Active Medium Acceleration

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Outline

- *Acceleration concepts*
- *Wake in a medium*
- *Frank-Hertz , LASER & Acceleration*
- *Space-charge wave in resonant medium*
- *Wake & Saturation*
- *Dynamics & Saturation*
- *Active enhancement of the Q-factor*
- *BNL-ATF PASER experiment*

Classification of Acceleration Concepts

Inverse Radiation Processes

- *Slow Wave*
  - *- Cerenkov*
  - *- Smith-Purcell*
- *Fast Wave:*
  - *- FEL*
  - *- Cyclotron*
- *Transition Radiation*
- *Laser*

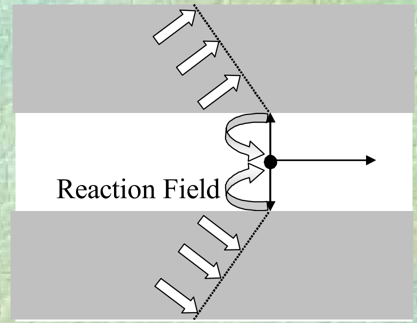
Wakes

- *Laser Wake-Field*
- *Plasma Wake-Field*
- *Plasma Beat Wave*
- *Dielectric Wake-Field*

Wake in a medium

Passive Dielectric

- *Cerenkov Radiation*
- *Decelerating Force*

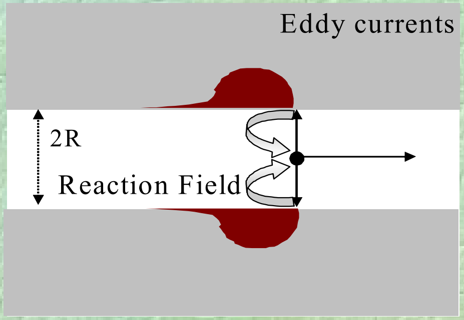


$$E_{\mathrm{dec}} = \frac{q}{4\pi\varepsilon_0 R^2} \times 2$$

$$\gamma \gg 1$$

Resistive Material

- *Eddy Currents*
- *Decelerating Force*

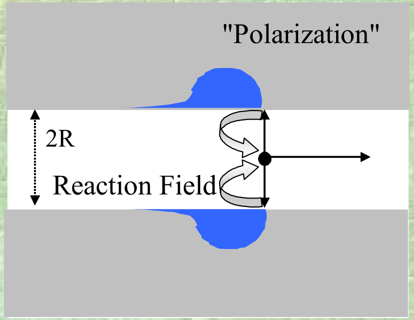


$$E_{\mathrm{dec}} = \frac{q}{4\pi\varepsilon_0 R^2} \times 2$$

$$\sqrt{\frac{(\gamma\beta)^3}{\sigma\eta_0 R}} \gg 1$$

Active Medium

- *Negative Resistivity*
- *Induced Polarization*
- *Accelerating Force*

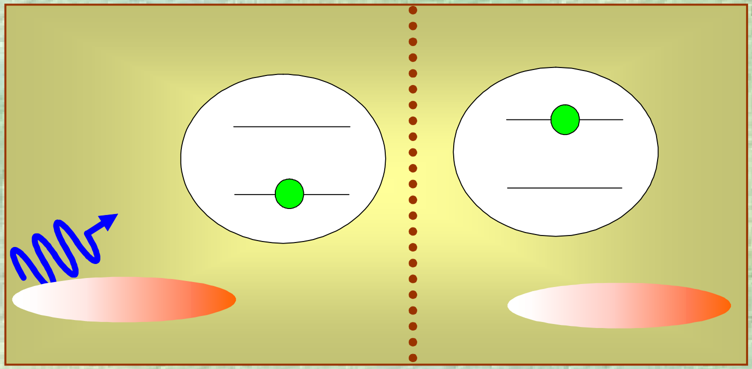


Narrow band

L. Schachter, PRE, 53, p.6427(1996)

Frank-Hertz, LASER and Acceleration

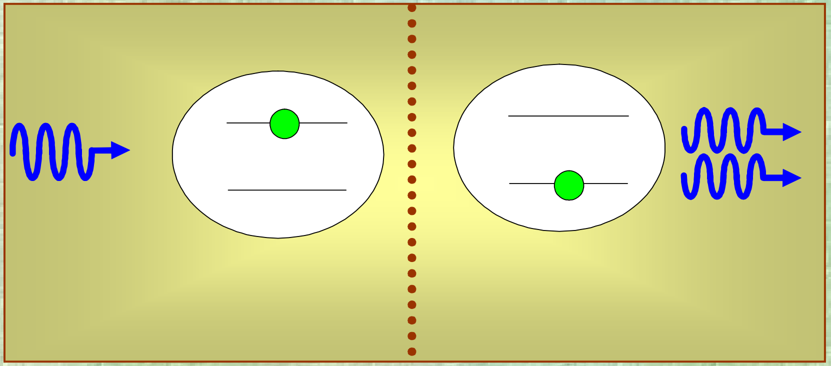
Frank-Hertz



L. Schachter,
Phys. Lett. A., 205, p.355(1995)

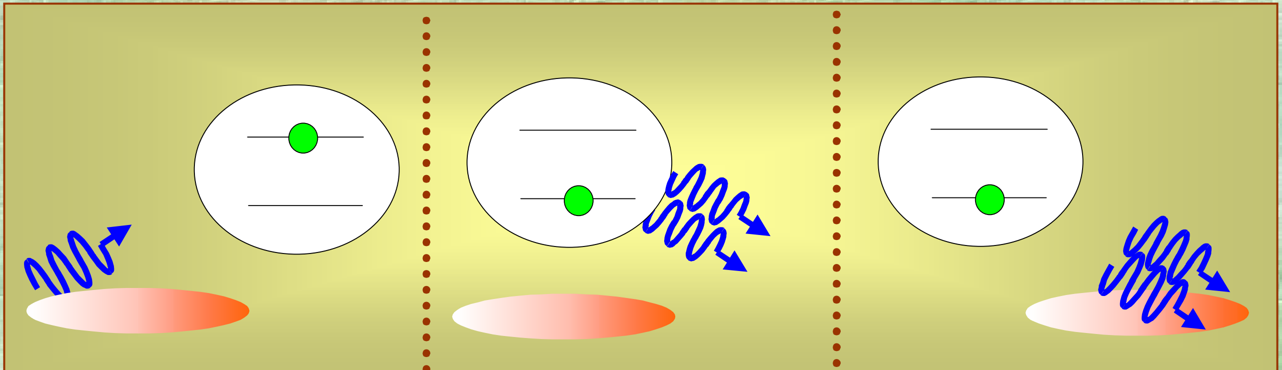
LASER

Light Amplification
by Stimulated
Emission of Radiation



PASER

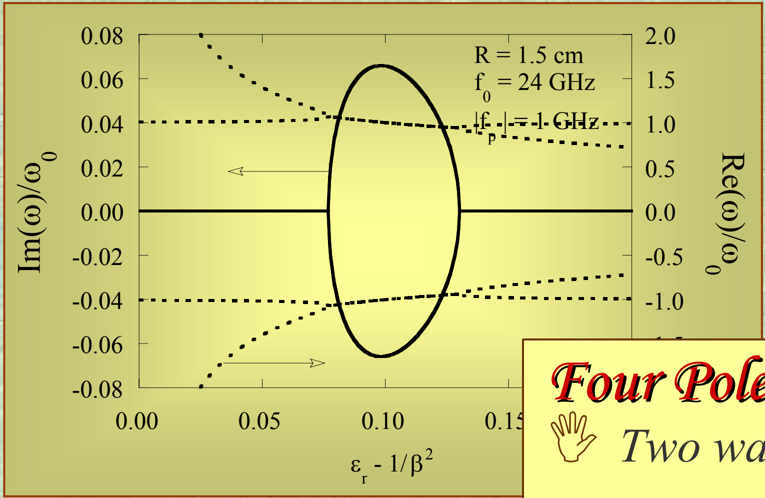
Particle Acceleration
by Stimulated
Emission of Radiation



