Electrons Acceleration in Active Medium

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Outline

- Overview & Motivation
- PASER: Particle Acceleration by Stimulated Emission of Radiation
- Wake Amplification
- Acceleration in a growing wake
- Acceleration & Saturation
- Summary

Acceleration & Saturation

Interaction of a single-mode with a bunch of electrons

$$\left.\begin{aligned}
&\frac{d}{d\xi} a = \alpha \left\langle e^{-j\chi_i} \right\rangle \\
&\frac{d}{d\xi} \gamma_i = -\frac{1}{2}\left[a\, e^{j\chi_i} + c.c. \right] \\
&\frac{d}{d\xi} \chi_i = \Omega \left(\frac{1}{\beta_i} - \frac{1}{\beta_p} \right)
\end{aligned}\right\} \Rightarrow \frac{d}{d\xi}\left[\underbrace{\langle \gamma_i \rangle - 1}_{Kinetic\ Energy} + \frac{|a|^2}{\underbrace{2\alpha}_{EM\ Energy}} \right] = 0$$

Energy Conservation

Acceleration & Saturation

Energy conservation in the presence of Active Medium

Photon Density

$$\frac{d}{d\xi}\left[\underbrace{\langle\gamma_i\rangle - 1}_{\text{Kinetic Energy}} + \underbrace{\frac{|a|^2}{2\alpha}}_{\text{EM Energy}} + \underbrace{\frac{N_{ph}\hbar\omega}{N_e mc^2}}_{\text{Energy in Medium}}\right] = 0$$

Electron Density

Acceleration & Saturation

The effect on the population inversion

$$\left.\begin{aligned} \frac{d}{d\xi} a &= \alpha \left\langle e^{-j\chi_i} \right\rangle + \left(\frac{1}{2} \sigma N_{ph} d \right) a \\ \frac{d}{d\xi} \gamma_i &= -\frac{1}{2} \left[a e^{j\chi_i} + c.c. \right] \end{aligned}\right\} \Rightarrow \quad \frac{d}{d\xi} \left[\left\langle \gamma_i \right\rangle - 1 + \frac{|a|^2}{2\alpha} \right] = \left(\frac{|a|^2}{2\alpha} \right) \left(\sigma N_{ph} d \right)$$

$$\frac{d}{d\xi} \left[\left\langle \gamma_i \right\rangle - 1 + \frac{|a|^2}{2\alpha} + \frac{N_{ph} \hbar\omega}{N_e mc^2} \right] = 0$$

Inversion equation

$$\Rightarrow \quad \frac{d}{d\xi} N_{ph} = -\left(\frac{|a|^2}{2\alpha} \right) \left(\sigma d N_e \frac{mc^2}{\hbar\omega} \right) N_{ph}$$

Acceleration & Saturation

Summary of governing equations

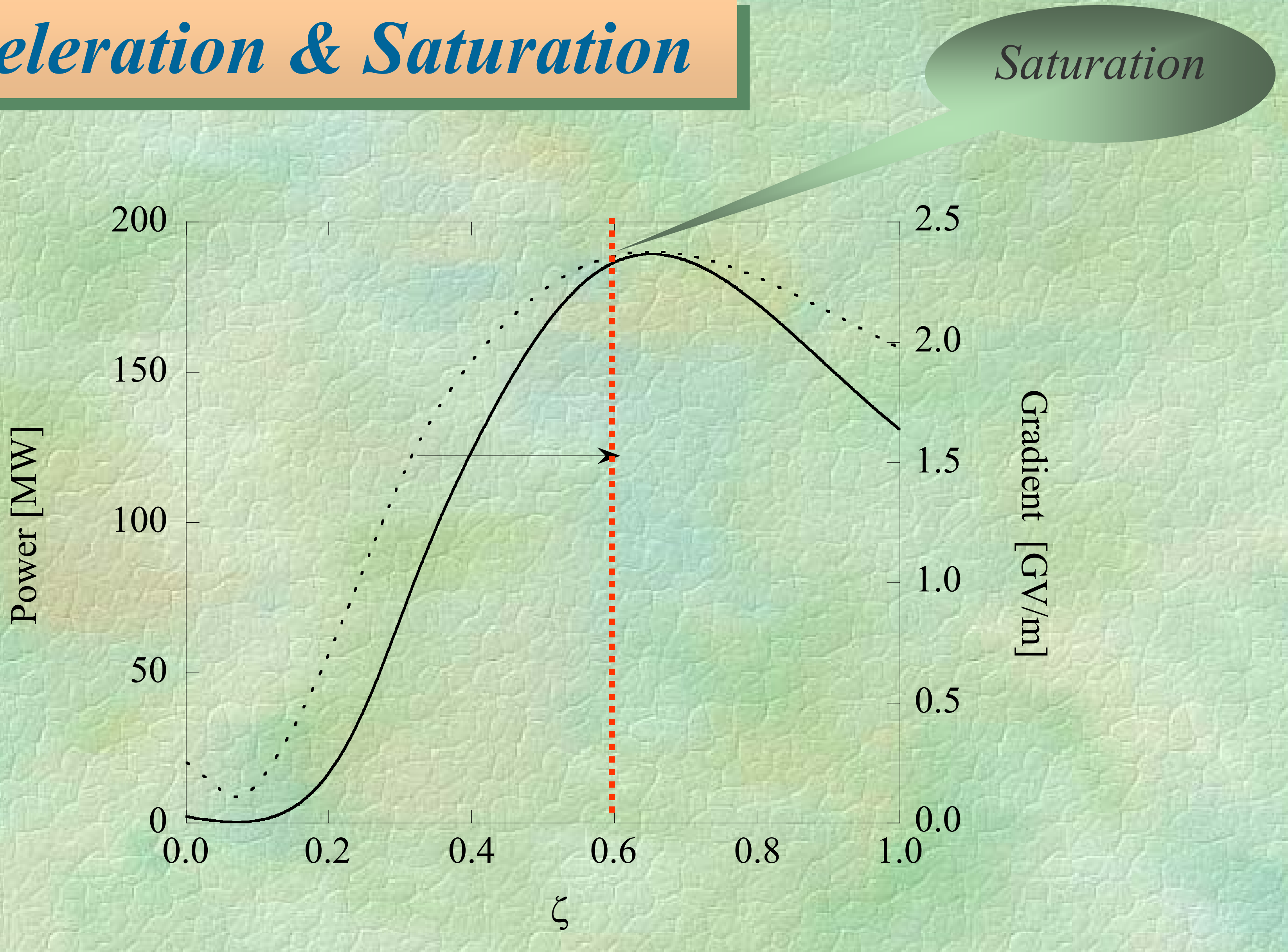
$$\left.\begin{aligned}
&\frac{d}{d\xi}a = \alpha\left\langle e^{-j\chi_i}\right\rangle + \left(\frac{1}{2}\sigma N_{ph} d\right)a \\
&\frac{d}{d\xi}\gamma_i = -\frac{1}{2}\left[a\, e^{j\chi_i} + c.c.\right] \\
&\frac{d}{d\xi}\chi_i = \Omega\left(\frac{1}{\beta_i} - \frac{1}{\beta_p}\right) \\
&\frac{d}{d\xi}N_{ph} = -\left(\frac{|a|^2}{2\alpha}\right)\left(\sigma\, d\, N_e \frac{mc^2}{\hbar\omega}\right)N_{ph}
\end{aligned}\right\} \Rightarrow \frac{d}{d\xi}\left[\underbrace{\left\langle\gamma_i\right\rangle - 1}_{Kinetic\ Energy} + \underbrace{\frac{|a|^2}{2\alpha}}_{EM\ Energy} + \underbrace{\frac{N_{ph}\hbar\omega}{N_e\, mc^2}}_{Energy\ in\ Medium}\right] = 0$$

Acceleration & Saturation

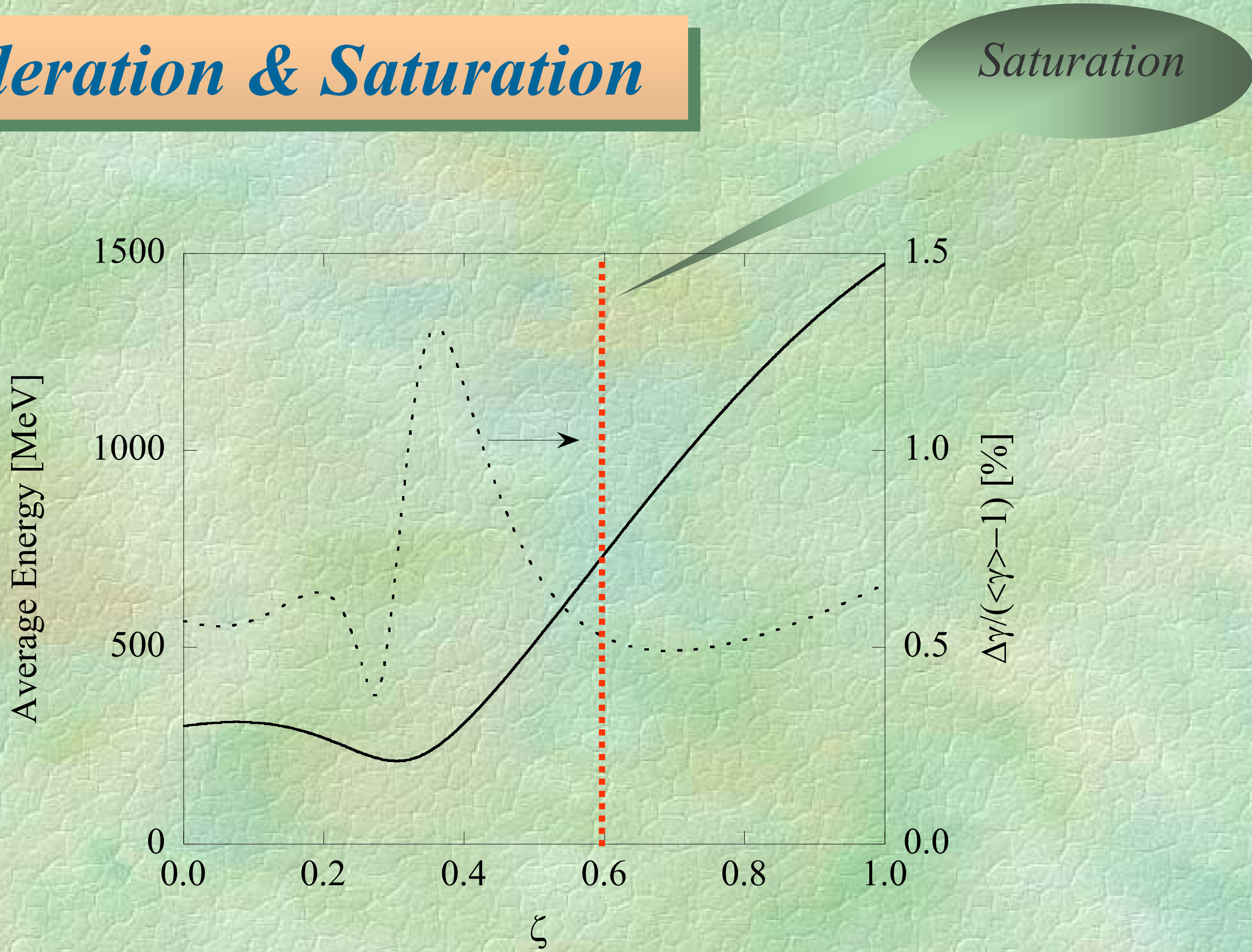
Simulation parameters:

