 65% efficiency is achievable



High Order Mode Suppression

- Preserving a high internal radius (>6mm) we solved one problem but we generated another: high order modes.
- Solution open structure



Summary

- For less than 5% energy spread at the input, a KA system should operate as X-band one.
- B=0.5T should suffice to confine a 3mm radius beam which produces radiation with 30% efficiency in a 6mm internal radius.
- Allowing the interaction impedance to vary in space facilitates increase of efficiency of up to 65% (R_b=2mm).