Wake in a resonant medium

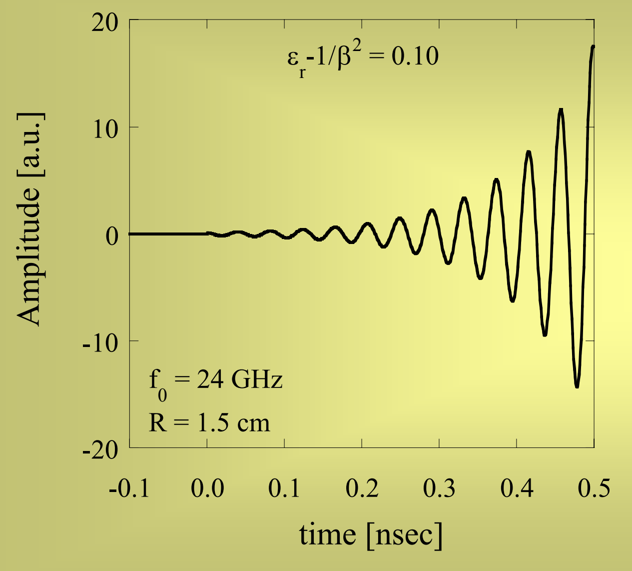
$$\varepsilon(\omega) \simeq \varepsilon_r + \frac{\omega_p^2}{\omega_0^2 - \omega^2 + 2j\omega\omega_1}$$

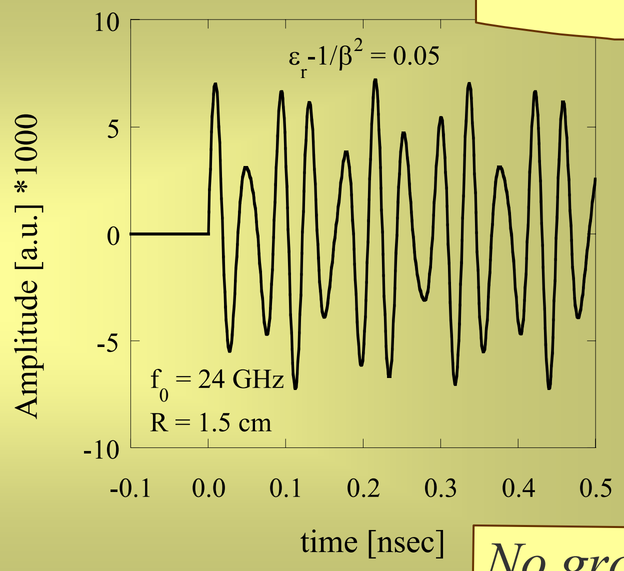
$$A_z \propto \int_{-\infty}^{\infty} d\omega \frac{e^{j\omega(t-z/v)}}{\omega^2\left[\varepsilon(\omega) - \beta^{-2}\right] - \omega_{c,s}^2}$$



Four Poles:

- *Two waveguide modes*
- *Two "resonance" modes*





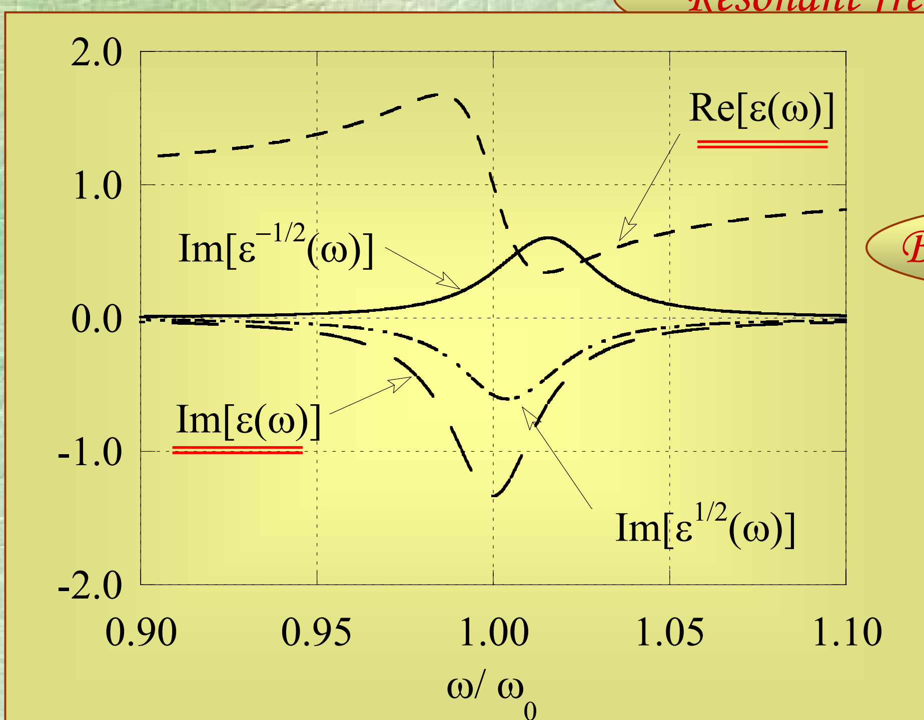
No growth off-resonance

L. Schachter, PRE, 62, p.1252 (2000)

Space-charge wave in a resonant medium

Basic Concept:

- *Space-charge wave propagating in a lossy waveguide leads to resistive-wall instability.*
- *Resonant absorption of atoms or molecules may be used for amplification or acceleration*



Gas plasma frequency

Resonant frequency

$$\varepsilon(\omega) = \varepsilon_r + \frac{\omega_{p,g}^2}{\omega_0^2 - \omega^2 + 2j\omega\omega_1}$$

Line width

Beam plasma frequency

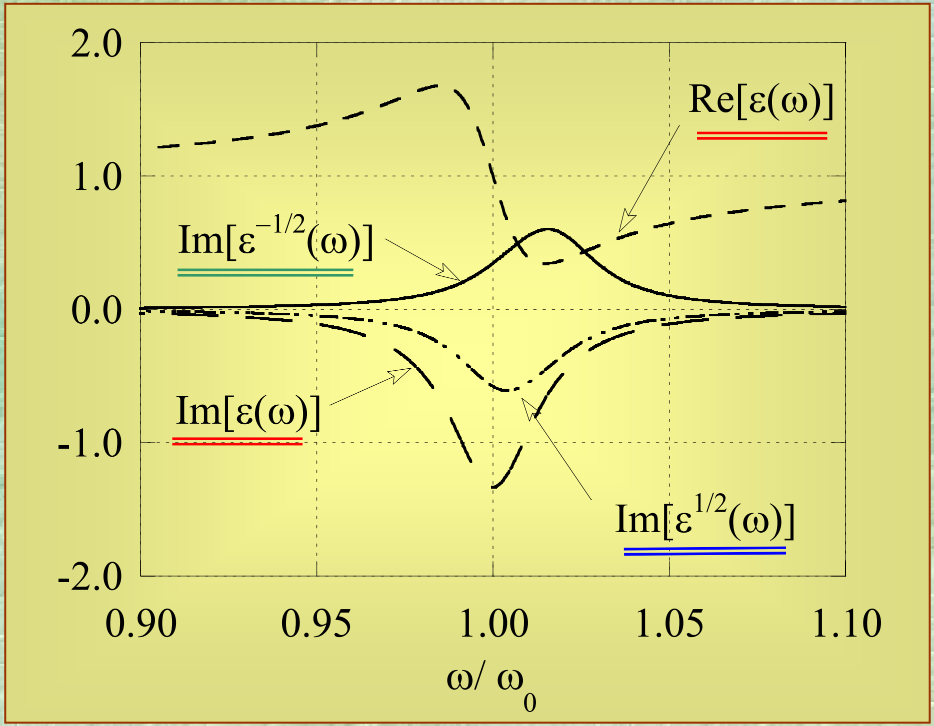
$$J_z(\omega,k) = -j\omega\varepsilon_0 \frac{\omega_{p,b}^2}{(\omega - kV)^2} E_z(\omega,k)$$