λ [μm]	1.06
α	7x 10^3
N_e [m^{-3}]	10^5
Energy [MeV]	300
N_{ph} [m^{-3}]	10^{25}
P_{in} [MW]	2
σ [m^2]	10^{-24}

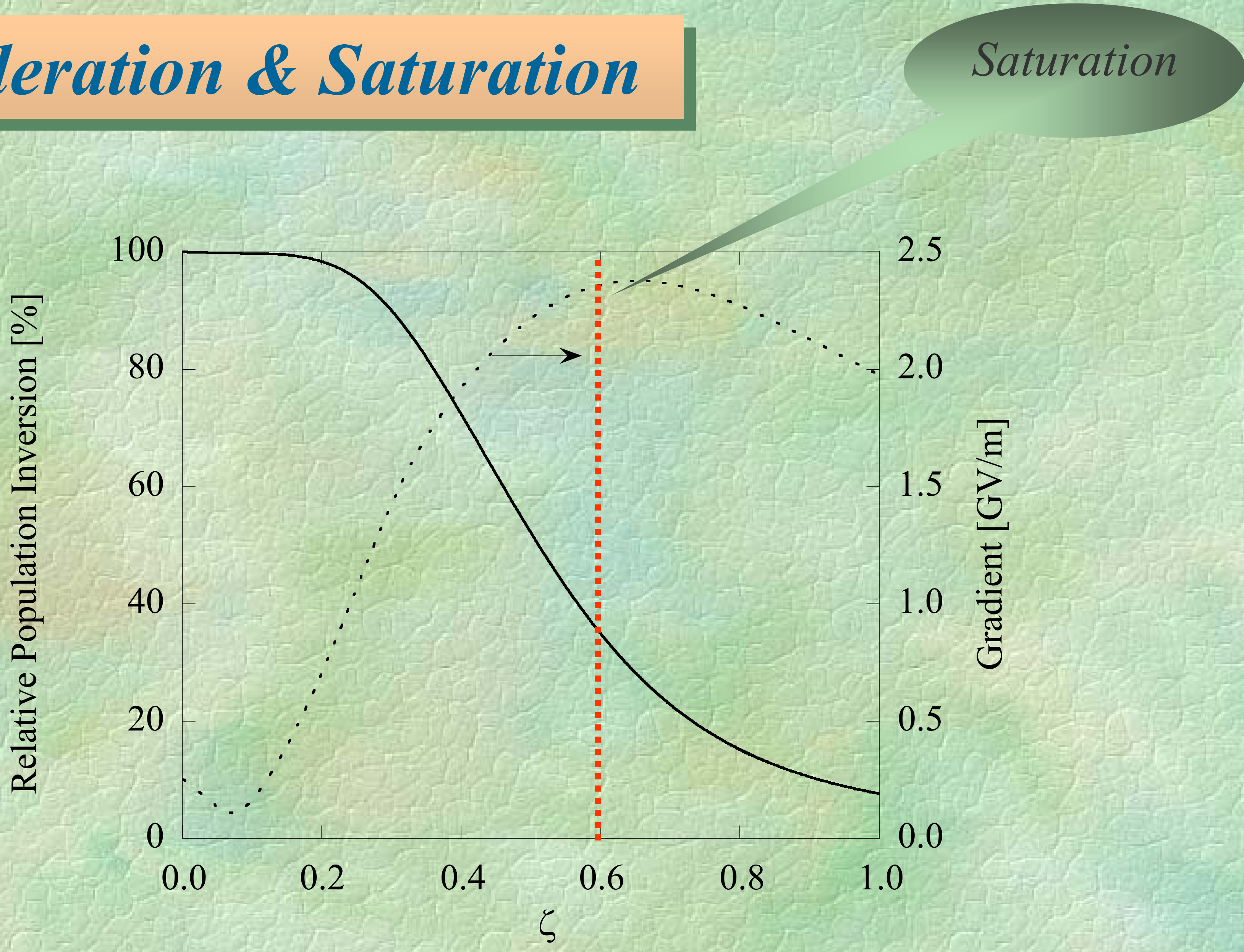
Acceleration & Saturation



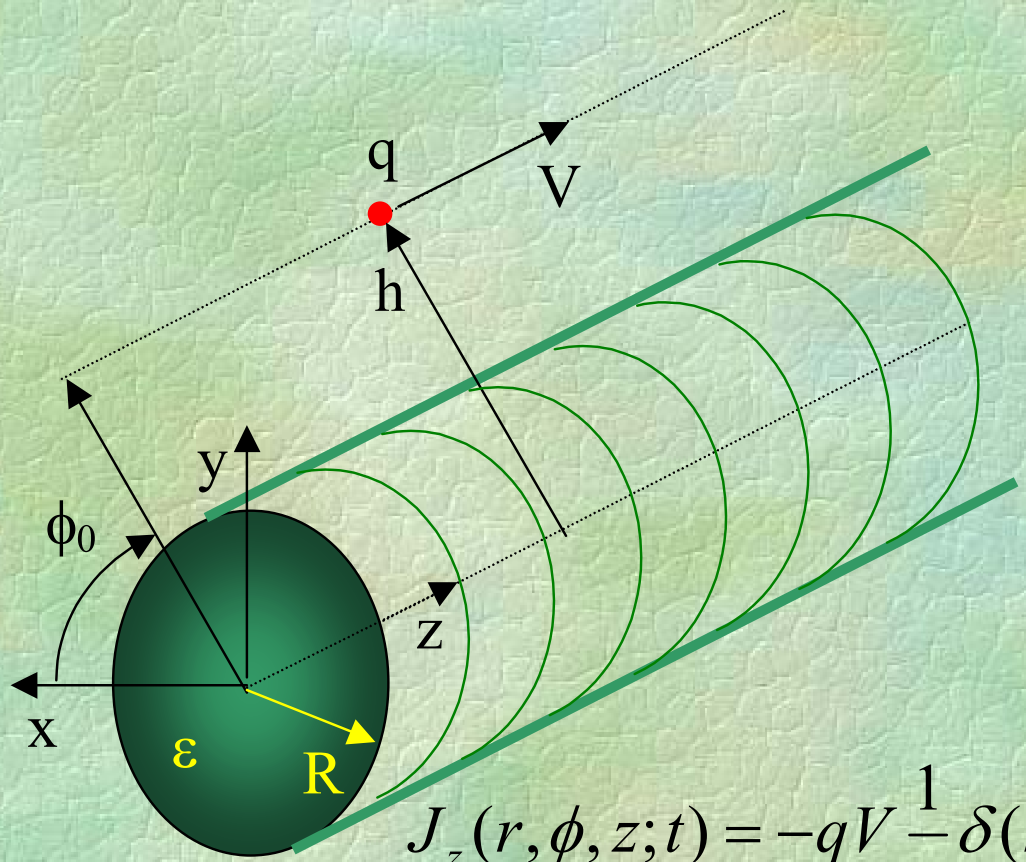
Acceleration & Saturation



Acceleration & Saturation



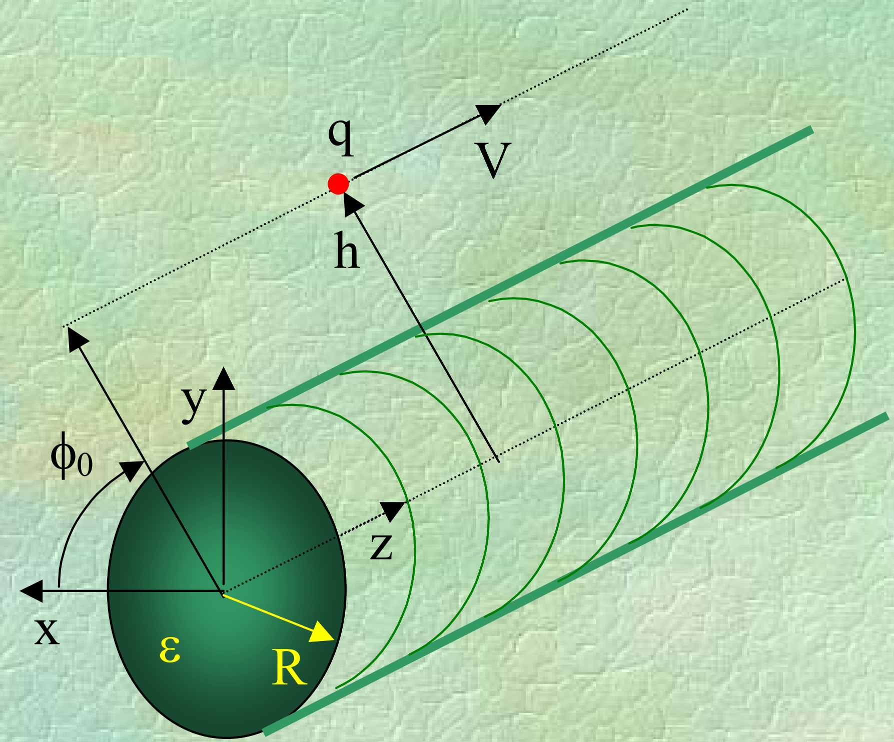
Point-charge along a Dielectric Cylinder



$$J_z(r,\phi,z;t) = -qV\frac{1}{r}\delta(r-h)\delta(\phi-\phi_0)\delta(z-Vt)$$

$$J_z(r,\phi,z;\omega) = -q\frac{1}{2\pi r}\delta(r-h)\delta(\phi-\phi_0)\exp(-j\frac{\omega}{V}z)$$

