Electrons Acceleration in Active Medium

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Outline

- Overview & Motivation
- PASER: Particle Acceleration by Stimulated Emission of Radiation
- Wake Amplification
- Acceleration in a growing wake
- Acceleration & Saturation
- Summary

Acceleration & Saturation

Interaction of a single-mode with a bunch of electrons

$$\left.\begin{aligned}
&\frac{d}{d\xi} a = \alpha \left\langle e^{-j\chi_i} \right\rangle \\
&\frac{d}{d\xi}\gamma_i = -\frac{1}{2}\left[a\, e^{j\chi_i} + c.c. \right] \\
&\frac{d}{d\xi}\chi_i = \Omega\left(\frac{1}{\beta_i} - \frac{1}{\beta_p} \right)
\end{aligned}\right\} \Rightarrow \frac{d}{d\xi}\left[\underbrace{\left\langle \gamma_i \right\rangle - 1}_{Kinetic\ Energy} + \underbrace{\frac{|a|^2}{2\alpha}}_{EM\ Energy} \right] = 0$$

Energy Conservation

Acceleration & Saturation

Energy conservation in the presence of Active Medium

Photon Density

$$\frac{d}{d\xi}\left[\underbrace{\langle\gamma_i\rangle - 1}_{Kinetic\ Energy} + \underbrace{\frac{|a|^2}{2\alpha}}_{EM\ Energy} + \underbrace{\frac{N_{ph}\hbar\omega}{N_e mc^2}}_{Energy\ in\ Medium}\right] = 0$$

Electron Density

Acceleration & Saturation

The effect on the population inversion

$$\left.\begin{aligned} \frac{d}{d\xi}a &= \alpha\left\langle e^{-j\chi_i}\right\rangle + \left(\frac{1}{2}\sigma N_{ph} d\right)a \\ \frac{d}{d\xi}\gamma_i &= -\frac{1}{2}\left[a e^{j\chi_i} + c.\,c.\right] \end{aligned}\right\} \Rightarrow \begin{aligned} &\frac{d}{d\xi}\left[\left\langle\gamma_i\right\rangle - 1 + \frac{|a|^2}{2\alpha}\right] = \left(\frac{|a|^2}{2\alpha}\right)\left(\sigma N_{ph} d\right) \\ &\frac{d}{d\xi}\left[\left\langle\gamma_i\right\rangle - 1 + \frac{|a|^2}{2\alpha} + \frac{N_{ph}\hbar\omega}{N_e mc^2}\right] = 0 \end{aligned}$$

Inversion equation

$$\Rightarrow \quad \frac{d}{d\xi}N_{ph} = -\left(\frac{|a|^2}{2\alpha}\right)\left(\sigma d N_e \frac{mc^2}{\hbar\omega}\right)N_{ph}$$

Acceleration & Saturation

Summary of governing equations

$$\left.\begin{aligned}
&\frac{d}{d\xi}a = \alpha\left\langle e^{-j\chi_i}\right\rangle + \left(\frac{1}{2}\sigma N_{ph} d\right)a \\
&\frac{d}{d\xi}\gamma_i = -\frac{1}{2}\left[a\,e^{j\chi_i} + c.c.\right] \\
&\frac{d}{d\xi}\chi_i = \Omega\left(\frac{1}{\beta_i} - \frac{1}{\beta_p}\right) \\
&\frac{d}{d\xi}N_{ph} = -\left(\frac{|a|^2}{2\alpha}\right)\left(\sigma\, d\, N_e \frac{mc^2}{\hbar\omega}\right)N_{ph}
\end{aligned}\right\} \Rightarrow \frac{d}{d\xi}\left[\underbrace{\left\langle\gamma_i\right\rangle - 1}_{\text{Kinetic Energy}} + \underbrace{\frac{|a|^2}{2\alpha}}_{\text{EM Energy}} + \underbrace{\frac{N_{ph}\hbar\omega}{N_e\, mc^2}}_{\text{Energy in Medium}}\right] = 0$$