$$\omega_{p,b}^2 = \frac{e^2 n_0}{m\varepsilon_0\gamma^3}$$

L. Schachter, Phys. Lett. A, 277, p.65 (2000)

Space-charge wave in a resonant medium

$$\underbrace{\left[\varepsilon(\omega)\frac{\omega^2}{c^2}-k^2\right]}_{TEM}\underbrace{\left[1-\frac{\omega_{p,b}^2}{(\omega-kV)^2}\frac{1}{\varepsilon(\omega)}\right]}_{SC}=0$$



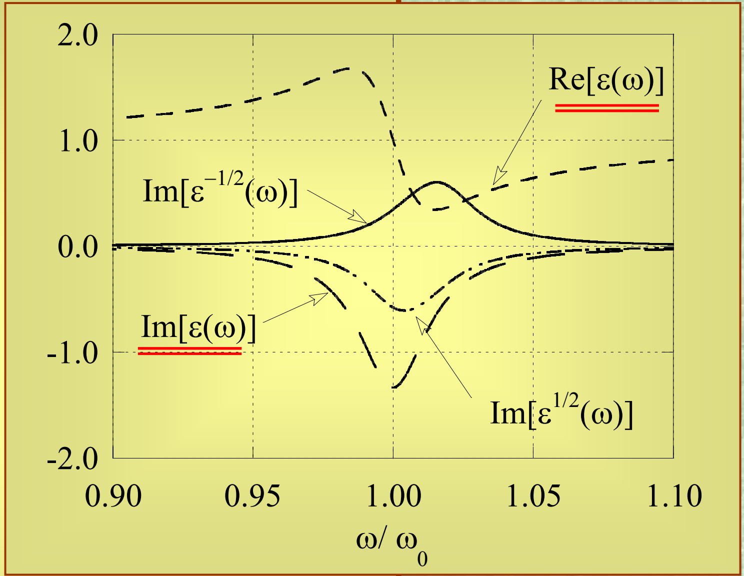
- TEM: $\mathrm{Im}(k)=\frac{\omega}{c}\mathrm{Im}\left[\sqrt{\varepsilon(\omega)}\right]$
- SC: $\mathrm{Im}(k)=\frac{\omega_{p,b}}{v}\mathrm{Im}\left[\frac{1}{\sqrt{\varepsilon(\omega)}}\right]$

L. Schachter, Phys. Lett. A, 277, p.65 (2000)

Space-charge wave in a resonant medium

Example: Amplifier

- μ=2.5 Debye
- $\omega_0/2\pi$ =125 GHz
- $\omega_1/2\pi$ =10MHz
- $\mathrm{Im}(k_{EM}) = 3.36\mathrm{cm}^{-1}$
- $\omega_{p,b}/2\pi$ = 2 GHz
- $\mathrm{Im}(k_{SC}) = 0.054\mathrm{cm}^{-1}$ => 0.47dB/cm
- 100W input, 110cm interaction =>10MW



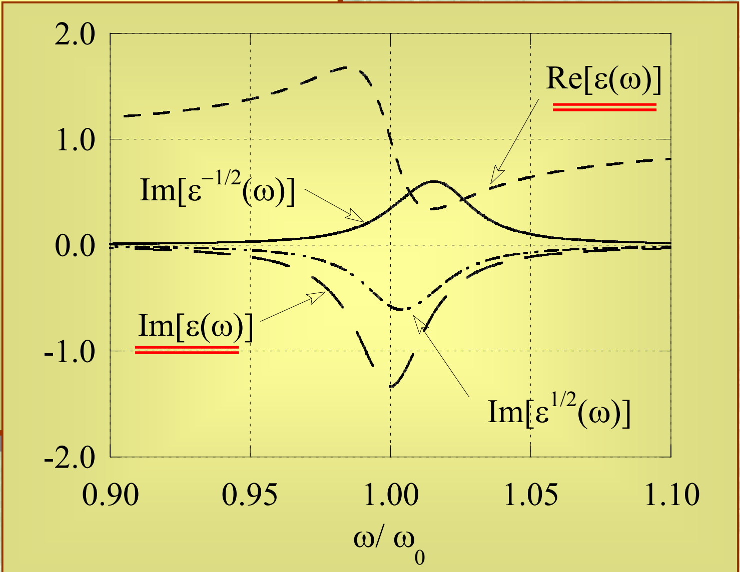
L. Schachter, Phys. Lett. A, 277, p.65 (2000)

Space-charge wave in a resonant medium

Example: Accelerator

- Same as above
- $\mathrm{Im}(k_{EM}) = 3.36\mathrm{cm}^{-1}$
- $\omega_{p,b}/2\pi = 2$ GHz
- $\mathrm{Im}(k_{SC}) = 0.054\mathrm{cm}^{-1}$ => 0.47dB/cm
- 100W input => 0.15MV/m
- 110cm interaction =>45MV/m

