Bragg Acceleration Structures

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Outline

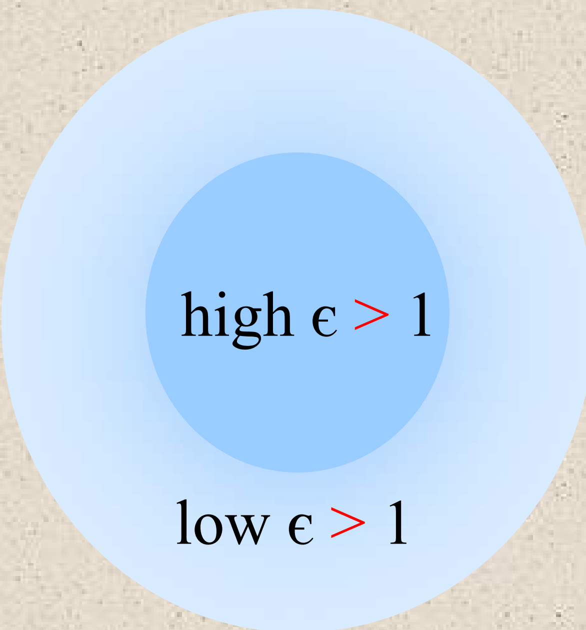
- Motivation and basic concept
- Confinement
- Accelerator parameters
- Wakes
- Efficiency

Motivation

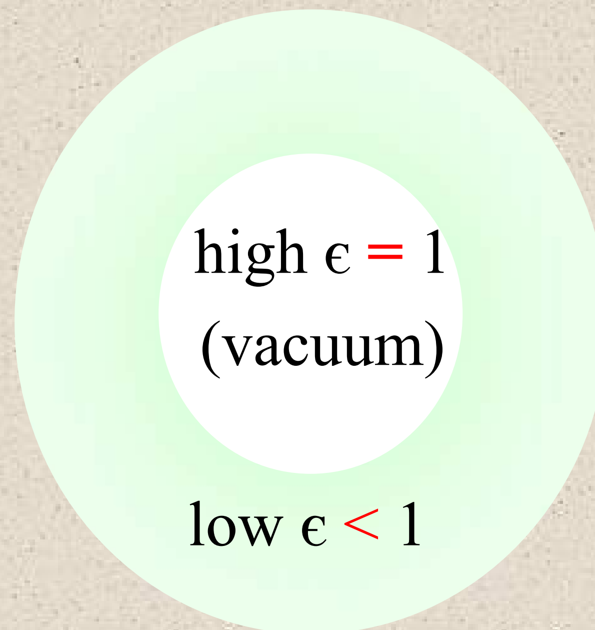
- Shorter and cheaper accelerators.
- Availability of high power lasers.
- Dielectrics sustain higher fields than metals.
- Fabrication: harness technology developed by communication or semiconductors industry.
- Need vacuum tunnel – confinement can not be achieved as in optical fibers – **Bragg waveguide!**

EM confinement

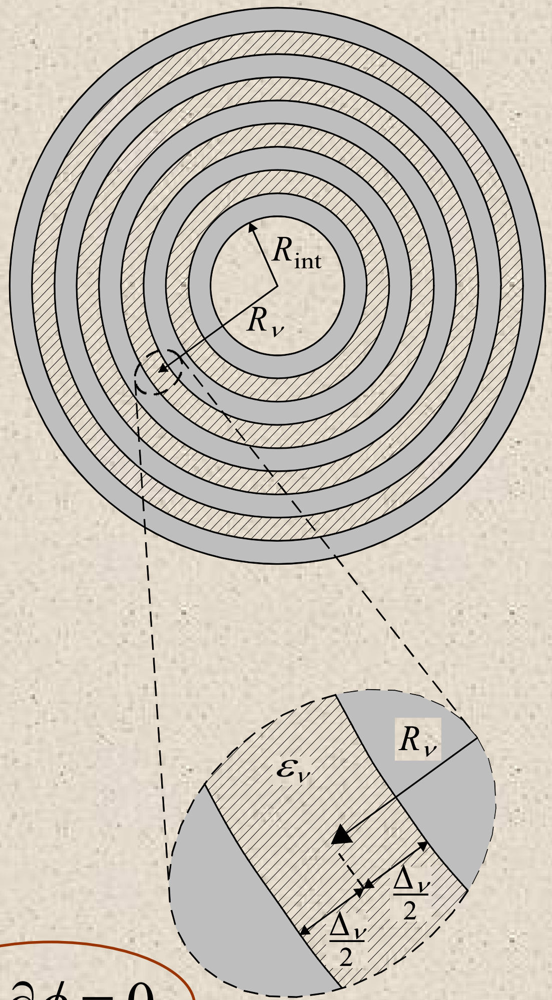
Optical fiber



Optical accelerator

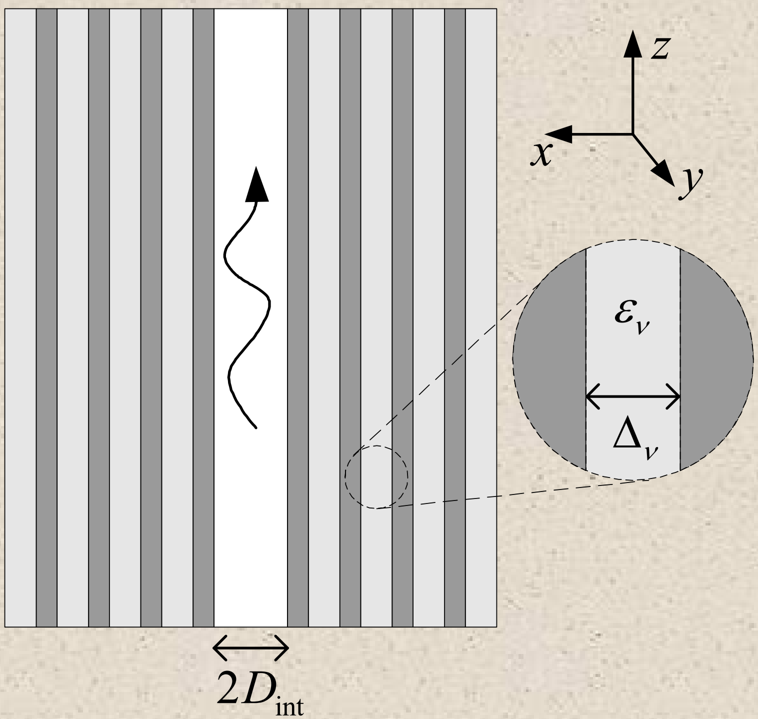


Cylindrical Bragg Fiber



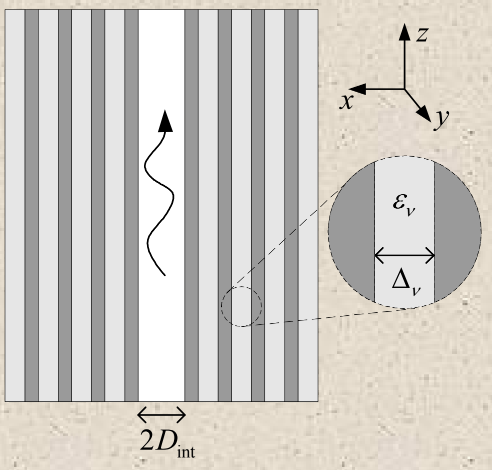
$\partial / \partial \phi \equiv 0$

Planar Bragg Waveguide



$\partial / \partial y \equiv 0$

Evanescent wave: Infinite case



Application to Bragg waveguides

- Yeh *et al.*, Opt. Commun. **19**, 427–430, (1976).
- Yeh *et al.*, JOSA, **68**, 1196–1201, (1978).

