Wake of a Charge Moving in the Vicinity of a Dielectric Structure

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Outline

- Motivation
- Configurations
- Charged Line & Dielectric Cylinder
- Formulation
- Simulation Results
- Summary

Motivation

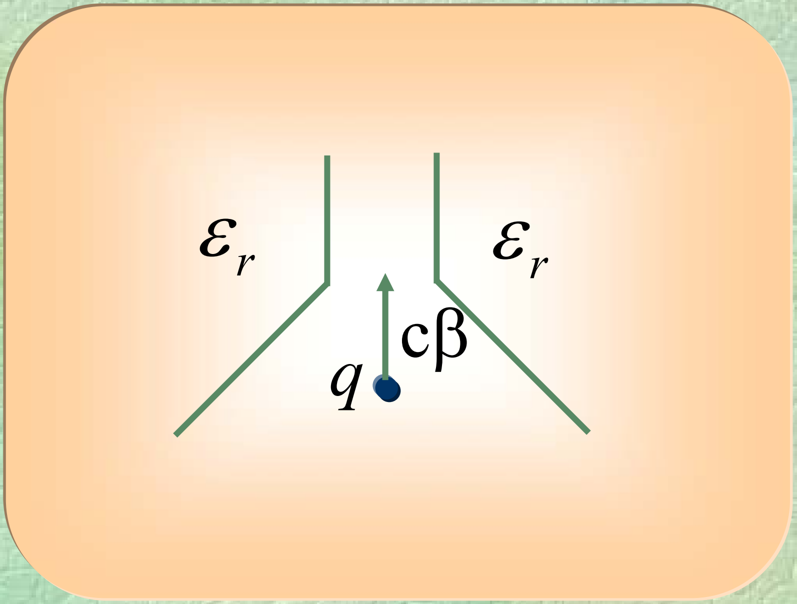
- Vacuum acceleration with lasers entails *relativistic* motion of a bunch in the vicinity of metallic or dielectric structures
- *Non-relativistic* electrons are used for ultra-microscopy of surfaces at nano-meter scale
- It is necessary to determine the characteristics of the *wakes* generated by a bunch in a variety of configurations

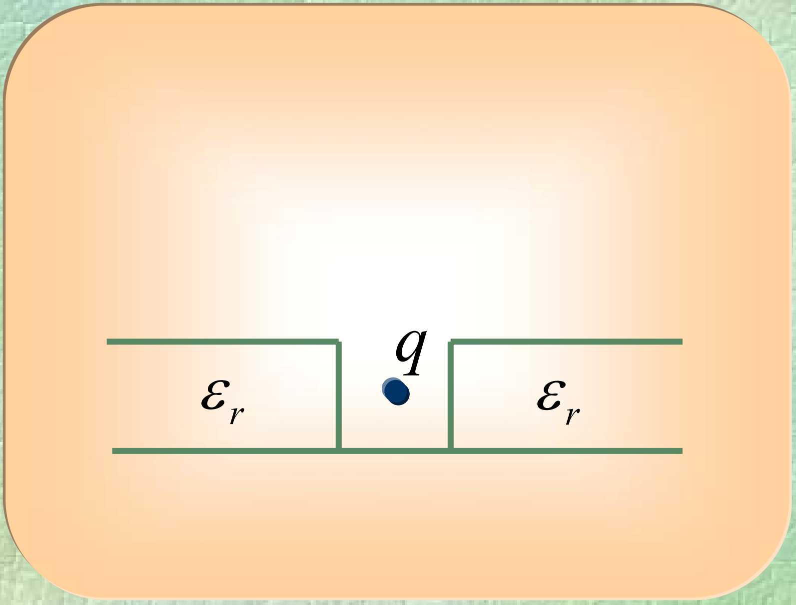
Motivation: Applications

- Ultra-microscopy (Chemistry, Biology)
- X-ray sources (Medicine, Biology, Chemistry, Physics)
- Spectroscopy (Biology, Chemistry)
- Particle accelerators (Medicine, Physics)