4nsec,1mm beam



L. Schachter, Phys. Lett. A, 277, p.65 (2000)

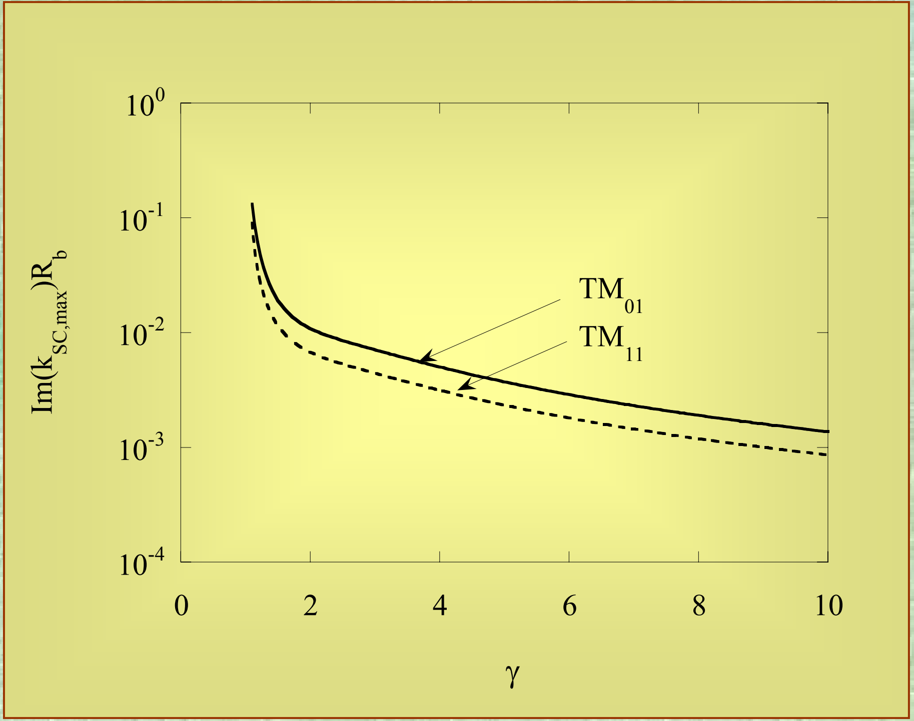
Space-charge wave in a resonant medium

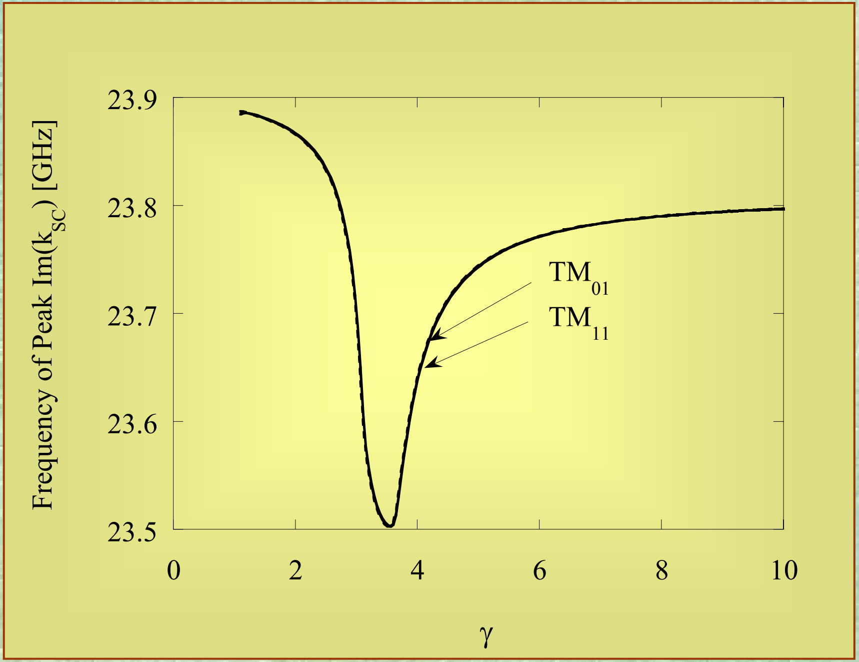
Asymmetric Modes

$$\left[\varepsilon(\omega)\frac{\omega^2}{c^2}-k^2\right]\left[1-\frac{\omega_{p,b}^2}{(\omega-kV)^2}\frac{1}{\varepsilon(\omega)}\right]=\frac{p_{n,s}^2}{R^2}$$

TM_{01}: $p_{01}=2.4048$

TM_{11}: $p_{11}=3.832$



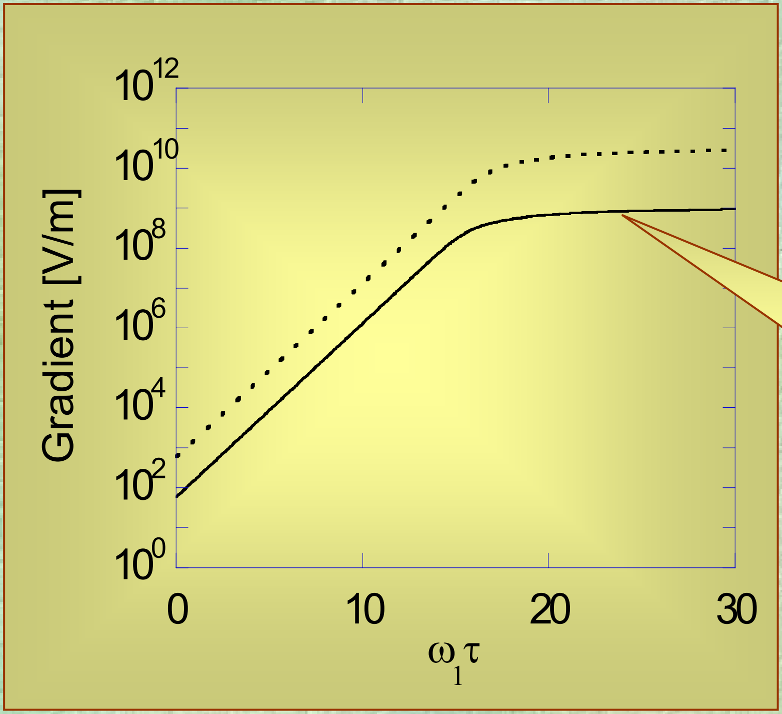


L. Schachter, Phys. Lett. A, 277, p.65 (2000)

Wake & Saturation

Amplified Wake

Quasi-linear approach



$$E_{sat} \simeq \sqrt{\frac{3}{T_1 T_2}}\,\frac{\hbar}{p} \underset{\substack{p\sim 10^{-31}[Cm] \\ \sqrt{T_1 T}\sim 0.1[n\sec]}}{\longrightarrow} 10[\mathrm{MV/m}]$$

L. Schachter, PRL, 83, p.92 (1999)

Self-consistent dynamics

Interaction of a single-mode with a bunch of electrons

$$\left.\begin{aligned}
&\frac{d}{d\xi}\, a = \alpha \left\langle e^{-j\chi_i} \right\rangle \\
&\frac{d}{d\xi}\, \gamma_i = -\frac{1}{2}\left[a\, e^{j\chi_i} + c.c. \right] \\
&\frac{d}{d\xi}\, \chi_i = \Omega \left(\frac{1}{\beta_i} - \frac{1}{\beta_{\mathrm{p}}} \right)
\end{aligned}\right\} \Rightarrow \frac{d}{d\xi}\left[\underbrace{\left\langle \gamma_i \right\rangle - 1}_{Kinetic\ \ Energy} + \underbrace{\frac{|a|^2}{2\alpha}}_{EM\ \ \ Energy} \right] = 0$$

Energy conservation

L. Schachter, PRL, 87, 134802-1 (2001)

Self-consistent dynamics

Energy conservation in the presence of Active Medium

Photon Density

$$\frac{d}{d\xi}\left[\underbrace{\langle\gamma_i\rangle-1}_{Kinetic\ \ Energy}+\underbrace{\frac{|a|^2}{2\alpha}}_{EM\ \ \ Energy}+\underbrace{\frac{n_{ph}\hbar\omega}{n_e mc^2}}_{Energy\ \ in\ \ Medium}\right]=0$$

Electron Density

L. Schachter, PRL, 87, 134802-1 (2001)

Self-consistent dynamics

The effect on the population inversion

$$\left.\begin{aligned} \frac{d}{d\xi} a &= \alpha \left\langle e^{-j\chi_i} \right\rangle + \left(\frac{1}{2}\sigma n_{ph} d\right) a \\ \frac{d}{d\xi}\gamma_i &= -\frac{1}{2}\left[a e^{j\chi_i} + c.c. \right] \end{aligned}\right\} \Rightarrow \frac{d}{d\xi}\left[\left\langle \gamma_i \right\rangle - 1 + \frac{|a|^2}{2\alpha} \right] = \left(\frac{|a|^2}{2\alpha}\right)\left(\sigma n_{ph} d\right)$$

$$\frac{d}{d\xi}\left[\left\langle \gamma_i \right\rangle - 1 + \frac{|a|^2}{2\alpha} + \frac{n_{ph}\hbar\omega}{n_e mc^2} \right] = 0$$

Inversion equation

$$\Rightarrow \frac{d}{d\xi} n_{ph} = -\left(\frac{|a|^2}{2\alpha}\right)\left(\sigma d n_e \frac{mc^2}{\hbar\omega}\right) n_{ph}$$