T – Unit cell Transition matrix of incoming and outgoing amplitudes of transverse waves

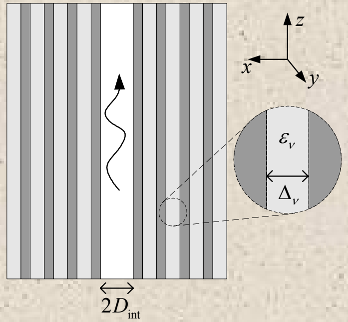
Eigen-value problem $\left|T - e^{-jKL} I\right| = 0$

L–periodicity, *K*–propagation coefficient

Dispersion relation $\cos(KL) = \frac{1}{2}(T_{11} + T_{22})$

Confinement condition $\left(\frac{T_{11} + T_{22}}{2}\right)^2 > 1$

Optimal Confinement



Confinement condition

$$\left(\frac{T_{11}+T_{22}}{2}\right)^2 = \left(\frac{(Z_1+Z_2)^2}{4Z_1Z_2}\cos(\chi_1+\chi_2) - \frac{(Z_1-Z_2)^2}{4Z_1Z_2}\cos(\chi_1-\chi_2)\right)^2$$

$$\chi_{1,2} \triangleq 2\pi\frac{\Delta_{1,2}}{\lambda_0}\sqrt{\varepsilon_{1,2}-1}$$

$$\Delta_{1,2} = \frac{\lambda_0}{4\sqrt{\varepsilon_{1,2}-1}}$$

Quarter λ structure !!

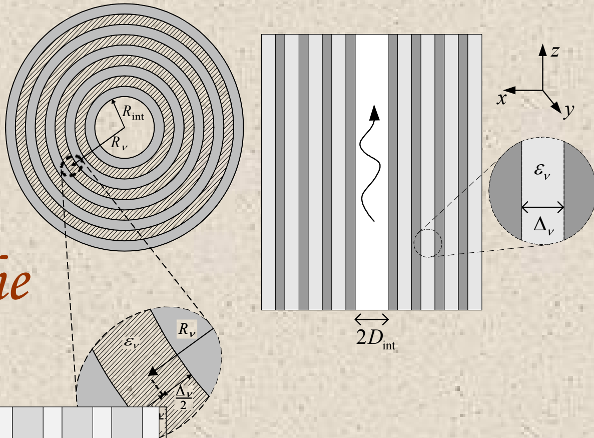
$$\left|e^{-jKL}\right|^n = \begin{cases} \left(\frac{Z_1}{Z_2}\right)^n & Z_1 < Z_2 \\ \left(\frac{Z_2}{Z_1}\right)^n & Z_1 > Z_2 \end{cases}$$

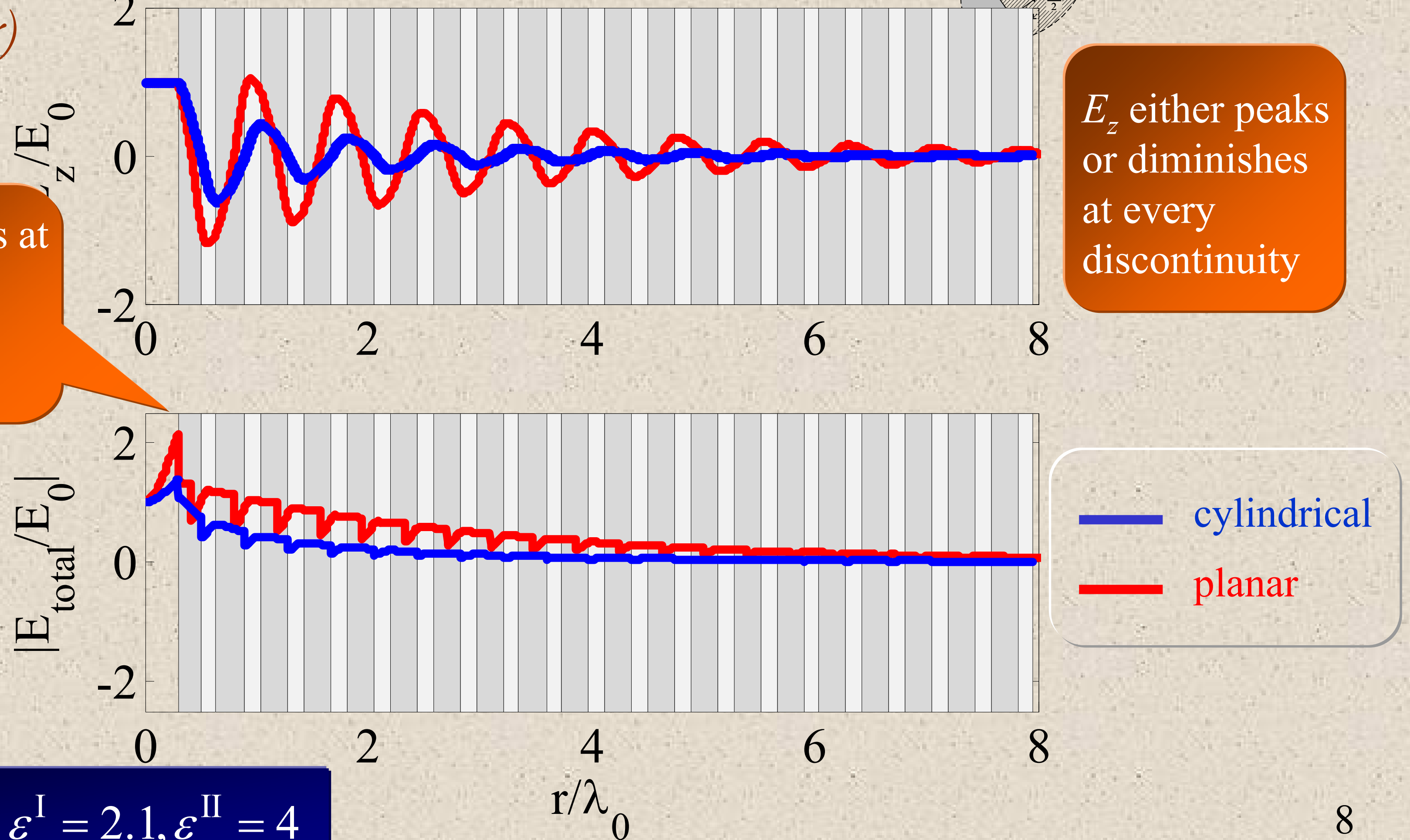
$$\left.\begin{matrix} Z_1 > Z_2 \\ x \simeq nL \end{matrix}\right\} \Rightarrow \left(\frac{Z_2}{Z_1}\right)^{2n} \simeq \left(\frac{Z_2}{Z_1}\right)^{2x/L} \triangleq \exp\left(-2\frac{x}{x_c}\right)$$

$$x_c = \frac{\lambda_0}{4}\left(\frac{1}{\sqrt{\varepsilon_1-1}} + \frac{1}{\sqrt{\varepsilon_2-1}}\right)\left|\ln^{-1}\left(\frac{\varepsilon_1\sqrt{\varepsilon_2-1}}{\varepsilon_2\sqrt{\varepsilon_1-1}}\right)\right|$$

Field Confinement

First dielectric layer matches between the eigen-mode of the vacuum tunnel and the eigen-mode of the Bragg structure (stub tunner)