Primary Field



Ignoring the dielectric cylinder

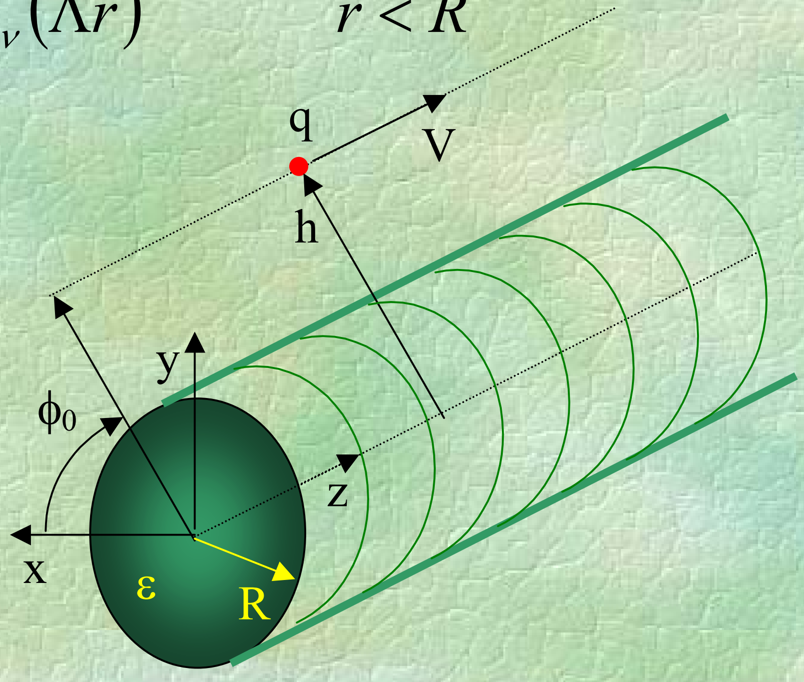
$$A_z^{(p)}(r,\phi,z;\omega) = \frac{q\mu_0}{(2\pi)^2} e^{-j\frac{\omega}{V}z} \sum_{\nu=-\infty}^{\infty} e^{j\nu(\phi-\phi_0)} \begin{cases} I_\nu\left(\frac{|\omega|}{c}\frac{h}{\gamma\beta}\right) K_\nu\left(\frac{|\omega|}{c}\frac{r}{\gamma\beta}\right) & r > h \\ K_\nu\left(\frac{|\omega|}{c}\frac{h}{\gamma\beta}\right) I_\nu\left(\frac{|\omega|}{c}\frac{r}{\gamma\beta}\right) & r < h \end{cases}$$

Secondary Field

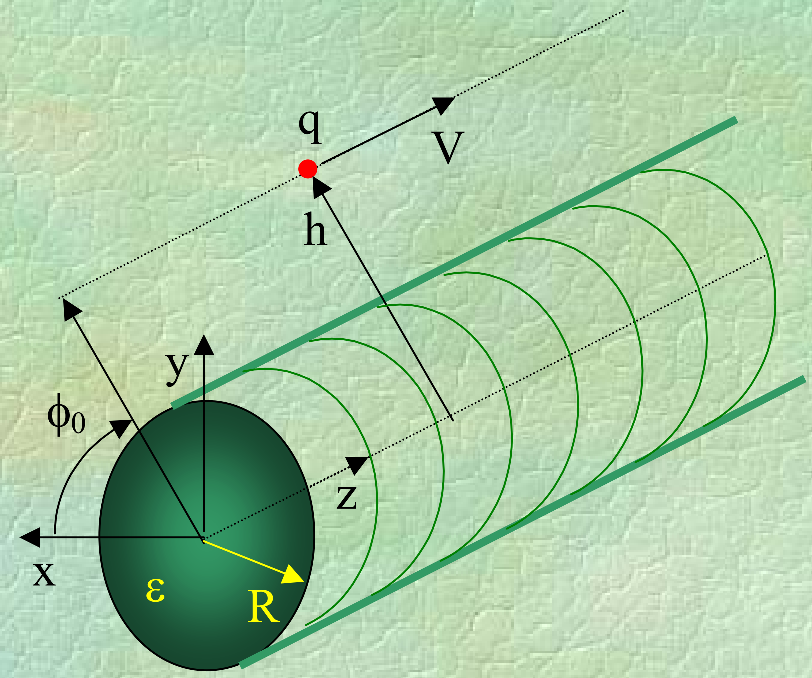
$$E_z^{(s)}(r,\phi,z;\omega) = e^{-j\frac{\omega}{V}z} \sum_{\nu=-\infty}^{\infty} e^{j\nu(\phi-\phi_0)} \begin{cases} A_\nu K_\nu\left(\dfrac{|\omega|}{c}\dfrac{r}{\gamma\beta}\right) & r > R \\ B_\nu J_\nu(\Lambda r) & r < R \end{cases}$$

$$H_z^{(s)}(r,\phi,z;\omega) = e^{-j\frac{\omega}{V}z} \sum_{\nu=-\infty}^{\infty} e^{j\nu(\phi-\phi_0)} \begin{cases} C_\nu K_\nu\left(\dfrac{|\omega|}{c}\dfrac{r}{\gamma\beta}\right) & r > R \\ D_\nu J_\nu(\Lambda r) & r < R \end{cases}$$

$$\Lambda = |\omega| \sqrt{\varepsilon - 1/\beta^2}\, / c$$



Decelerating Field



$$E = E_z^{(p)}\left(r=h, \phi=\phi_0, z=Vt, t\right)$$

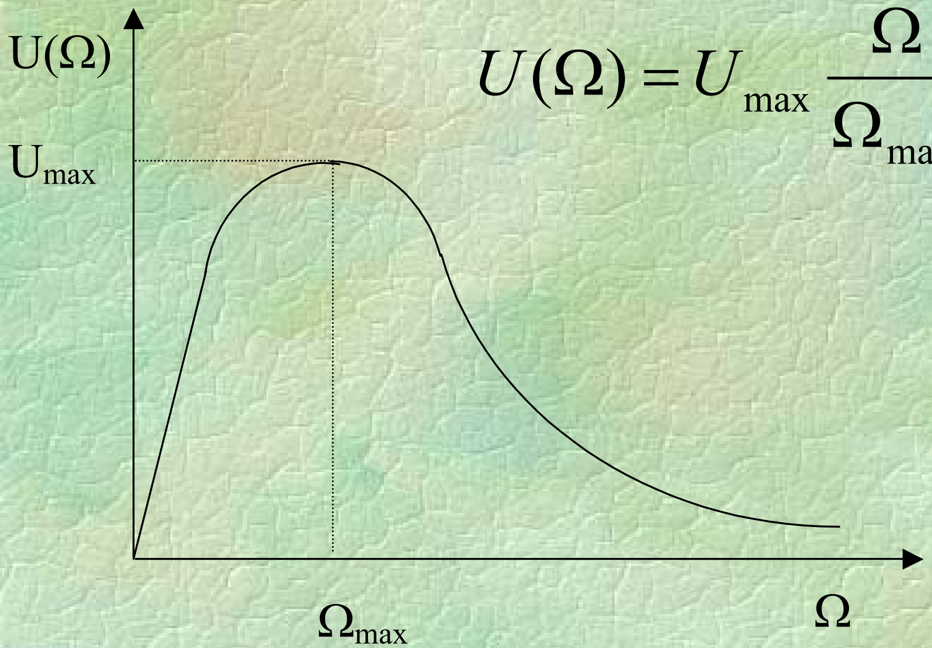
$$= \frac{q\mu_0}{(2\pi)^2} \sum_{\nu=-\infty}^{\infty} \int_{-\infty}^{\infty} d\omega\,(-j\omega) \left[\frac{K_\nu\left(\frac{|\omega|}{c}\frac{h}{\gamma\beta}\right)}{\overset{o}{K}_\nu\left(\frac{|\omega|}{c}\frac{R}{\gamma\beta}\right)} \right]^2 \frac{N_\nu(\omega,\gamma)}{D_\nu(\omega,\gamma)}$$

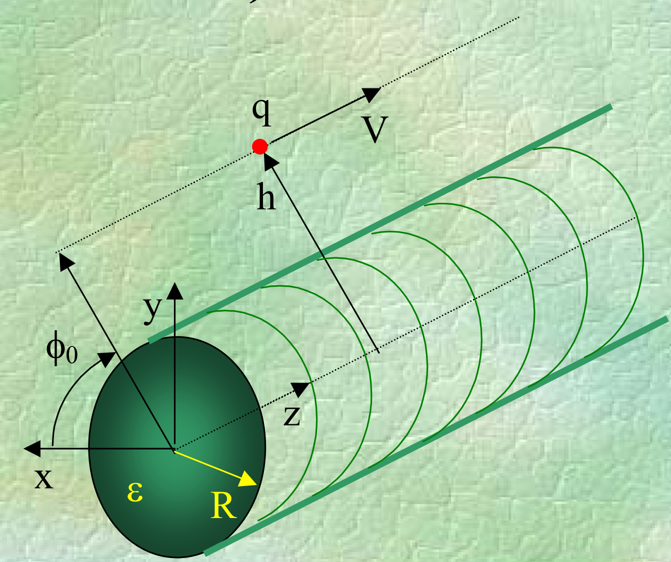
Decelerating Field: $v = 0$, $\gamma >> 1$

$$E_0 = \frac{q}{4\pi\varepsilon_0 R^2}\frac{4}{\gamma^2}\frac{\varepsilon}{\varepsilon - 1}\int_0^\infty d\Omega \sum_{s=1}^{\infty} U(\Omega)\delta\left(\Omega - \Omega_s\right)$$

$$\Omega_s = p_s / \sqrt{\varepsilon - 1}$$

$$U(\Omega) = U_{\max}\frac{\Omega}{\Omega_{\max}}\exp\left(-\frac{\Omega - \Omega_{\max}}{\Omega_{\max}}\right)$$