L. Schachter, PRL, 87, 134802-1 (2001)

Self-consistent dynamics

$$\left.\begin{aligned}
&\frac{d}{d\xi}a = \alpha\left\langle e^{-j\chi_i}\right\rangle + \left(\frac{1}{2}\sigma n_{ph} d\right)a \\
&\frac{d}{d\xi}\gamma_i = -\frac{1}{2}\left[ae^{j\chi_i} + c.c.\right] \\
&\frac{d}{d\xi}\chi_i = \Omega\left(\frac{1}{\beta_i} - \frac{1}{\beta_p}\right) \simeq 0\left(\gamma^{-2}\right) \\
&\frac{d}{d\xi}n_{ph} = -\left(\frac{|a|^2}{2\alpha}\right)\left(\sigma d n_e \frac{mc^2}{\hbar\omega}\right)n_{ph}
\end{aligned}\right\} \Rightarrow \frac{d}{d\xi}\left[\underbrace{\left\langle\gamma_i\right\rangle - 1}_{Kinetic\ \ Energy} + \underbrace{\frac{|a|^2}{2\alpha}}_{EM\ \ \ Energy} + \underbrace{\frac{n_{ph}\,\hbar\omega}{n_e mc^2}}_{Energy\ \ in\ \ Medium}\right] = 0$$

L. Schachter, PRL, 87, 134802-1 (2001)

Self-consistent dynamics

$$\lambda = 1[\mu\mathrm{m}],\ d = 300\lambda,\ Z_{\mathrm{int}} = 100[\Omega], N_{el} = 10^5 \Rightarrow I = \frac{eN_{el}}{T} = 4.8[\mathrm{A}],\ E_{acc} = 1[\mathrm{GV/m}]$$

$$\alpha = \frac{IZ_{\mathrm{int}}}{\mathrm{mc}^2/e}\left(\frac{d}{\lambda}\right)^2 \Rightarrow \boxed{\alpha = 84.5},\quad a \equiv \frac{eE_{acc}d}{mc^2} \Rightarrow \boxed{a \simeq 0.6}$$

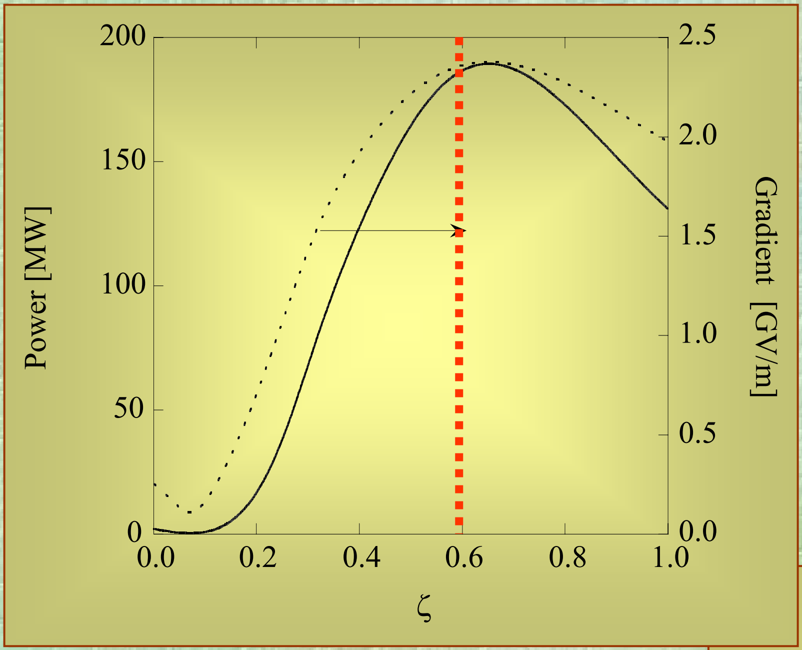
$$n_e \cong \frac{N_{el}}{\pi R_b^2 \lambda} = \frac{10^5}{\pi(0.1\lambda)^2\lambda} \simeq 0.3\times10^{25}\left[\mathrm{m}^{-3}\right]$$

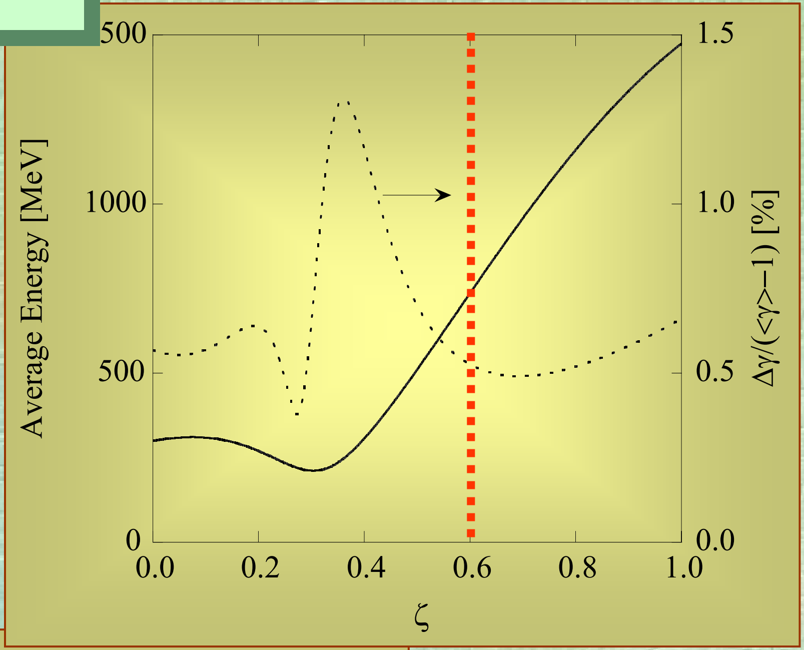
$$\sigma[Nd:YAG] = 5\times10^{-23}\left[\mathrm{m}^2\right],\ \frac{mc^2}{\hbar\omega} \simeq 0.4\times10^6$$

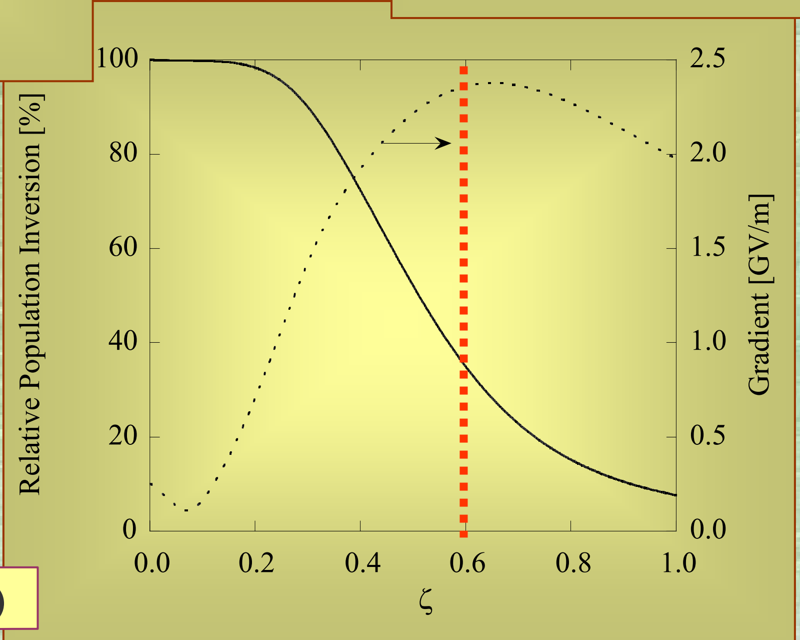
$$\frac{d}{d\xi}n_{ph} = -\underbrace{\left(\underbrace{\frac{|a|^2}{2\alpha}}_{2.1\times10^{-3}}\right)\left(\underbrace{\sigma d n_e \frac{mc^2}{\hbar\omega}}_{1.9\times10^4}\right)}_{40} n_{ph}$$

$$n_{ph} \simeq 2.5\times10^{25}\ [\mathrm{m}^{-3}] \Rightarrow w_{ph} \sim 5[\mathrm{J/cm}^3]$$