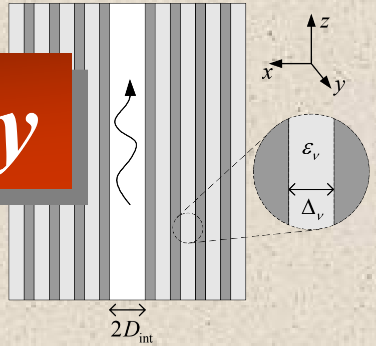


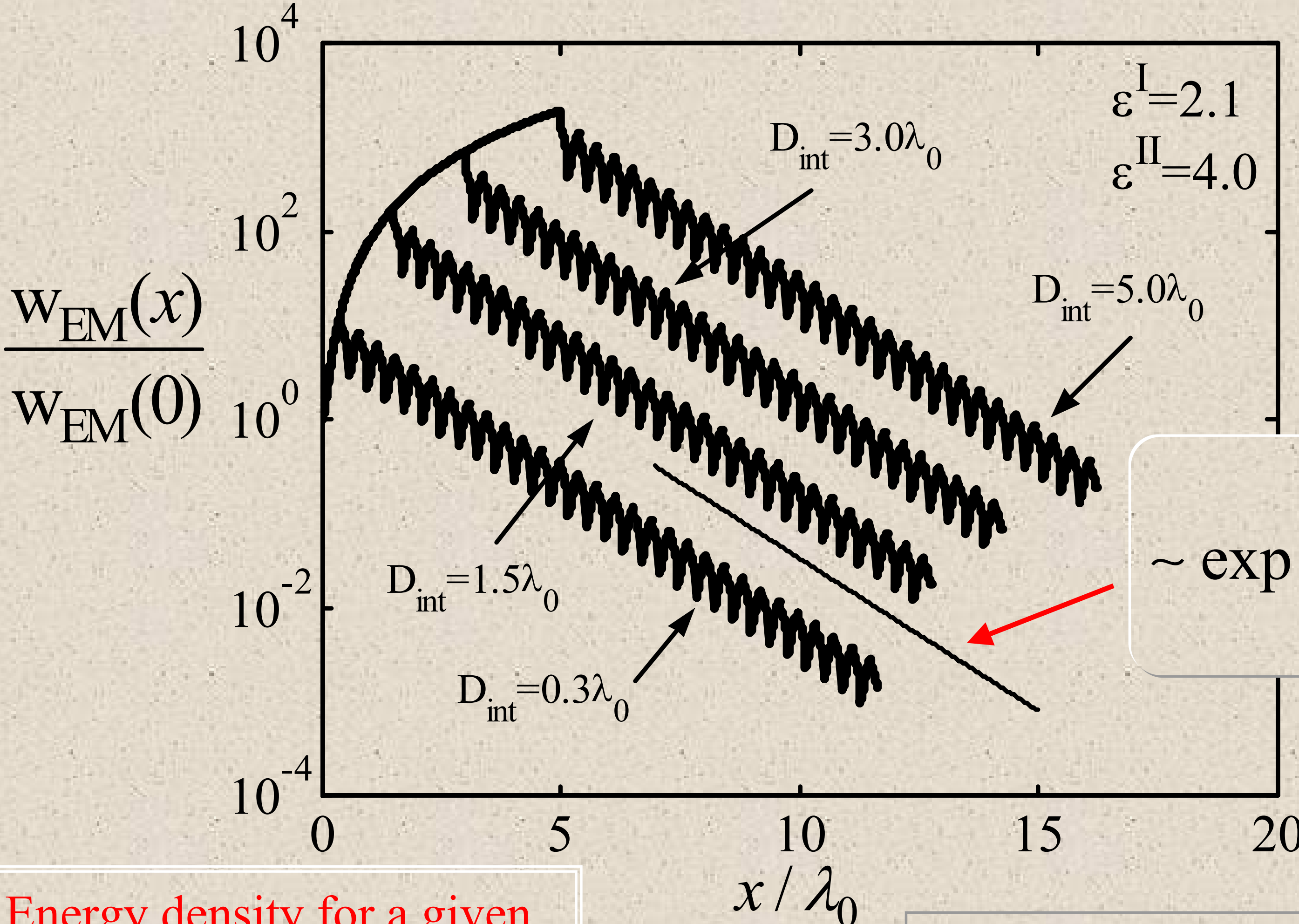
E_z either peaks or diminishes at every discontinuity