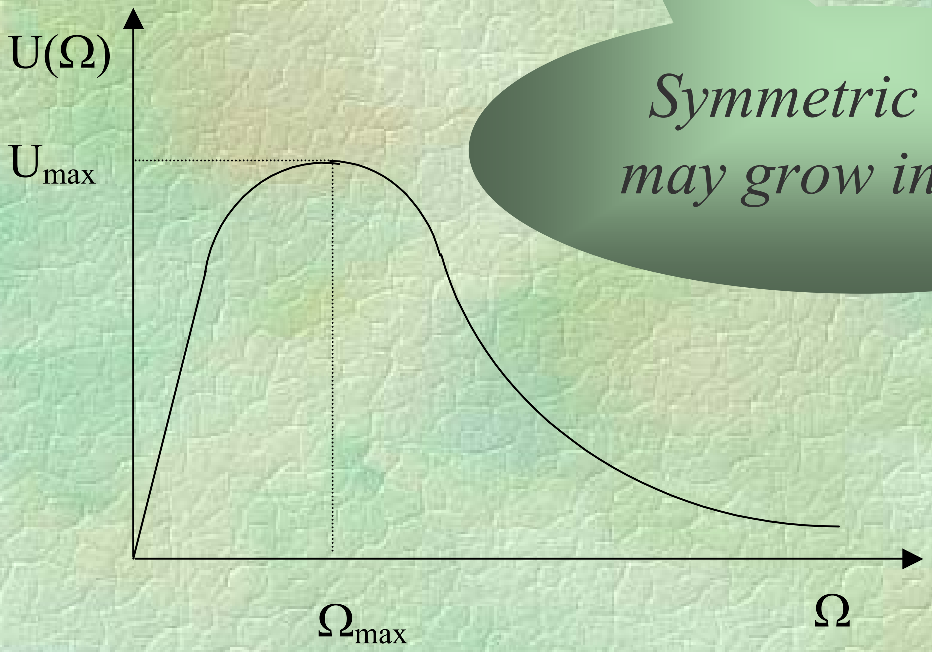


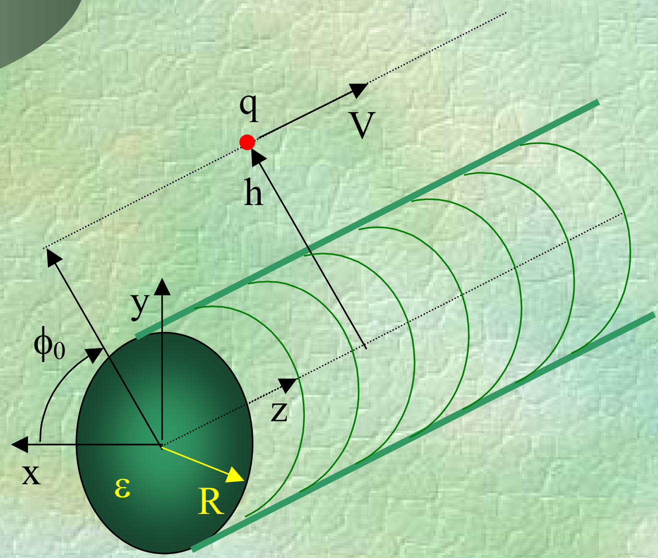


Decelerating Field: ν= 0, γ>>1

$$\Omega_s = p_s / \sqrt{\varepsilon - 1} \Rightarrow \frac{\mathrm{Im}(\omega)}{\omega_{res}} \approx \frac{\varepsilon_i}{2(\varepsilon_r - 1)^{3/2}}$$

Symmetric mode may grow in space





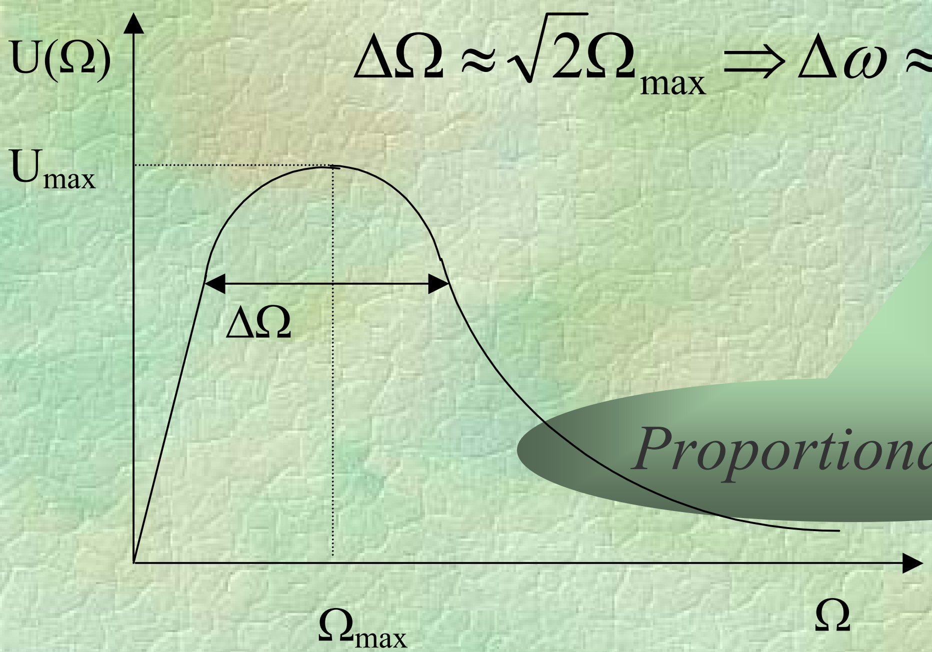
Decelerating Field: v= 0, γ>>1

$$E_0 \approx \frac{q/2\pi R}{2\pi\varepsilon_0 (h-R)} \frac{2\varepsilon}{\sqrt{\varepsilon - 1}} \frac{1}{\gamma}$$

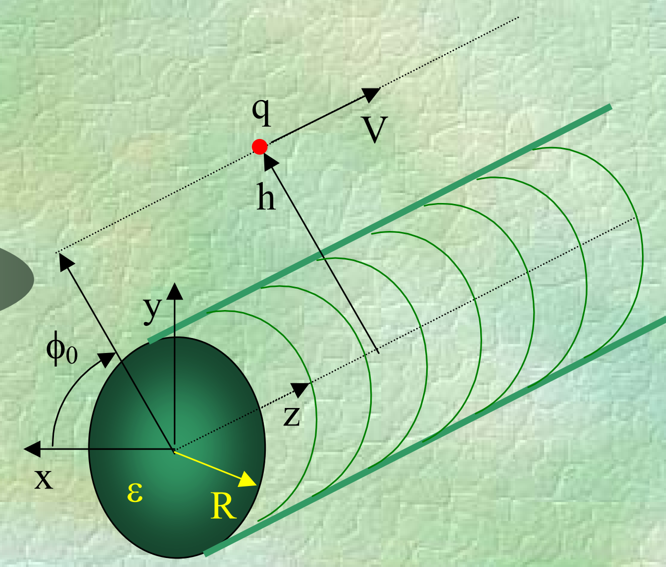
Inverse proportional to γ

$$\Omega_{max} \approx 0.4\gamma (h/R - 1)^{-3/4}$$

$$\Delta\Omega \approx \sqrt{2}\Omega_{max} \Rightarrow \Delta\omega \approx 0.6\gamma (c/R)\left(\frac{h}{R} - 1\right)^{-3/4}$$



Proportional to γ

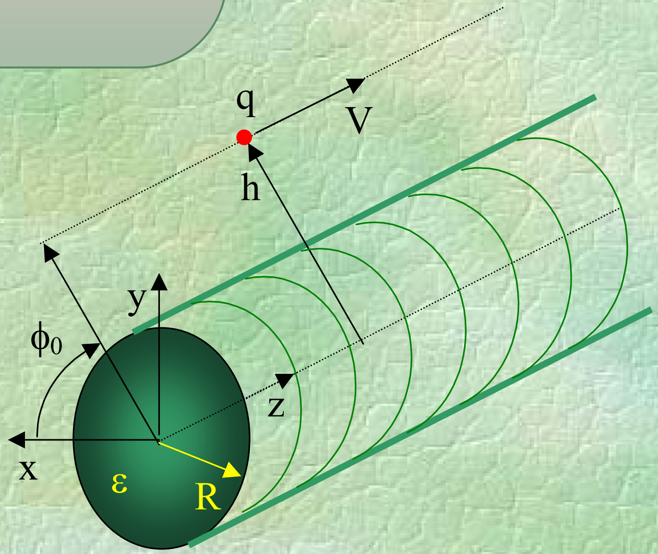


Decelerating Field: $\nu \neq 0$, $\gamma >> 1$

Independent of γ

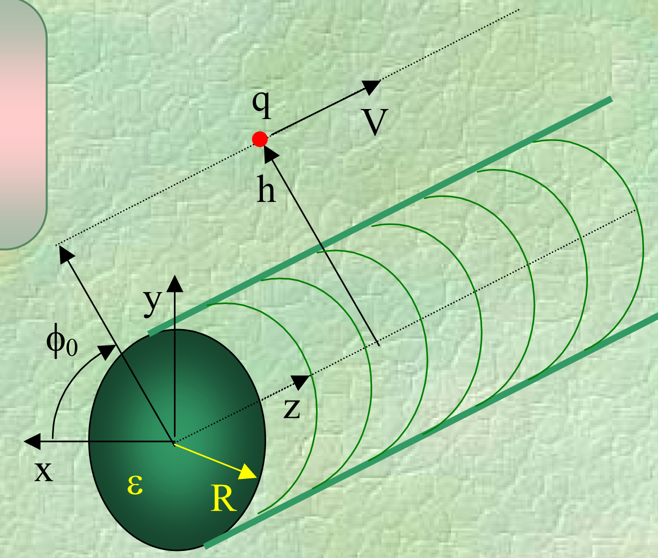
$$E \approx \frac{q}{4\pi\varepsilon_0 R^2}\left\{(-4)\sum_{\nu=1}^{\infty}\left[\frac{K_\nu(\nu h/R)}{K_\nu(\nu)}\right]^2\left[\frac{K_\nu(\nu)}{K'_\nu(\nu)}\right]^3\right\}$$

$$\approx \frac{q}{4\pi\varepsilon_0 R^2}\frac{0.36}{\left(\frac{h}{R}-1\right)^2}\exp\left(-1.7\frac{h-R}{R}\right)$$