Self-consistent dynamics



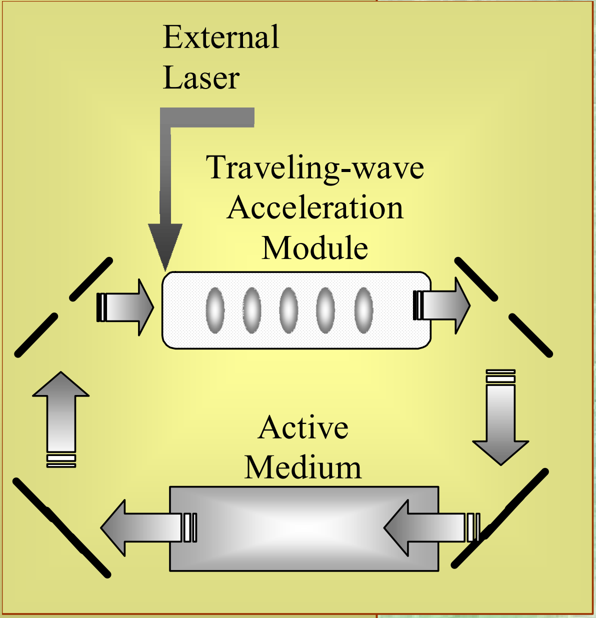




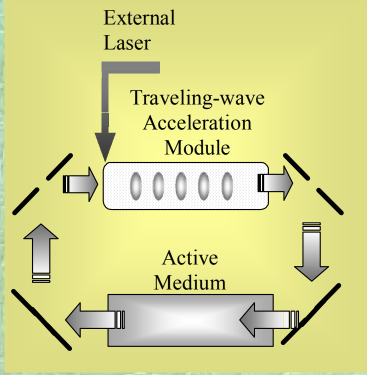
L. Schachter, PRL, 87, 134802-1 (2001)

Q-factor enhancement

- The energy stored in the cavity of a laser is much higher (Q) by the extracted energy.
- Significant fraction of the energy is wasted by the various optical components.
- Efficiency of a single acceleration module can be fairly small.
- Number of accelerated bunch in one micro-bunch → macro-bunch → quasi-coherent wake.
- Proposed concept: combine the cavity of the laser and the acceleration module, to one unit.
- Different perspectives:
  - "recycle" some of the energy
  - "active" enhancement of the Q-factor
  - "quasi-coherent" wake

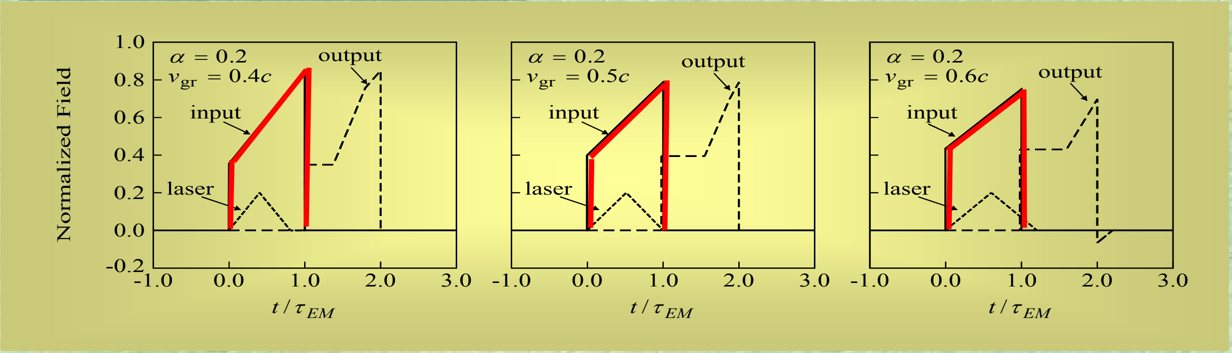


Q-factor enhancement



$\lambda M = d$

Tapered laser pulse



Conditions for self-consistent field:

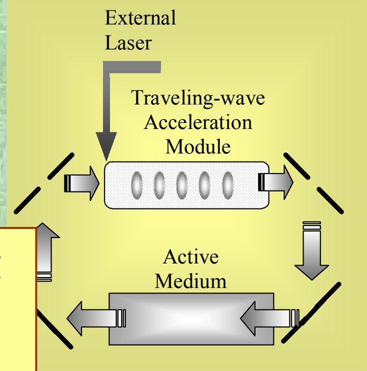
*(I) Amplifier compensates for all **radiation loss***

*(II) External laser compensates for **beam-loading***

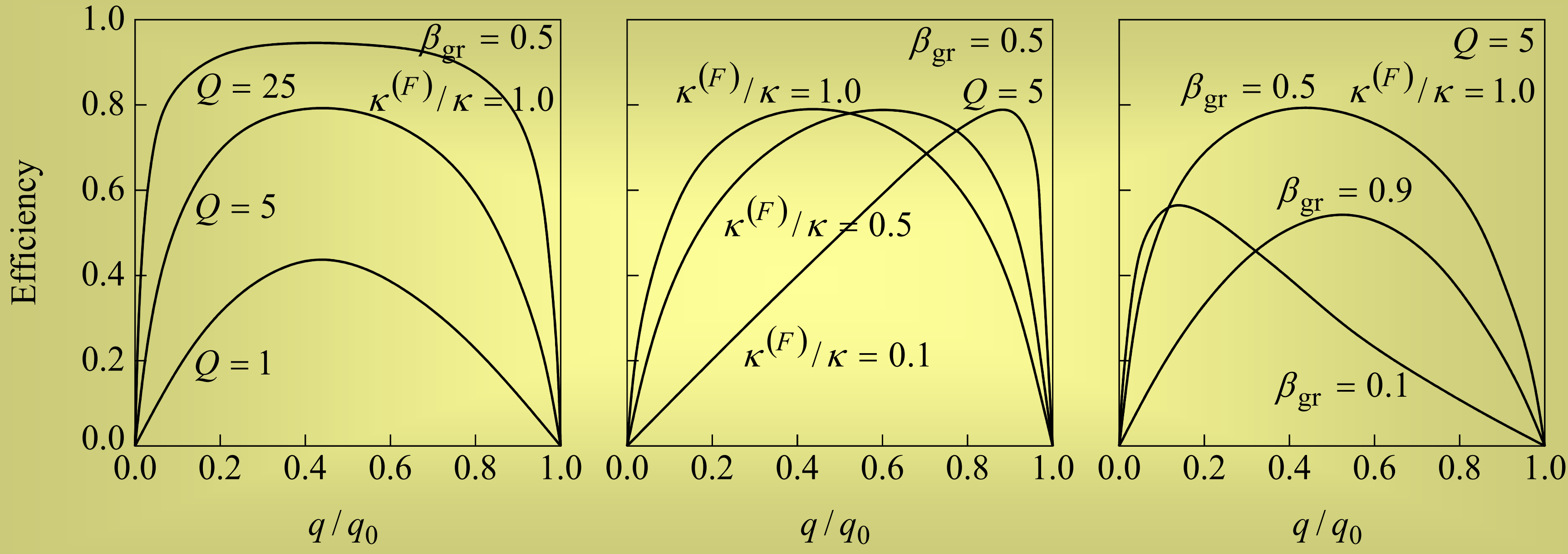
$$\eta = \frac{\Delta U_{KIN}}{U_{LASER} + U_{ACTIVE}} = \frac{1}{1 + \frac{1}{Q}\frac{U_{OUT}}{\Delta U_{KIN}}}$$