Maximum is at vacuum-dielectric interface

$$D_{int} = 0.3\lambda_0, \varepsilon^{I} = 2.1, \varepsilon^{II} = 4$$

Field Confinement – Energy Decay



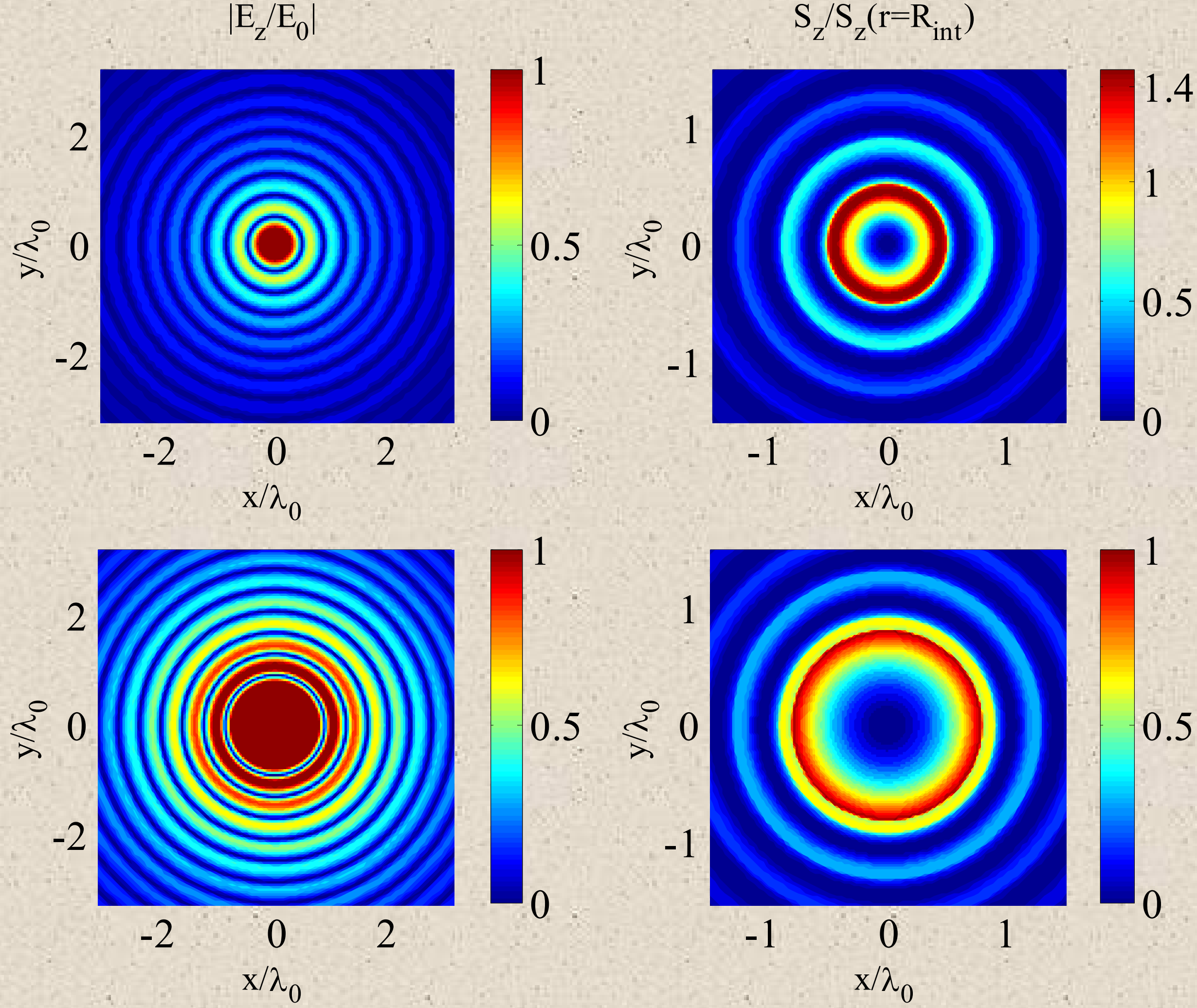


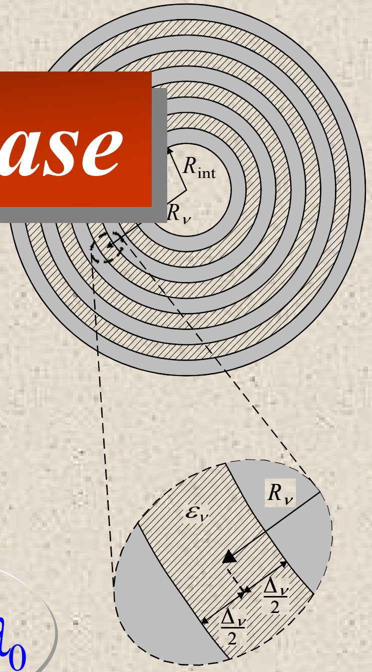
$$\sim \exp\left(-2\frac{x}{x_c}\right)$$

Energy density for a given E_0 increases for larger D_{int}/λ_0 !! Breakdown.

$$x_c = \frac{\lambda_0}{4}\left(\frac{1}{\sqrt{\varepsilon_1 - 1}} + \frac{1}{\sqrt{\varepsilon_2 - 1}}\right)\left|\ln^{-1}\left(\frac{\varepsilon_1\sqrt{\varepsilon_2 - 1}}{\varepsilon_2\sqrt{\varepsilon_1 - 1}}\right)\right|$$

Field Confinement – Cylindrical Case





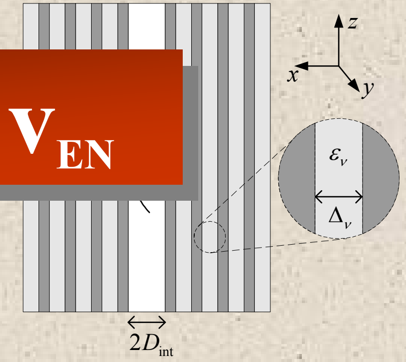
$R_{\text{int}} = 0.3\lambda_0$

$\varepsilon^{\text{I}} = 2.1$

$\varepsilon^{\text{II}} = 4$

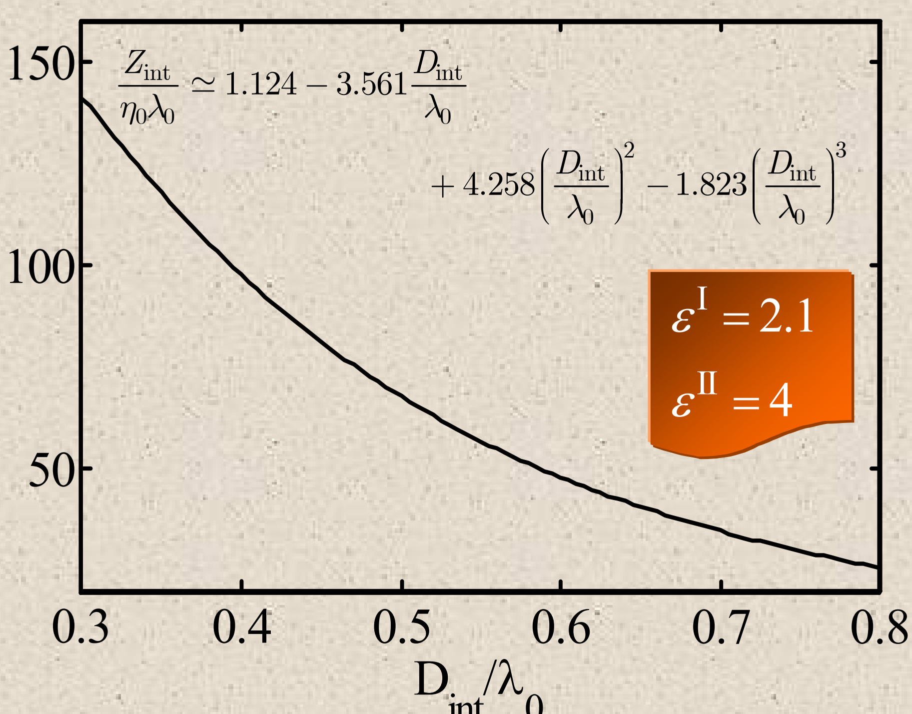
$R_{\text{int}} = 0.8\lambda_0$

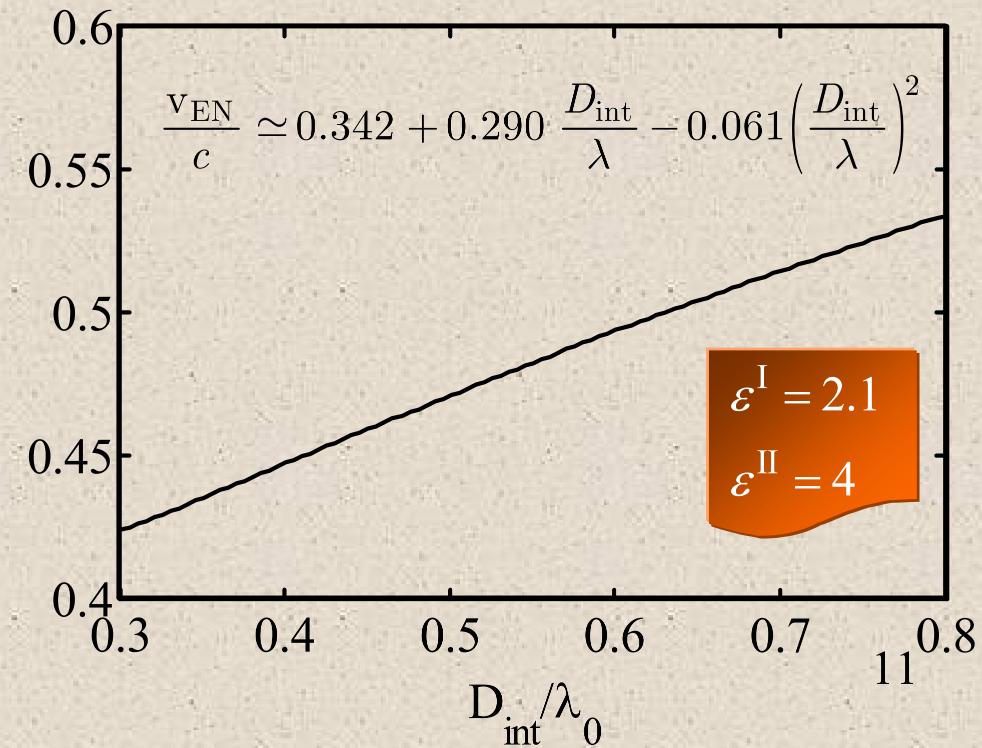
Accelerator Parameters – Z_{int} and v_{EN}