Decelerating Field: $\nu \neq 0$, $\gamma >> 1$

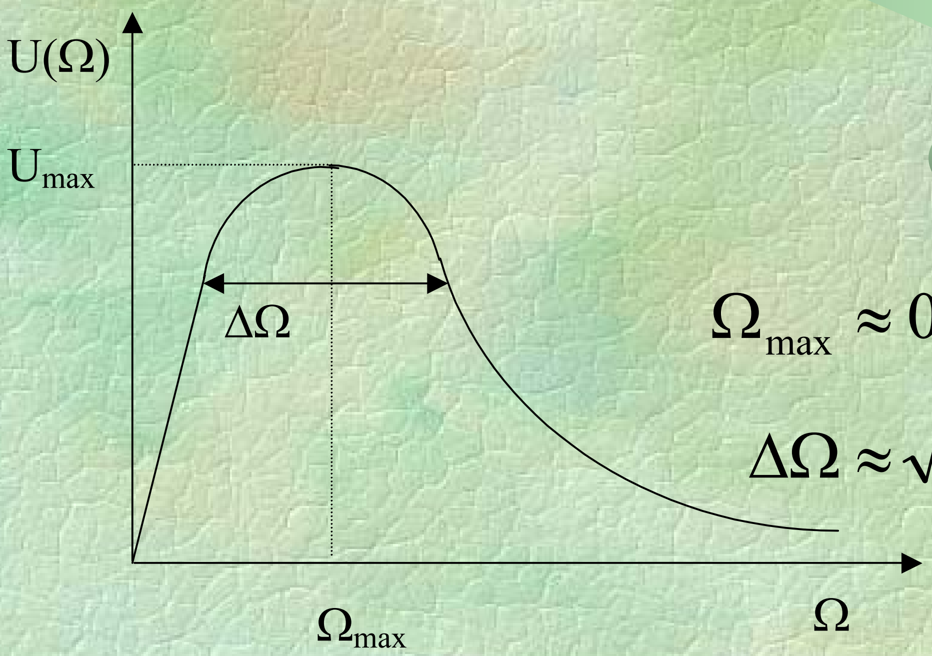
$$E' = \frac{q}{4\pi\varepsilon_0 R^2}\int_0^\infty d\Omega \sum_{\nu=1}^{\infty} U(\Omega)\delta(\Omega - \Omega_\nu)$$



$$\Omega_\nu = \nu\gamma$$

$$U(\Omega) = U_{\max}\frac{\Omega}{\Omega_{\max}}\exp\left(-\frac{\Omega - \Omega_{\max}}{\Omega_{\max}}\right)$$

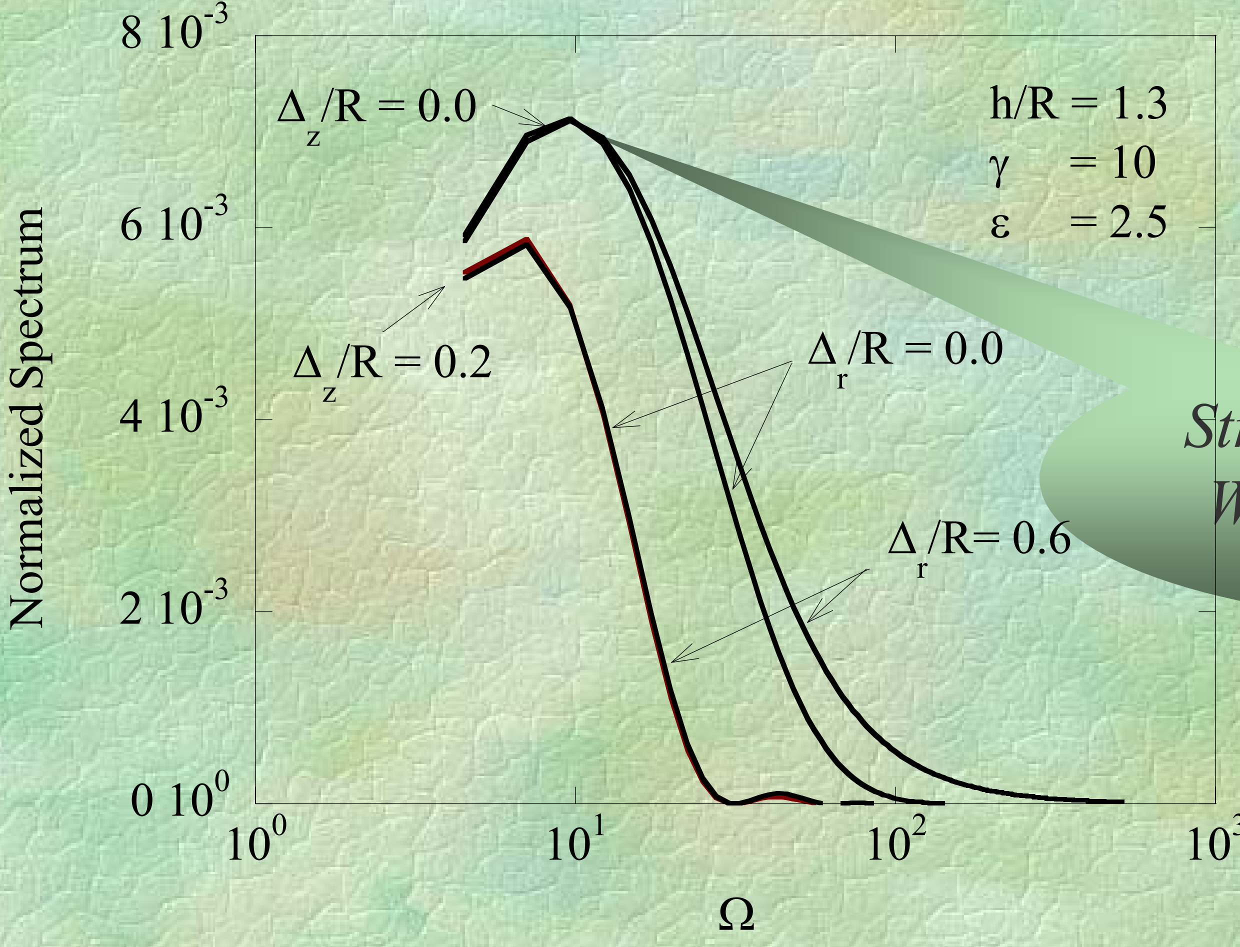
ε – independent;
no spatial growth



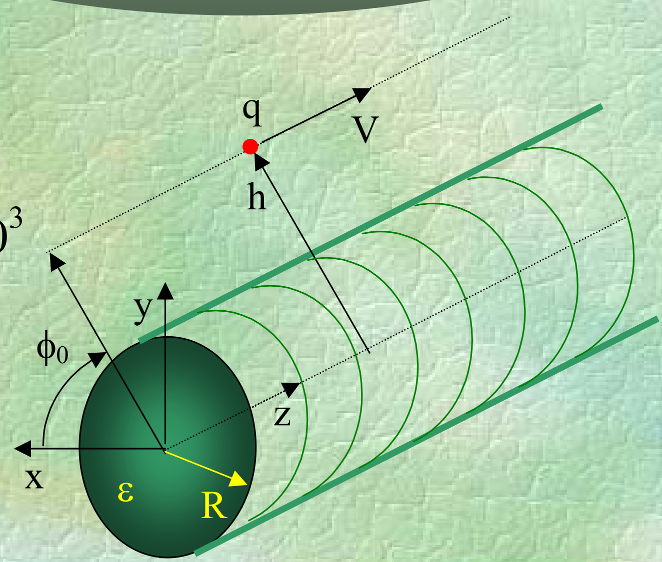
$$\Omega_{\max} \approx 0.5\gamma(h/R - 1)^{-1}$$

$$\Delta\Omega \approx \sqrt{2}\Omega_{\max} \Rightarrow \Delta\omega \approx 0.7\gamma(c/R)\left(\frac{h}{R} - 1\right)^{-1}$$

Decelerating Field: Finite size bunch (ν=0)