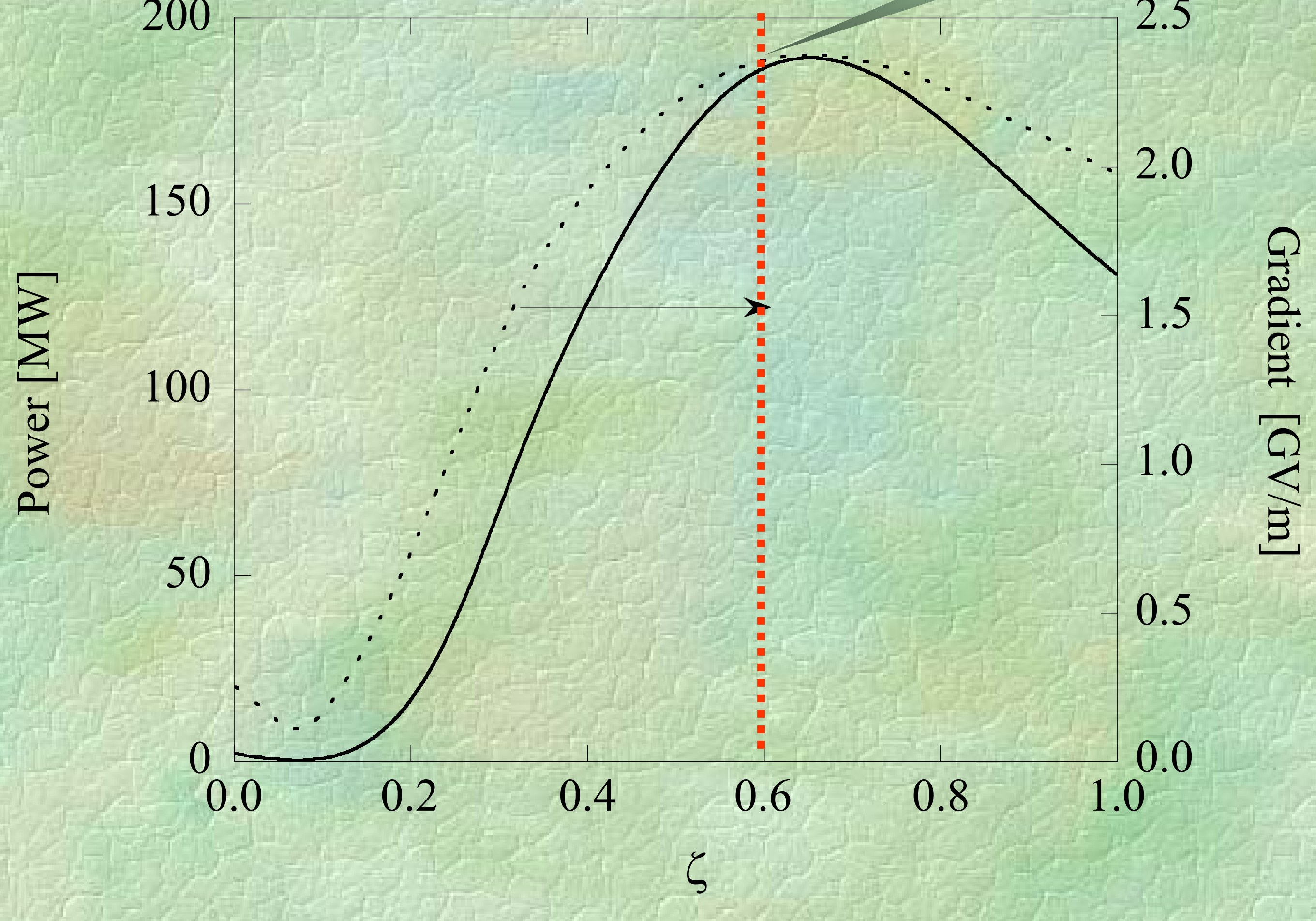
Acceleration & Saturation

Simulation parameters:

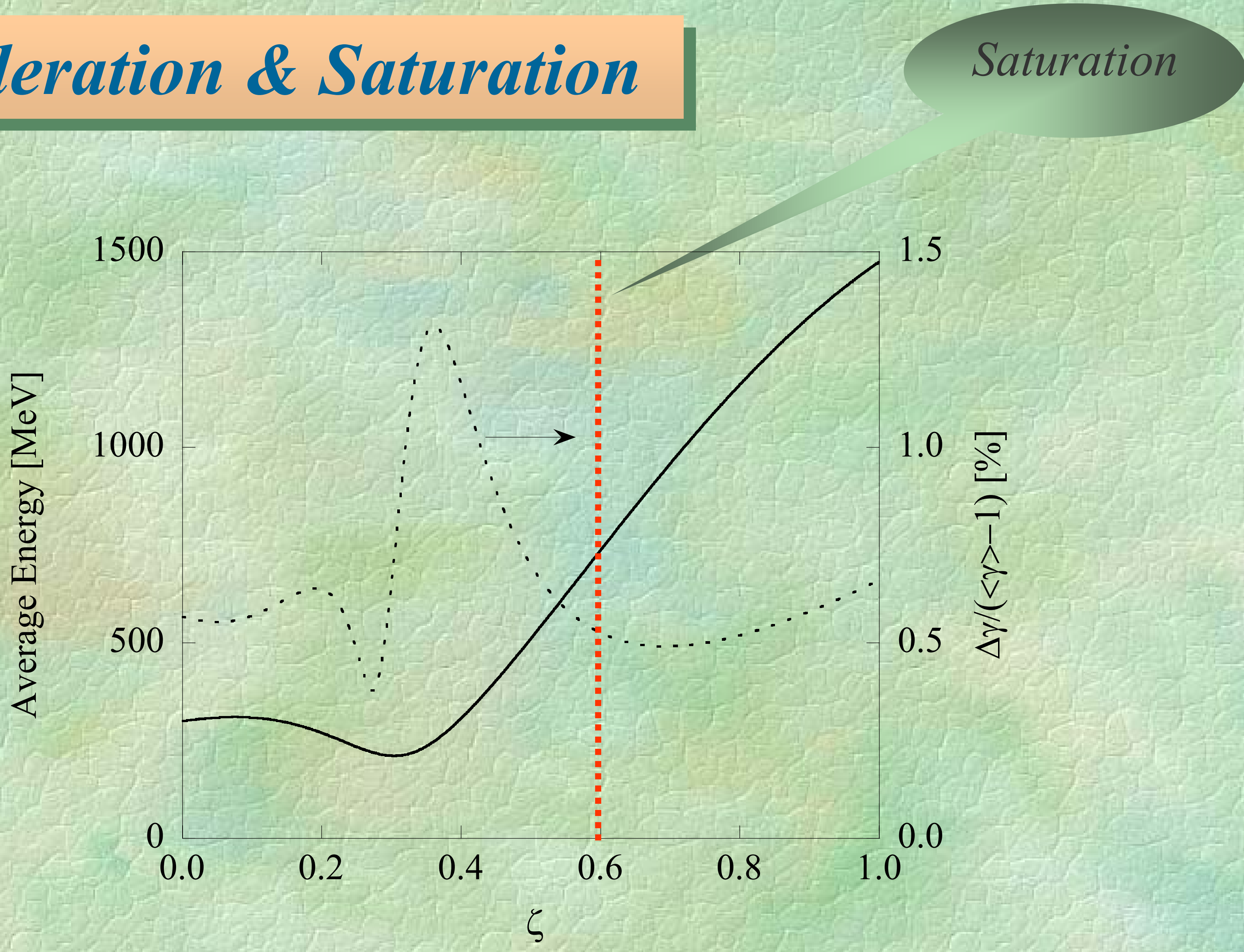
λ [μm]	1.06
α	$7\text{x } 10^3$
N_e [m^{-3}]	10^5
Energy [MeV]	300
N_{ph} [m^{-3}]	10^{25}
P_{in} [MW]	2
σ [m^2]	10^{-24}