$$Z_{\text{int}}\,[\Omega m] \triangleq \frac{(\lambda_0 E_0)^2}{P}$$

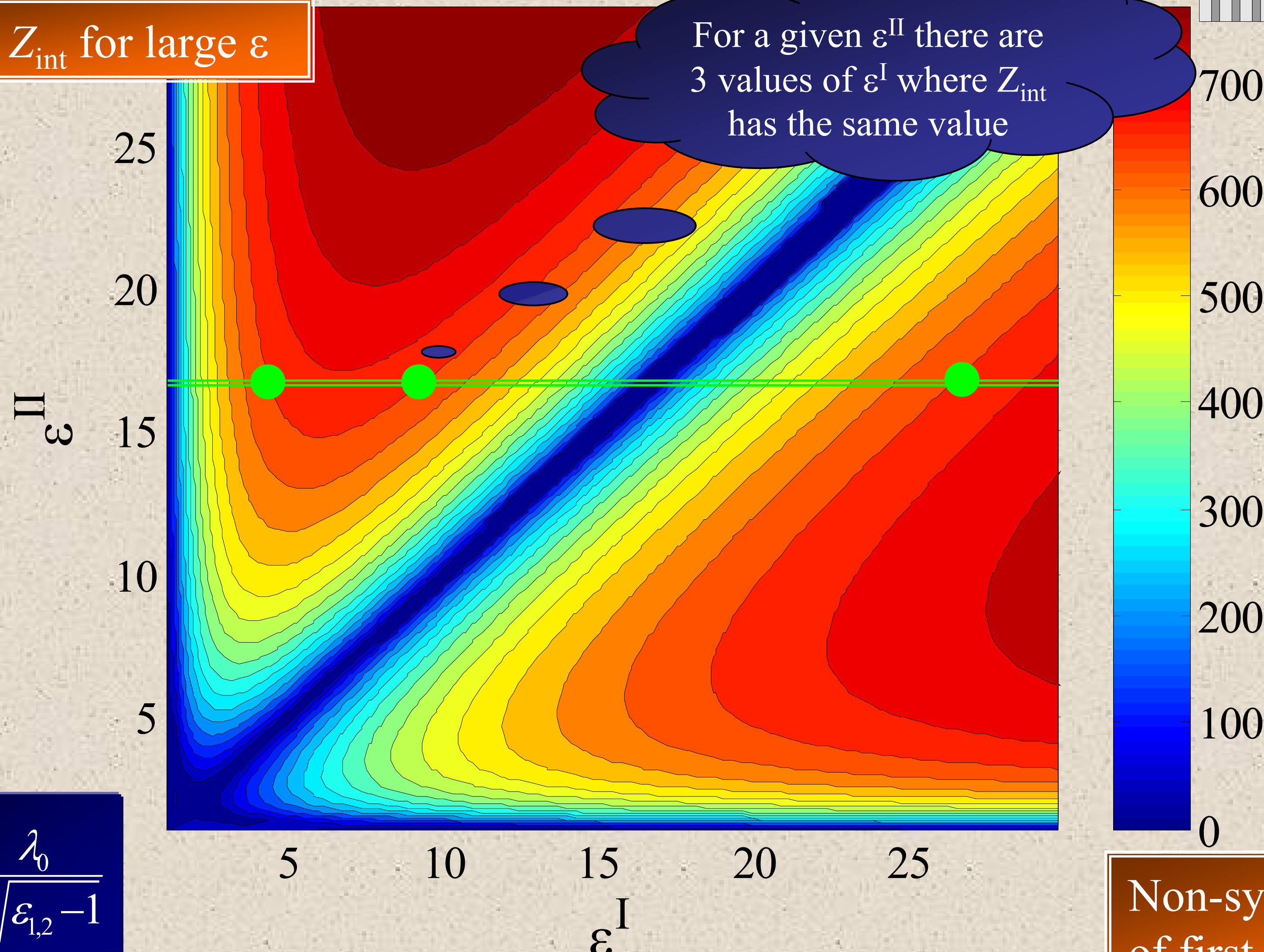
$$\frac{\mathrm{v}_{EN}}{c} \triangleq \frac{P}{cW}$$

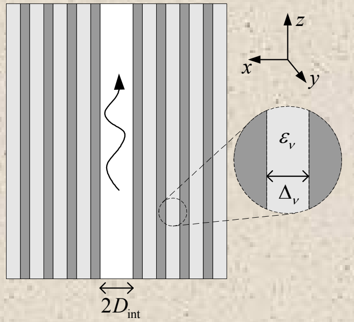




Accelerator Parameters – Z_{int}

High Z_{int} for large ε



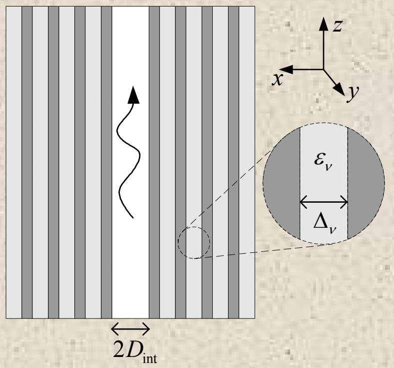


$D_{int} = 0.3\lambda_0$

$$\Delta_{1,2} = \frac{\lambda_0}{4\sqrt{\varepsilon_{1,2}-1}}$$

Non-symmetry: role of first layer

Accelerator Parameters – E_{max}



Planar

$$E_z = E_0 e^{-j\frac{\omega}{c}z}$$

$$E_x = j\frac{\omega}{c} x E_0 \, e^{-j\frac{\omega}{c}z}$$

$$\frac{E_{max}}{E_0} = \sqrt{1 + \left(2\pi \frac{D_{int}}{\lambda_0}\right)^2}$$

1 [GV/m]

2 [GV/m]

$$\frac{E_{acc}}{E_{max}} = 0.5$$

$D_{int} \simeq 0.28\lambda_0$

$R_{int} \simeq 0.55\lambda_0$

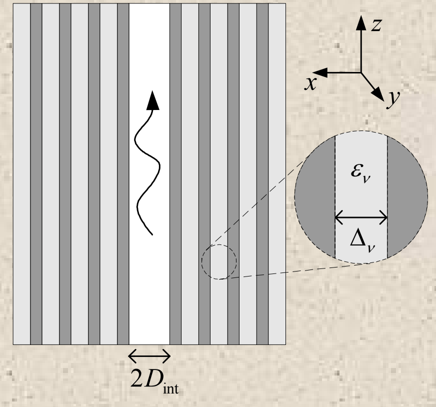
Cylindrical

$$E_z = E_0 e^{-j\frac{\omega}{c}z}$$

$$E_r = j\frac{\omega}{c}\frac{1}{2} r E_0 \, e^{-j\frac{\omega}{c}z}$$

$$\frac{E_{max}}{E_0} = \sqrt{1 + \left(\pi \frac{R_{int}}{\lambda_0}\right)^2}$$

Wake-Field



Moving line charge

q – charge per unit length

$$J_z(x,z,t) = -q\,\text{v}\,\delta(x)\delta(z-\text{v}t)$$

$$\beta \triangleq \text{v}/c$$

$$\gamma \triangleq 1/\sqrt{1-\text{v}^2/c^2}$$

$$\Gamma \triangleq \frac{|\omega|}{c\gamma\beta}$$

$$\Lambda \triangleq \frac{|\omega|}{c}\sqrt{\varepsilon_1-\beta^{-2}}$$

Primary potential

$$A_z^{(p)}(x,z,t) = -\frac{q\mu_0}{4\pi}\int_{-\infty}^{\infty} d\omega\, e^{j\omega(t-z/\text{v})}\frac{1}{\Gamma}e^{-\Gamma|x|}$$

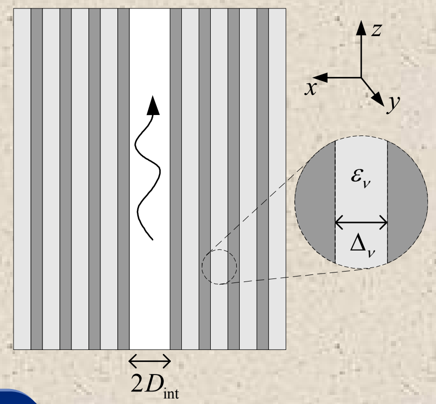
Secondary potential

$$A_z^{(s)}(x,z,t) = -\frac{q\mu_0}{4\pi}\int_{-\infty}^{\infty} d\omega\, e^{j\omega(t-z/\text{v})}\frac{1}{\Gamma}\begin{cases} B_0\cosh(\Gamma x) & |x|<D_{\text{int}} \\ -\dfrac{\varepsilon_1}{\gamma^2\beta^2\left(\varepsilon_1-\beta^{-2}\right)}\left(C_0 e^{-j\Lambda x}+D_0 e^{j\Lambda x}\right) & x>D_{\text{int}} \end{cases}$$

$$R_{11} \triangleq \frac{D_0}{C_0}e^{2j\Lambda D_{\text{int}}}$$

All waves have $k_z=\omega/\text{v}$!!