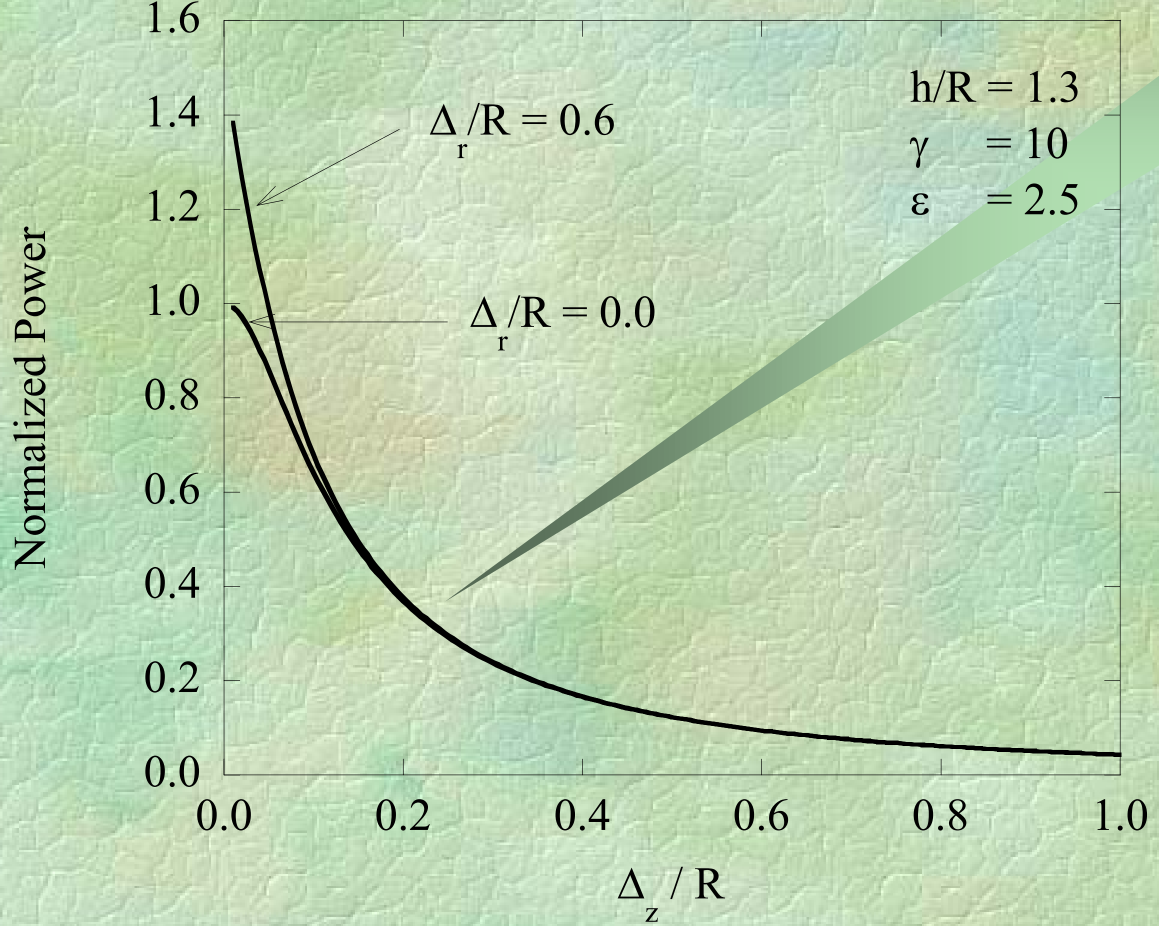


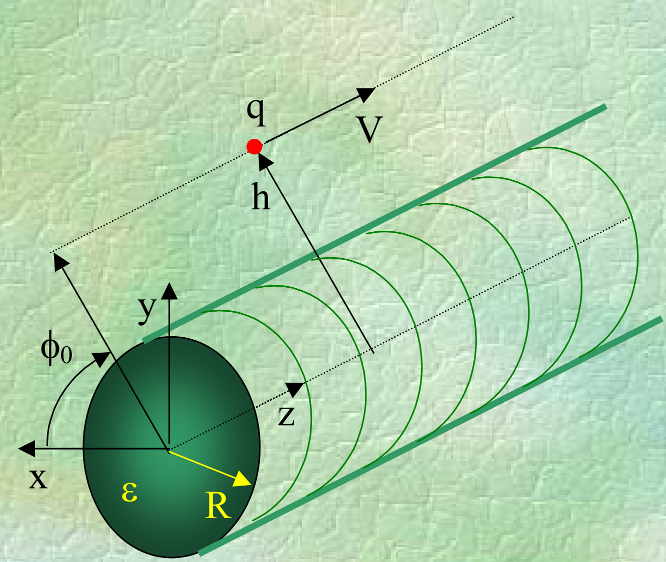
Strong Δ_z dependence
Weak Δ_r dependence



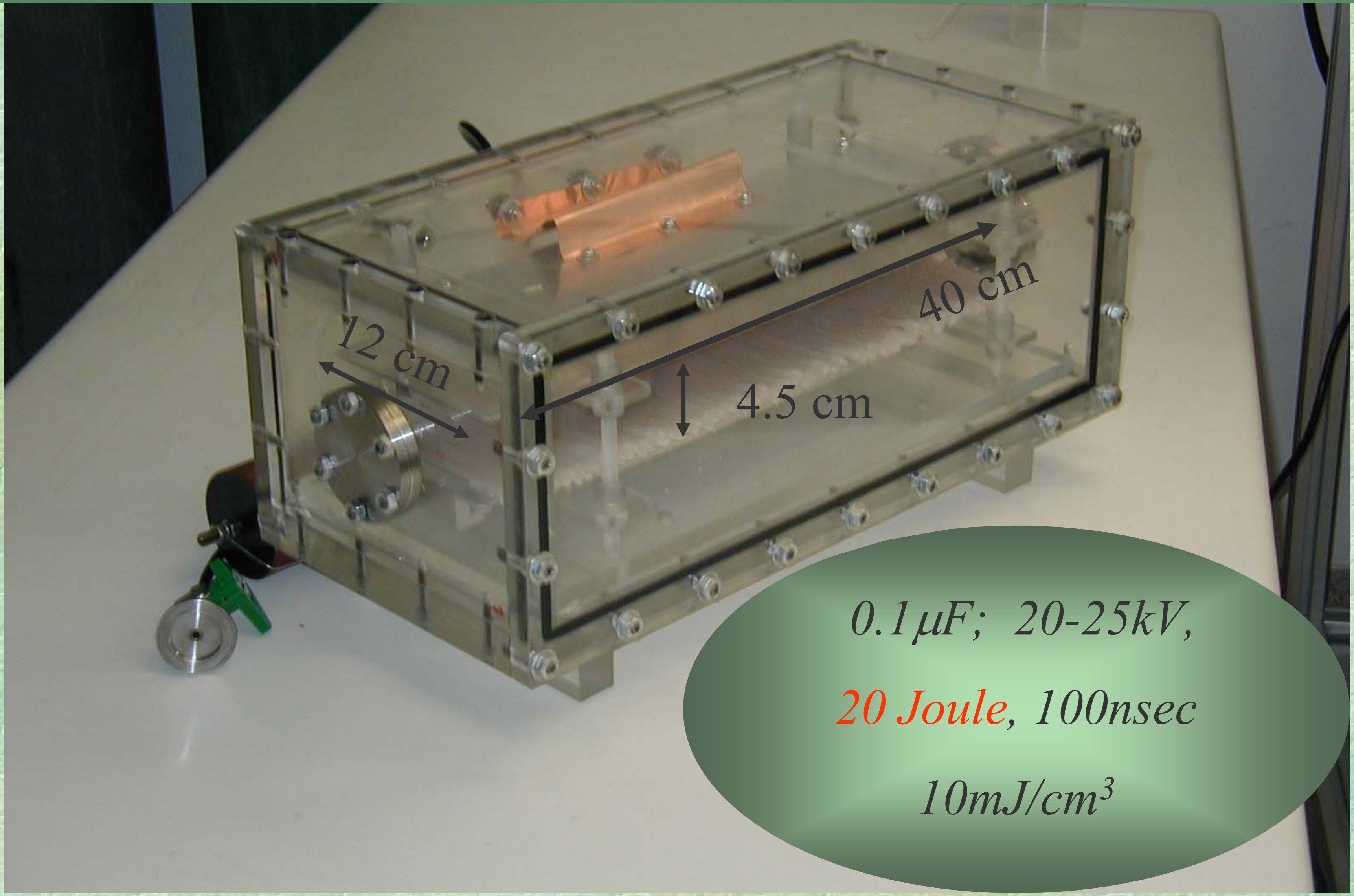
Decelerating Field: Finite size bunch (ν=0)