Acceleration & Saturation

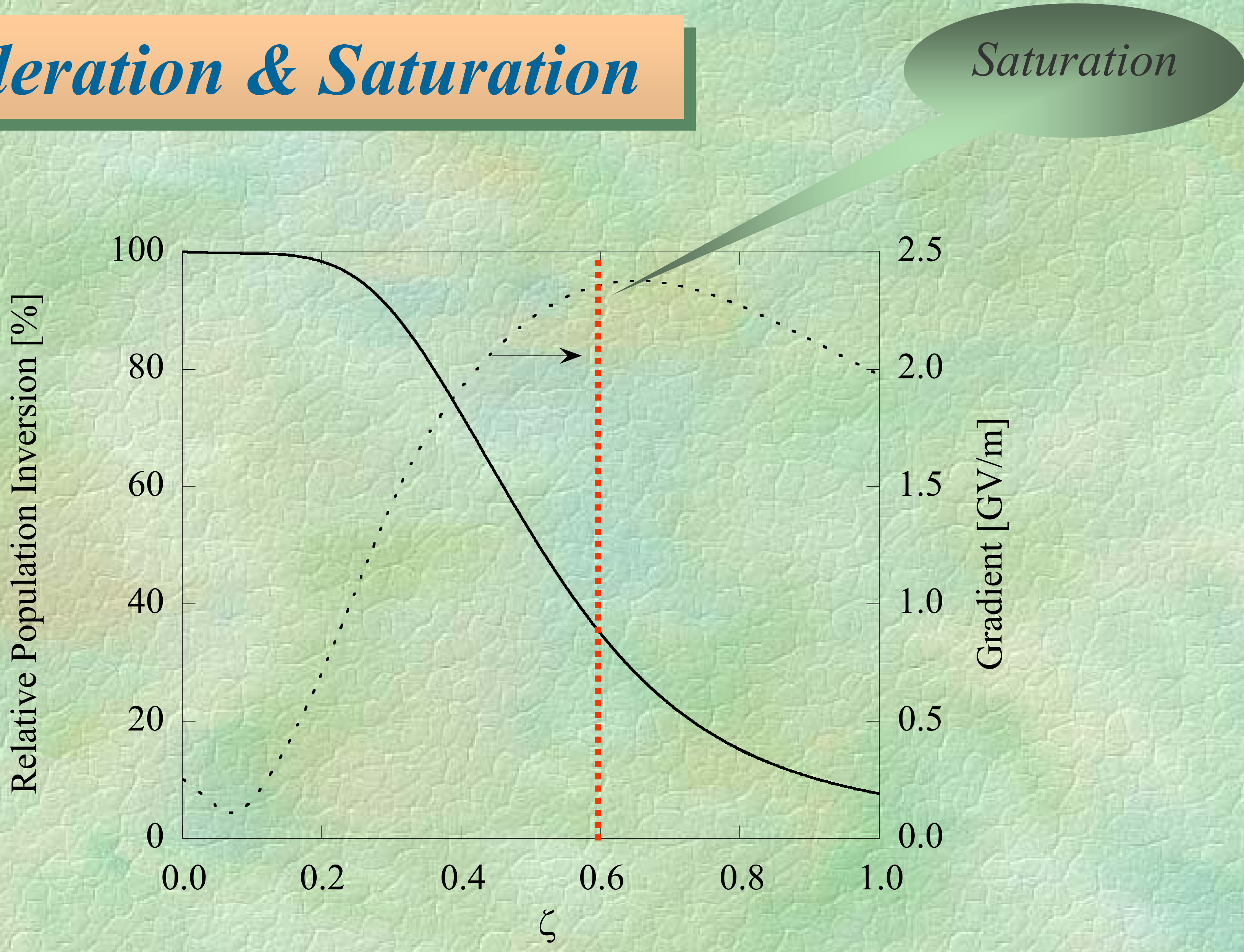
Saturation



Acceleration & Saturation



Acceleration & Saturation



Summary

- *PASER: Point-charge accelerated by energy stored in the medium*
- *Same energy amplifies a wake-field - Cerenkov*
- *Eigen-modes move at the speed of the bunch*
- *Inherent longitudinal e-field: interaction length*
- *Emittance growth < 0.1π mm-mrad*
- *Acceleration not affected by medium saturation*