Wake-Field On Axis



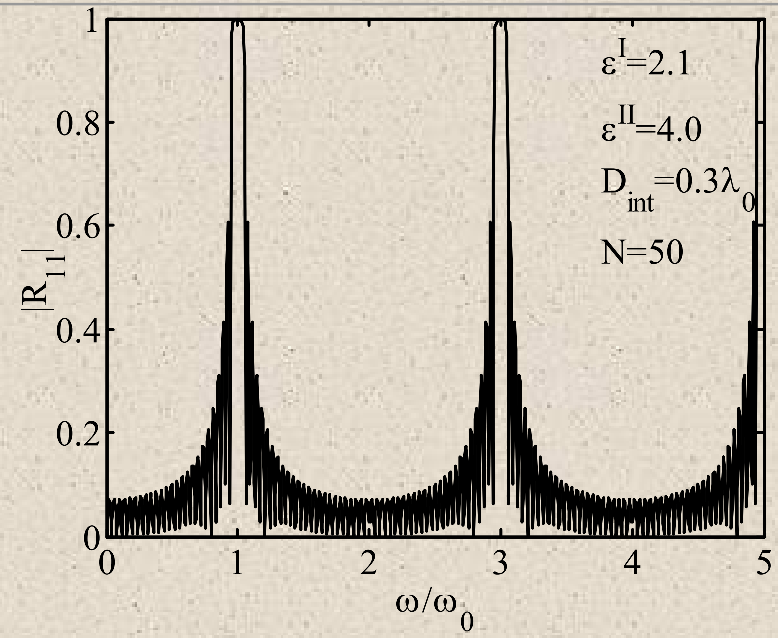
Ultra-relativistic wake-field on axis $\gamma \to \infty$

$$E_z^{(s)}(\bar{\tau}) = \frac{q}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{1}{2}\int_{-\infty}^{\infty} d\bar{\omega}\, e^{j\bar{\omega}\bar{\tau}} \frac{1+R_{11}(\bar{\omega})}{(1+j\bar{\omega})-(1-j\bar{\omega})R_{11}(\bar{\omega})}$$

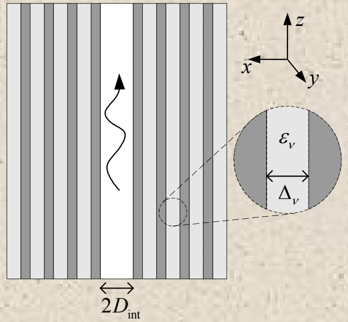
$$\bar{\omega} \triangleq \frac{\omega}{c} D_{\text{int}} \frac{\sqrt{\varepsilon_1 - 1}}{\varepsilon_1}$$

$$\bar{\tau} \triangleq \left(t - \frac{z}{c}\right)\frac{c}{D_{\text{int}}}\frac{\varepsilon_1}{\sqrt{\varepsilon_1 - 1}}$$

Reflection coefficient (analytic expression)