Weak Δ_r dependence





CO_2 Based System



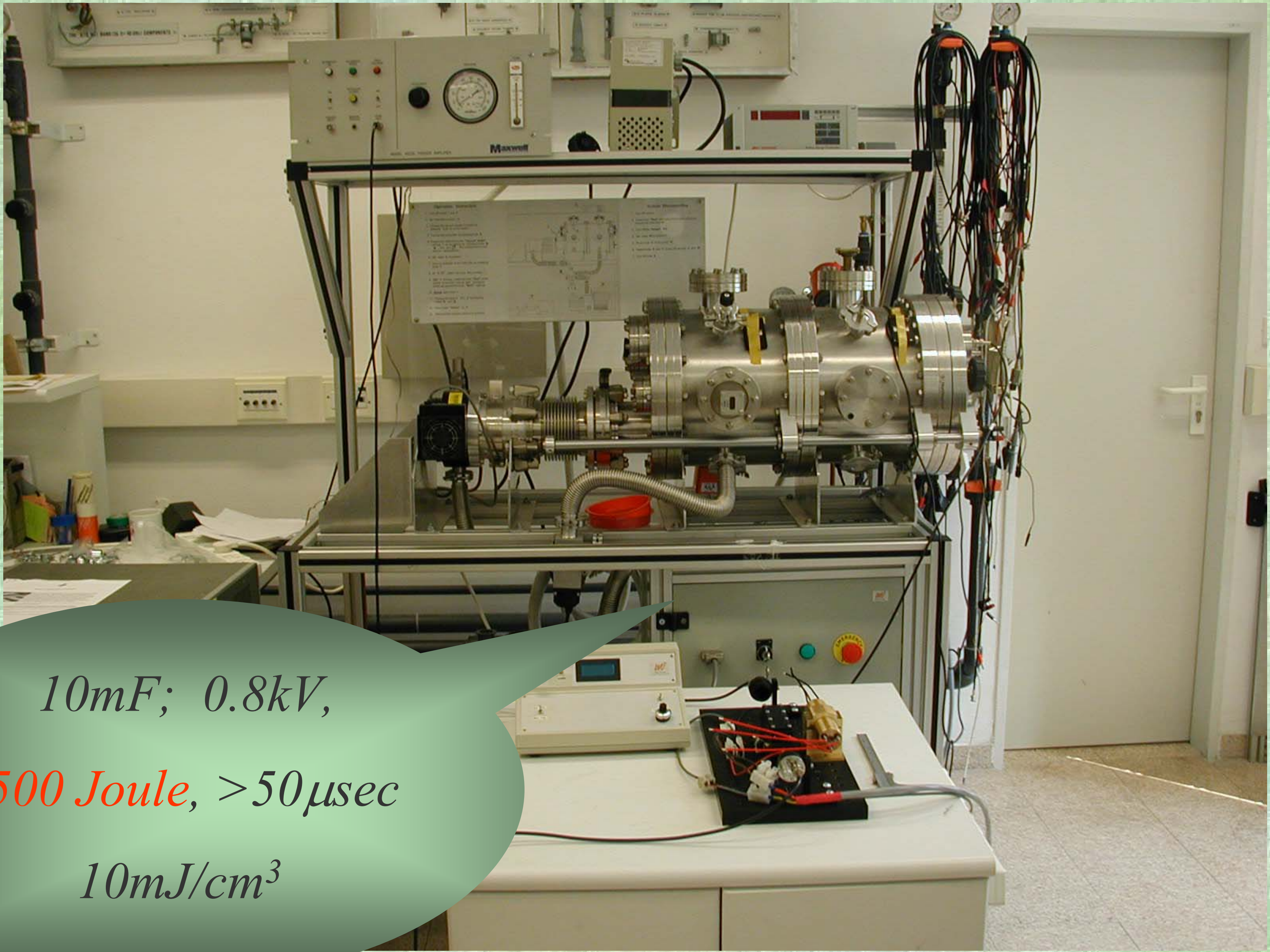
0.1 μF; 20-25kV,

20 Joule, 100nsec

10mJ/cm^3

CO_2 Based System

Nd:YAG Based System

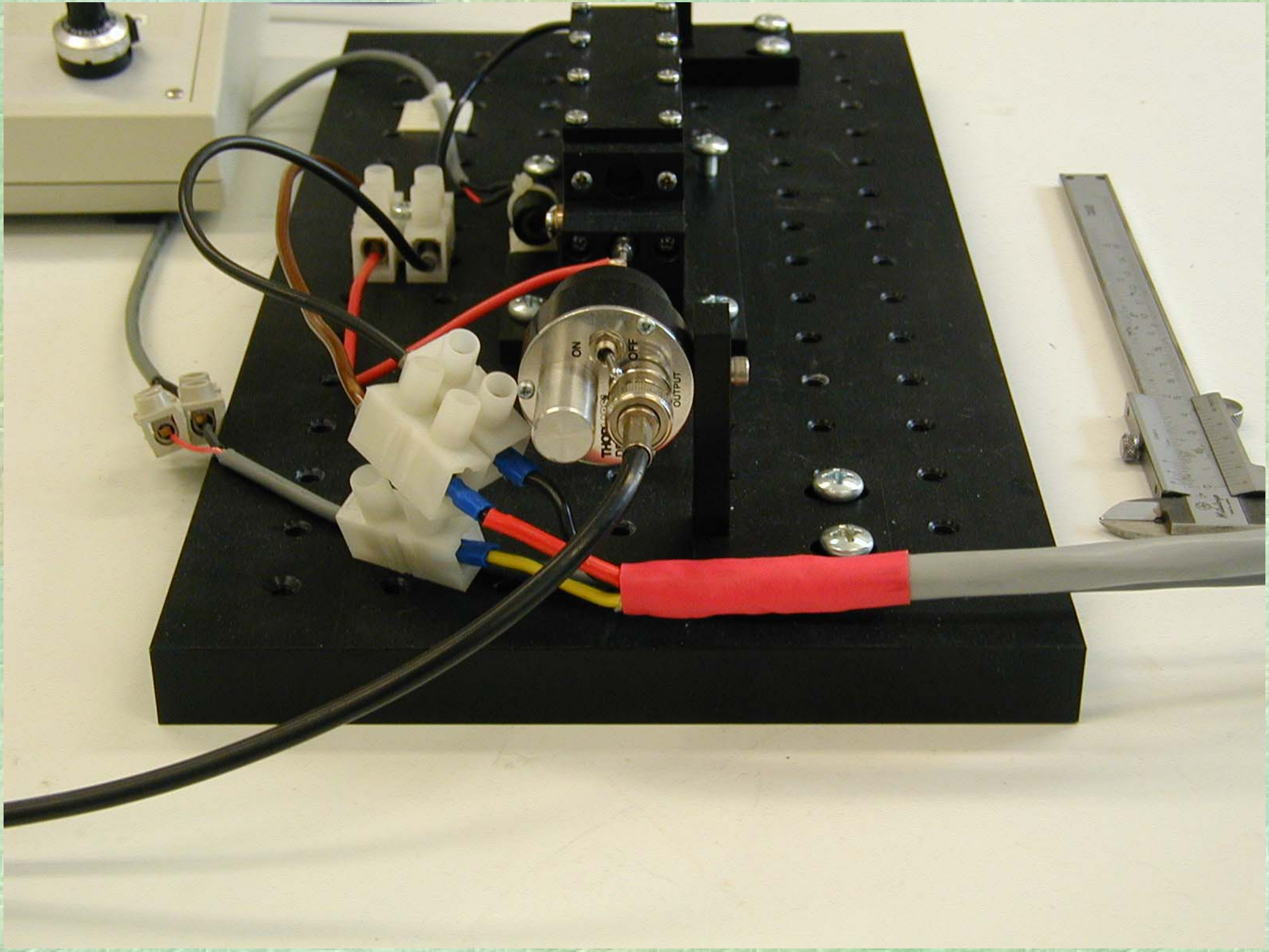


10mF; 0.8kV,

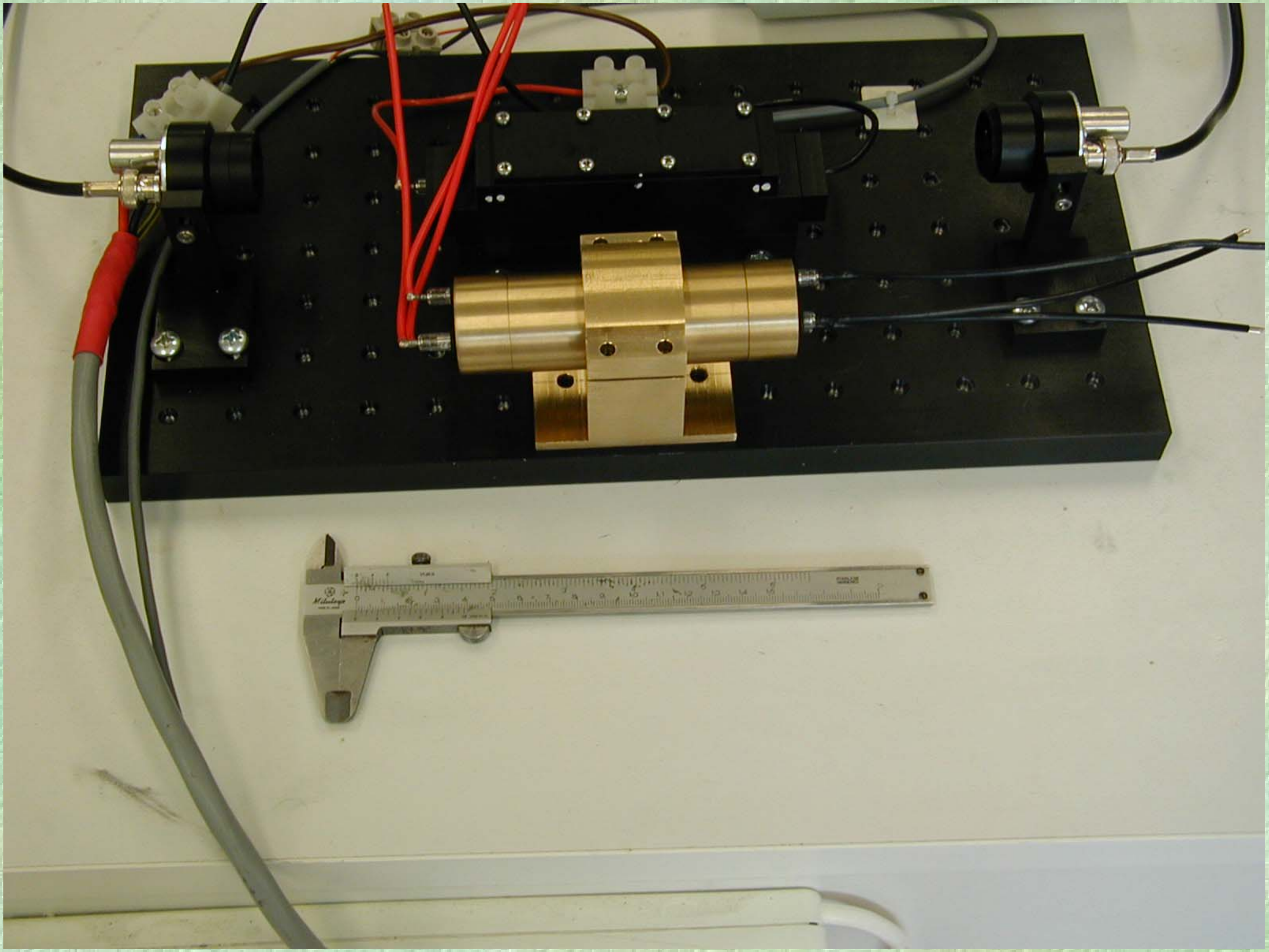
1500 Joule, >50μsec

10mJ/cm^3

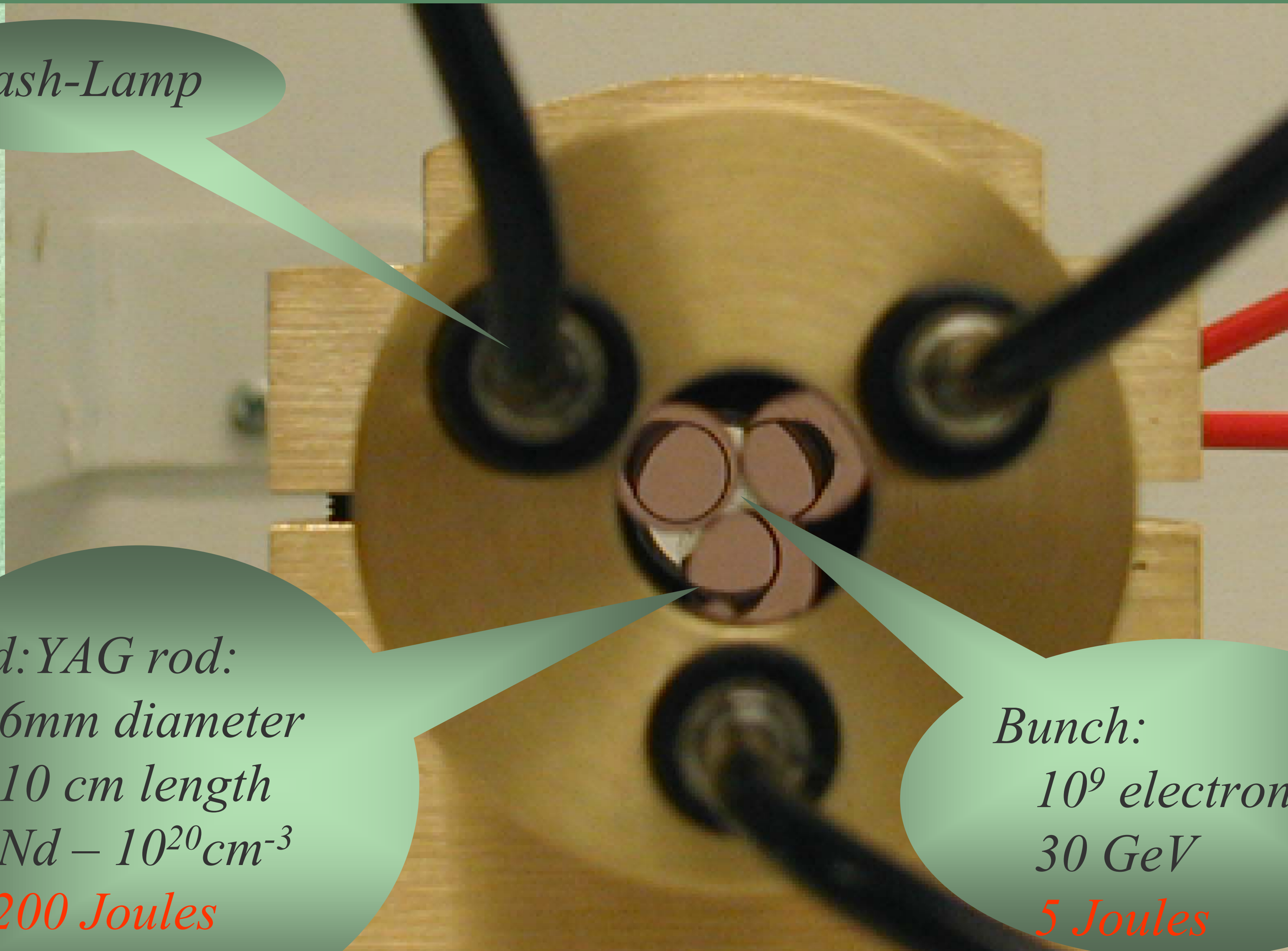
Nd:YAG Based System



Nd:YAG Based System



Nd:YAG Based System

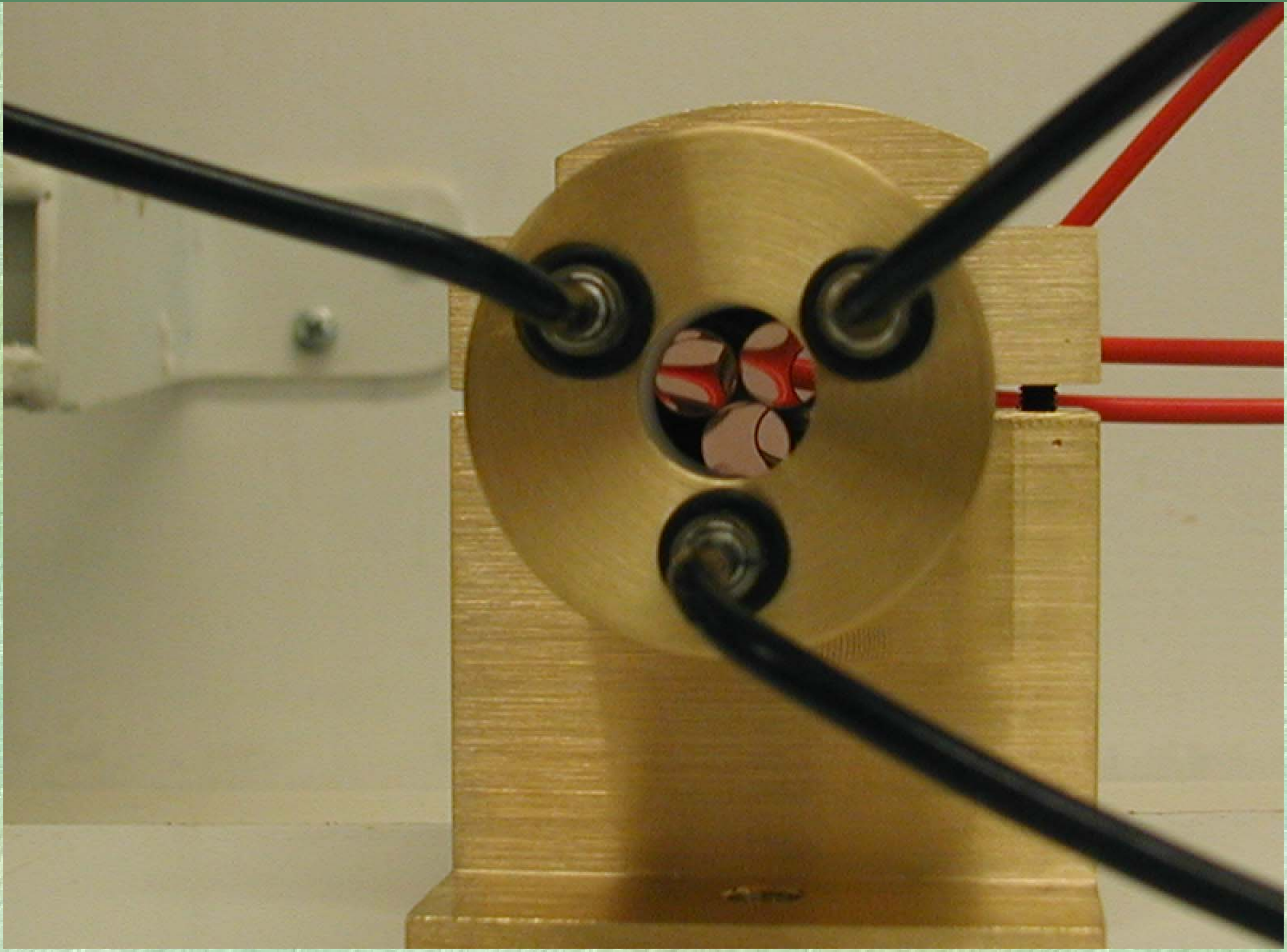


Flash-Lamp

Nd:YAG rod:
- 6mm diameter
- 10 cm length
- Nd – 10^{20} cm^{-3}
- 200 Joules

Bunch:
10^{9} electrons
30 GeV
5 Joules

Nd:YAG Based System



Field Analysis

$$F(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{j\omega\tau}\, d\omega}{\omega^2\left[\beta^{-2} - \varepsilon(\omega)\right] + \omega_s^2}$$

4 poles

$$\varepsilon(\omega) = \varepsilon_r - \chi \frac{\omega_0^2}{\omega_0^2 - \omega^2 + 2j\omega T_2^{-1}}$$

$$\varepsilon_c = \varepsilon_r - \beta^{-2}$$

$$\omega_s \approx \pi s \frac{c}{R}$$

Eigen-frequencies

Decaying waves

Growing waves

$$\omega_{-,+} \approx \omega_0 + jT_2^{-1}$$

$$\omega_{-,-} \approx -\omega_0 + jT_2^{-1}$$

$$\omega_{+,+} \approx \frac{\omega_s}{\sqrt{\varepsilon_c}}\left[1 - \frac{\chi}{2\varepsilon_c}\frac{\omega_0^2}{\omega_s^2/\varepsilon_c - \omega_0^2 - 2jT_2^{-1}\omega_s/\sqrt{\varepsilon_c}}\right]$$