L. Schachter, PRE (2004)

Q-factor enhancement



- Peak efficiency independent of κ_1/κ
- Sensitivity

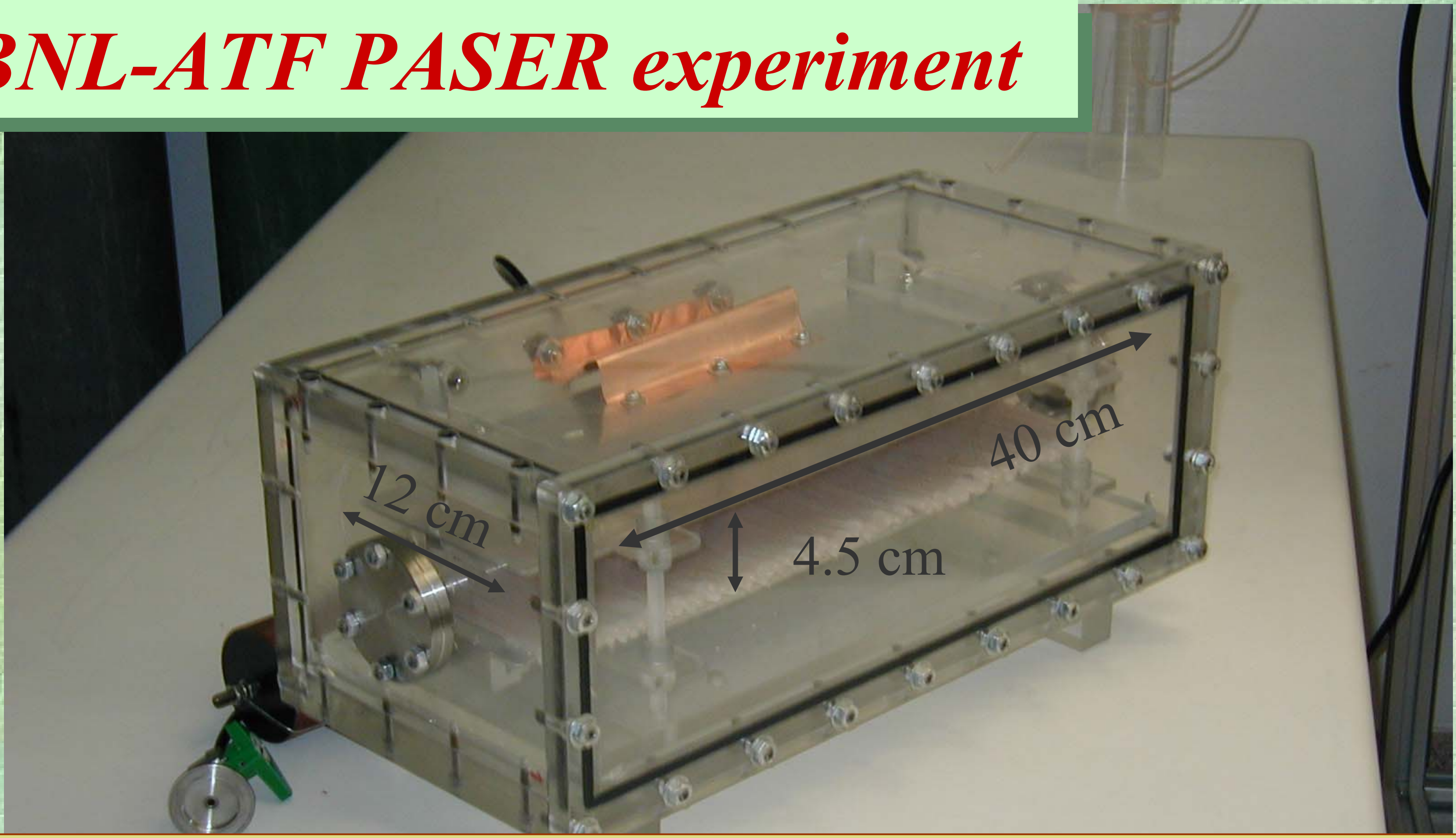


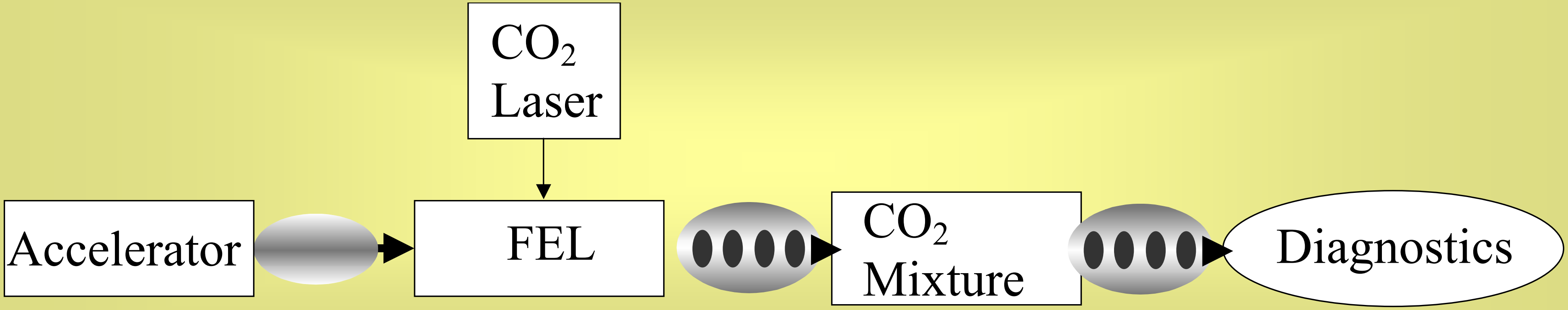
High Q:
(I) High efficiency
(II) Reduced sensitivity

- Peak efficiency dependent on β_{gr}
- Sensitivity

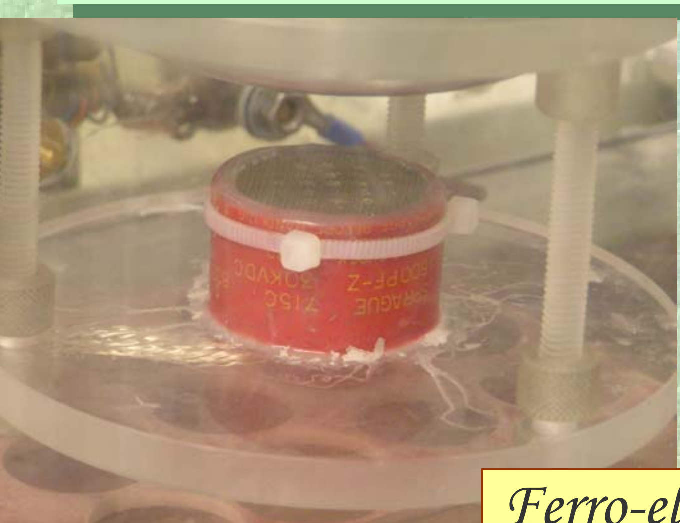
L. Schachter, PRE (2004)