Wake-field – t=0 picture

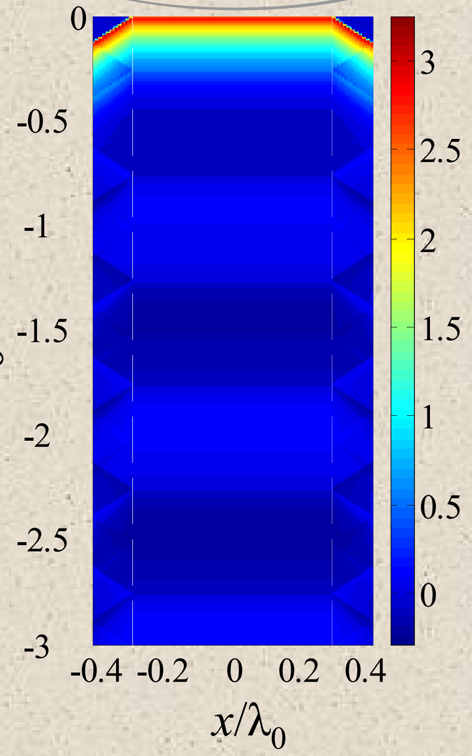


$D_{\text{int}} = 0.3\lambda_0$

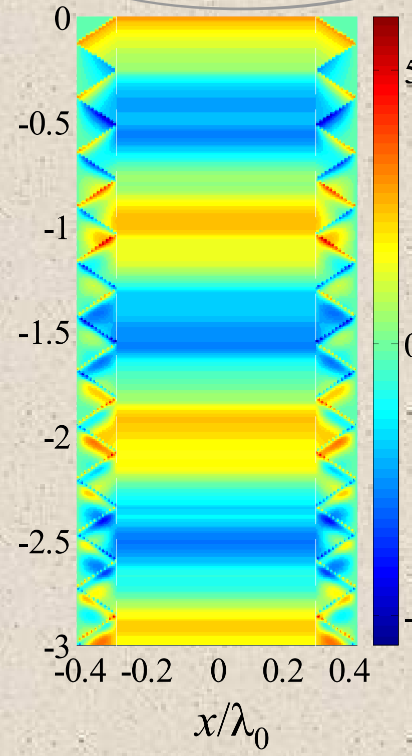
$\varepsilon^{\text{I}} = 2.1$

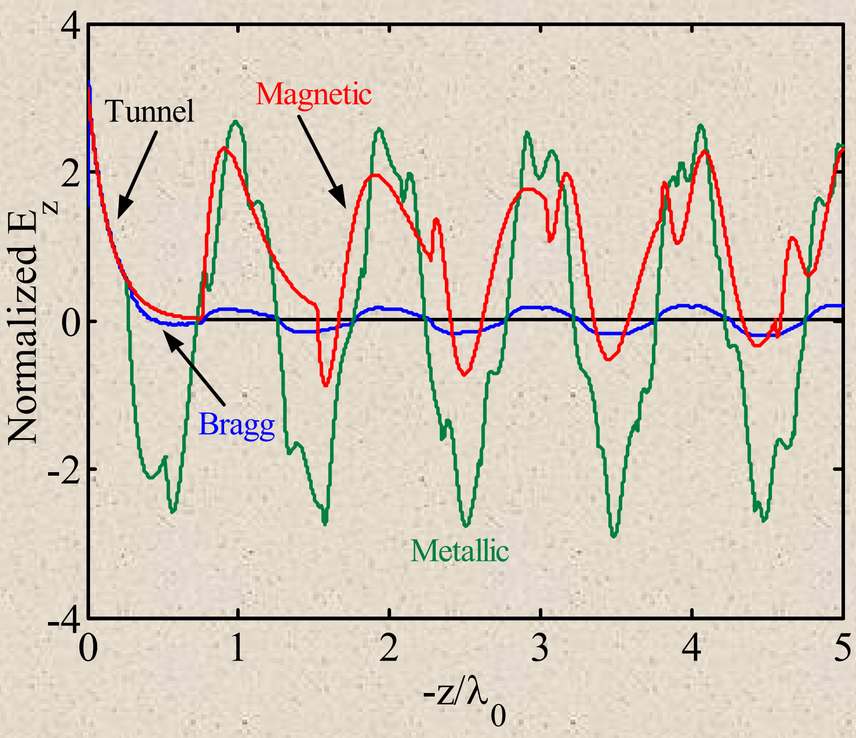
$\varepsilon^{\text{II}} = 4$

Bragg Structure

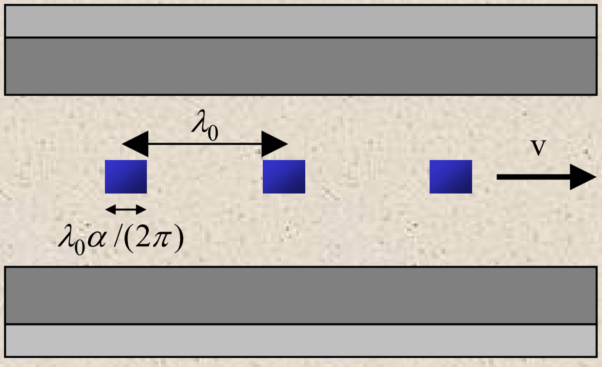


Dielectric Loaded





Emitted Power – Qualitative Approach



one line-charge

$$P = \frac{\mathrm{v} q^2}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{\pi}{2}$$

Total power neglecting mutual effects. $q = N_{el} q_{el}$, $\alpha = 0$

$$P = \frac{\mathrm{v} q_{el}^2 N_{el}^2 / M^2}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{\pi}{2} \times M = \frac{\mathrm{v} q_{el}{}^2}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{N_{el}^2}{M} \times \frac{\pi}{2}$$

$\sim 1/M$

One micro-bunch (line-charge)

$$P = \frac{\mathrm{v} q_{el}^2}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{\pi}{2} \times N_{el}^2$$

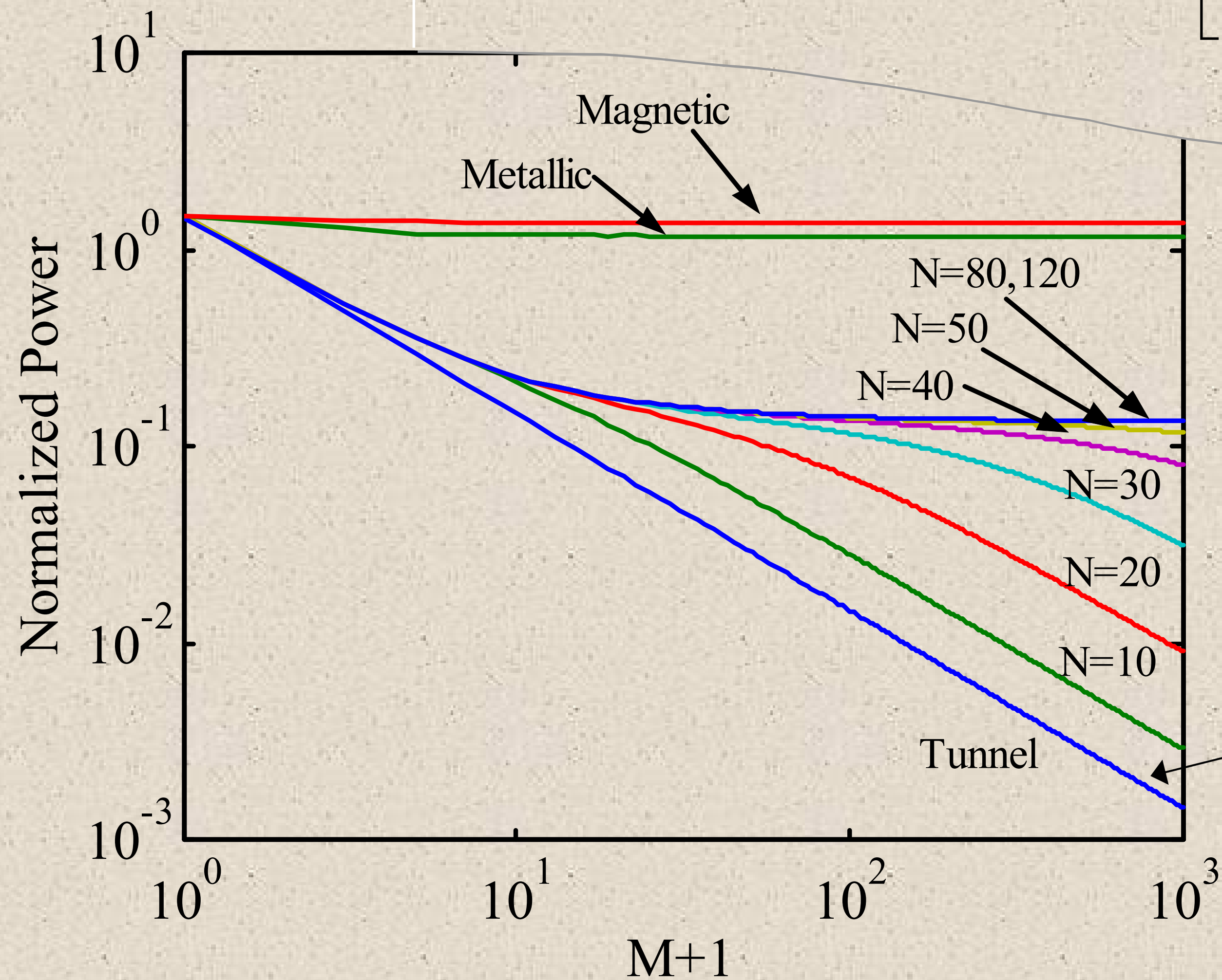
Randomly distributed

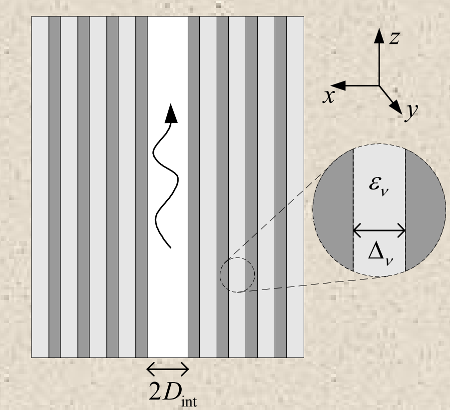
$$P = \frac{\mathrm{v} q_{el}^2}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{\pi}{2} \times N_{el}$$

$$P=\frac{vq^2}{2\pi\varepsilon_0 D_{\text{int}}}\frac{1}{2}\int_{-\infty}^{\infty} d\bar{\omega}\frac{1+R_{11}(\bar{\omega})}{(1+j\bar{\omega})-(1-j\bar{\omega})R_{11}(\bar{\omega})}$$

$$\times\,\text{sinc}^2\left[\frac{\alpha}{2}\frac{\bar{\omega}}{\bar{\omega}_0}\right]\frac{\text{sinc}^2\left[\pi\frac{\bar{\omega}}{\bar{\omega}_0}(M+1)\right]}{\text{sinc}^2\left[\pi\frac{\bar{\omega}}{\bar{\omega}_0}\right]}$$