$$\omega_{+,-} \approx \frac{-\omega_s}{\sqrt{\varepsilon_c}}\left[1 - \frac{\chi}{2\varepsilon_c}\frac{\omega_0^2}{\omega_s^2/\varepsilon_c - \omega_0^2 + 2jT_2^{-1}\omega_s/\sqrt{\varepsilon_c}}\right]$$

$$\varepsilon_C \equiv \varepsilon_r - \beta^{-2}$$

Growing mode

$$\frac{\mathrm{Im}(\omega_{+,+})}{\omega_0} = \frac{\chi}{4\varepsilon_c}\omega_0 T_2 \begin{cases} 1 & for \quad \omega_{s_0} = \omega_0 \\ \left(\frac{R}{cT_2}\right)^2 \frac{\sqrt{\varepsilon_c}}{(\pi \Delta s)^2} & for \quad \omega_s \neq \omega_0 \end{cases}$$

$$s_0 : \omega_{s_0} = \omega_0$$

$$\Delta s \equiv s - s_0$$

$$\left.\begin{array}{l} R \approx 6\,\mathrm{mm} \\ T_2 \approx 230\,\mu\sec \end{array}\right\} \Rightarrow \left(\frac{R}{cT_2}\right)^2 \approx 7 \times 10^{-15}$$

$\Rightarrow$ ***Single Mode !!***

Output Energy Spread

$$\sigma \approx 6\times10^{-19}\,\mathrm{cm}^2 \qquad \varepsilon = 3.312$$

$$\mathrm{N_{ph}} \approx 1.38\times10^{20}\,\mathrm{cm}^{-3} \qquad \mathrm{T_2} \approx 230\,\mu\,\mathrm{sec}$$

$$Z_{\mathrm{int}} \approx 1.8\times10^4\,\Omega \qquad P_{out} \approx 0.9TW$$

$$N_e \approx 10^9$$

$$\frac{\Delta\gamma}{\langle\gamma\rangle - 1} \approx 0.5\% \to 5\%$$

Summary

- *PASER: Point-charge accelerated by energy stored in the medium*
- *Same energy amplifies a Cerenkov wake-field*
- *Eigen-modes move at the speed of the bunch*
- *Inherent longitudinal E-field: interaction length*
- *Acceleration not affected by medium saturation*
- *Non-symmetric modes (γ>>1) are not amplified*