BNL-ATF PASER experiment

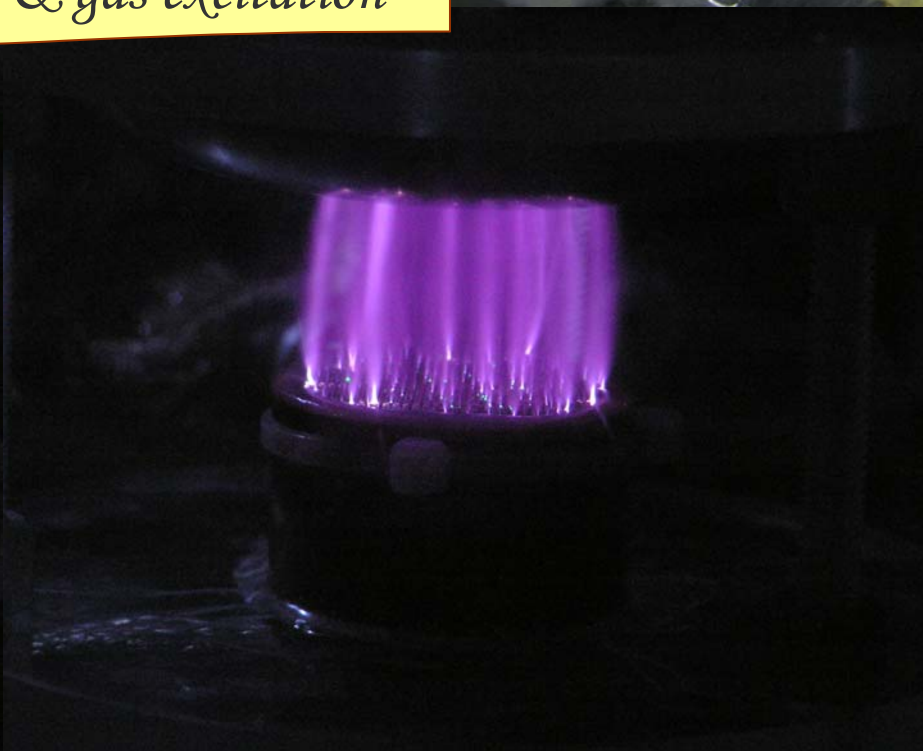




BNL-ATF PASER experiment



Ferro-electric cathode & gas excitation



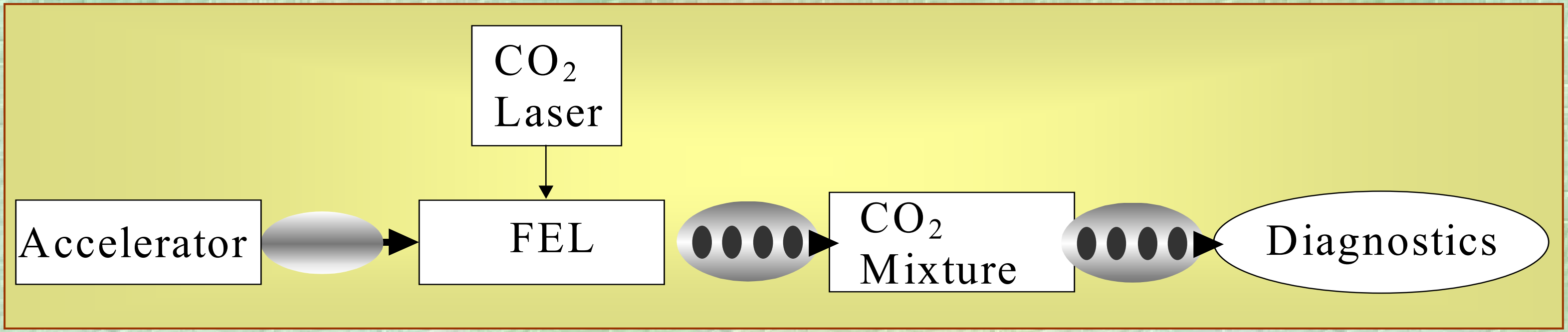
BNL-ATF PASER experiment

$$J_z(r,z,t) = -qV\sum_{\nu}\frac{1}{2\pi r}\delta\left(r-r_\nu\right)\delta\left[z-z_\nu-Vt\right] \Rightarrow$$

$$P = 2\pi\int dr\, r\int dz\, E_z(r,z,t)J_z(r,z,t) = -qV\sum_{\nu}E_z\left(r_\nu, z_\nu+Vt;t\right)$$

$$\frac{-P}{\dfrac{Q^2V}{4\pi\varepsilon_0\lambda_0^2}} = \frac{2}{\pi}\int_0^\infty dx\,\mathrm{sinc}^2\left(\frac{x}{2\beta}\frac{\Delta}{\lambda_0}\right)\frac{\mathrm{sinc}^2\left(\dfrac{x}{2\beta}M\right)}{\mathrm{sinc}^2\left(\dfrac{x}{2\beta}\right)}\mathrm{Re}\left\{jx\left(1-\frac{1}{\varepsilon\beta^2}\right)\mathbb{F}\left(\frac{x}{\beta}\frac{R_b}{\lambda_0}\sqrt{1-\beta^2\varepsilon}\right)\right\}$$

$$\mathbb{F}(u) \equiv \frac{2}{u^2}\left[1-2K_1(u)I_1(u)\right]$$



BNL-ATF PASER experiment

$$\eta_{Cer} \simeq -200 \frac{N_{el}}{\gamma\text{-}1} \frac{1}{M^2} \frac{r_{el} d}{R_b^2} \simeq -2.5[\%]$$

$N_{el} \simeq 10^{10}$, $d = 0.5\text{m}$, $\lambda_0 \sim 10.6\mu\text{m}$, $R_b \sim 100\mu\text{m}$, $\gamma \sim 139$, $L_{pulse} \sim 3\text{mm}$

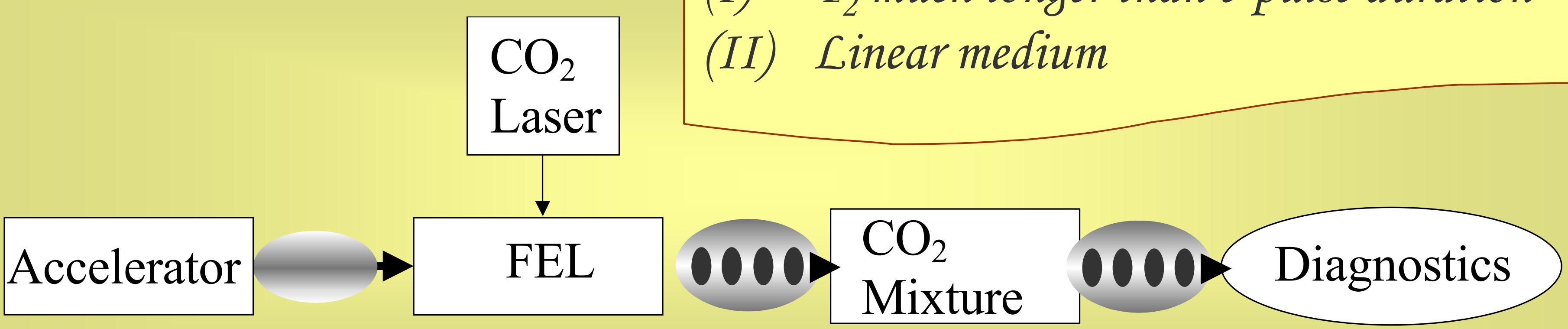
$$\eta_{Act} \simeq 14 \frac{N_{el}}{\gamma\text{-}1} \mu_1^2 \frac{w_{ph} \left(\pi r_{el}^2 d\right)}{\hbar\omega_0} \simeq 6.5[\%]$$

....., $w_{ph} \sim 10^3\,\text{J/m}^3$, $\mu_1 \sim 0.01$

Assumptions:

(I) T_2 much longer than e-pulse duration

(II) Linear medium



Summary

- *In an active medium a particle may be accelerated: Frank-Hertz, LASER and PASER*
- *Wake in an active medium may be amplified*
- *Space-charge wave in a resonant passive medium may become unstable (resonant absorption instability)*
- *Wake saturation in an active medium*
- *Self-consistent equations (e, field and active medium)*
- *Active enhancement of the Q-factor of an acceleration module*
- *PASER experiment – few percents energy change*