Emitted Power





Efficiency

- *Wake parameter:*

  Decelerating field for a given charge ………… $E_{\text{dec}} \equiv \kappa q$

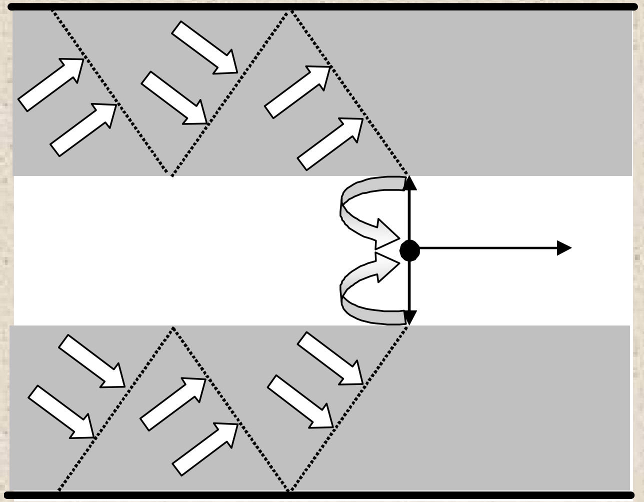
- *Beam-loading parameter:*

  Beam-loading of the accelerating mode ……….. $E_{\text{dec}}^{(\text{F})} \equiv \kappa_1 q$

$$E_z(r=0, \tau = t - z/c) \simeq q\kappa \sum_{n=1}^{\infty} W_n \cos(\omega_n \tau) 2h(\tau)$$

$$W_n = \left[\frac{2J_1(p_n R_b / R_{ext})}{p_n J_1(p_n)} \right]^2$$

$$\sum_{n=1}^{\infty} W_n = 1$$

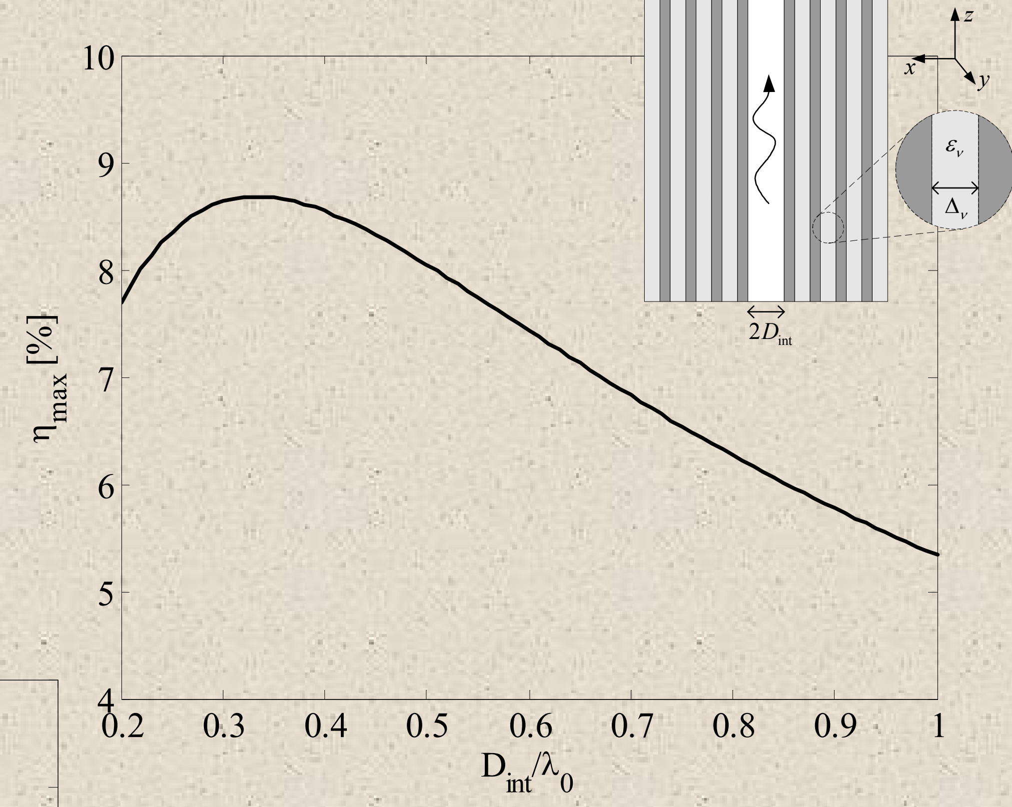


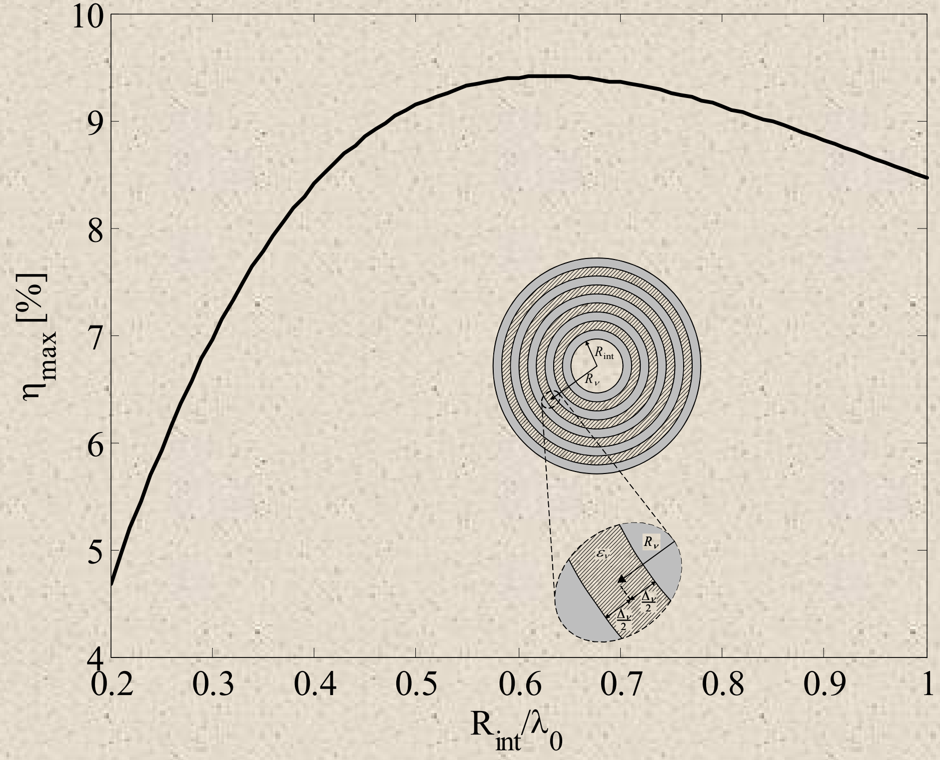
$$\kappa_1 \equiv \kappa W_1$$

$$\kappa_1 = \frac{\beta_{\text{gr}}}{1-\beta_{\text{gr}}} \frac{Z_{\text{int}}}{\sqrt{\mu_0/\varepsilon_0}} \frac{\pi}{4\pi\varepsilon_0\lambda^2}$$

$$\eta_{\max} = \frac{\kappa_1}{\kappa}$$

Efficiency





$\varepsilon^{I} = 2.1$

$\varepsilon^{II} = 4$

Summary

- Detailed design of Bragg acceleration structures – theoretical feasibility was introduced (PRE 2004)
- Structure parameters (interaction impedance, group velocity, maximal field).
- “Better” materials can dramatically improve performance.
- Analysis of wake-field – power decreases with the number of micro-bunches, and increases with the number of layers.
- Efficiency has an optimum within an internal dimension range of $0.3 \div 0.8\lambda_0$.

Summary - parameters

Silica-Zirconia structure, $R_{int}, D_{int} = 0.3 \div 0.8\lambda_0$

$\varepsilon^{I} = 2.1$

$\varepsilon^{II} = 4$

	Planar	**Cylindrical**
Z_{int} [Ω]	$147 \div 25\ \lambda_0/\Delta_y$	$268 \div 37$
β_{gr}	$0.42 \div 0.53$	$0.41 \div 0.48$
E_{acc}/E_{max}	$0.47 \div 0.20$	$0.73 \div 0.37$
κ	$1/4\varepsilon_0 D_{int}$	$1/2\pi\varepsilon_0 R_{int}^2$
η_{max} [%]	$8.7 \div 6.23$	$7 \div 9.4$