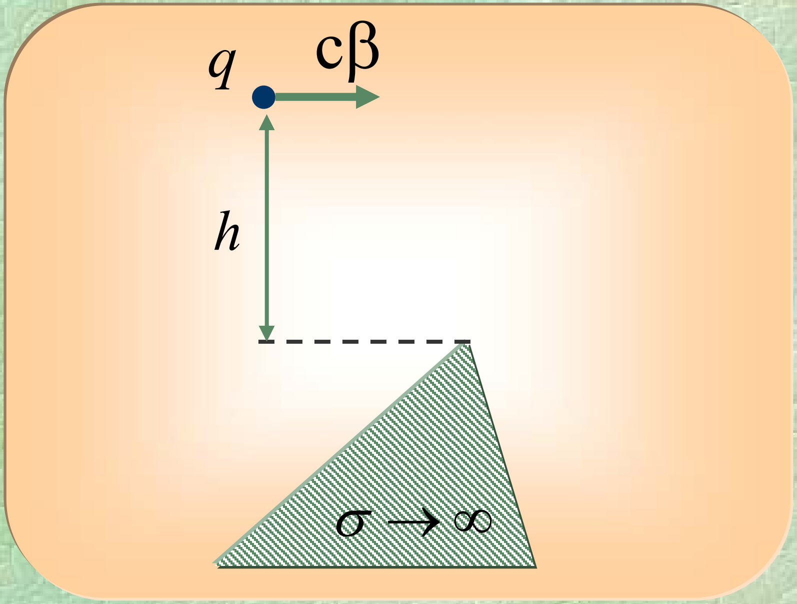
Configurations of Interest

Point Charge & Accelerating Structure



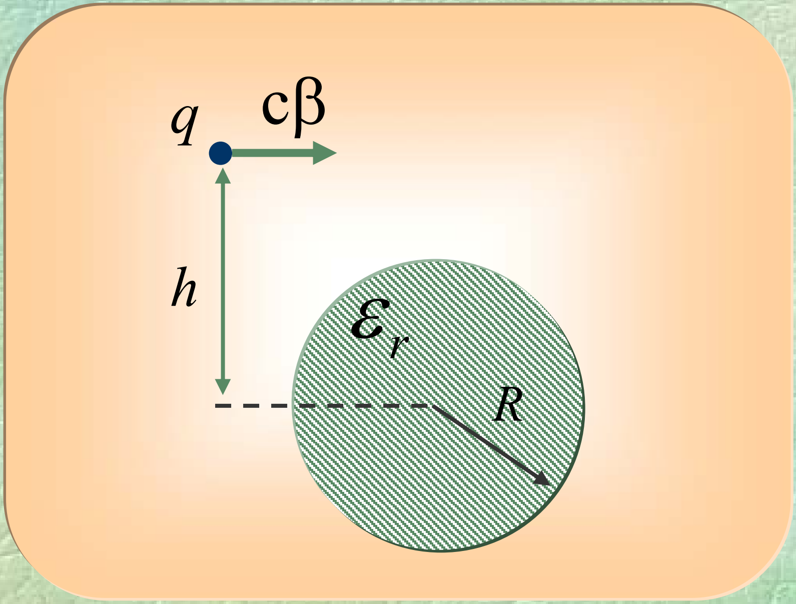


Configurations of Interest

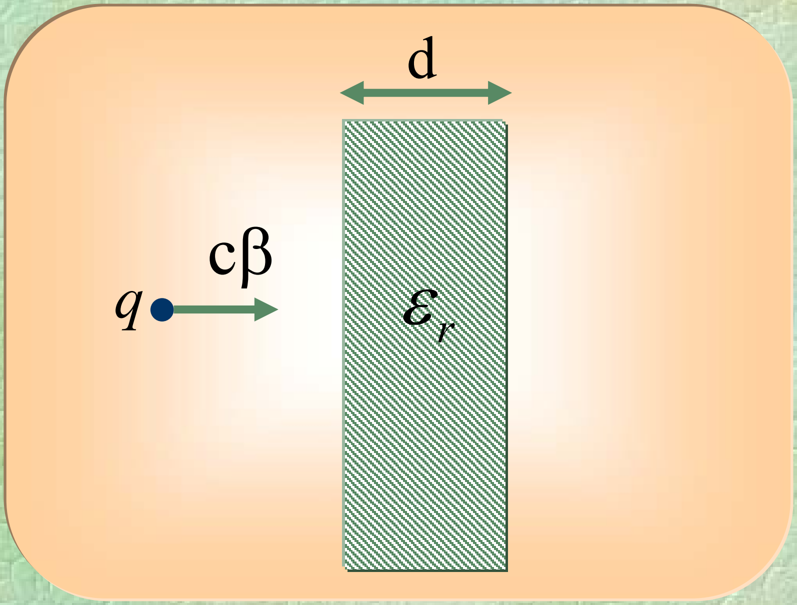


Point Charge & Metallic Edge

Point Charge & Dielectric Cylinder

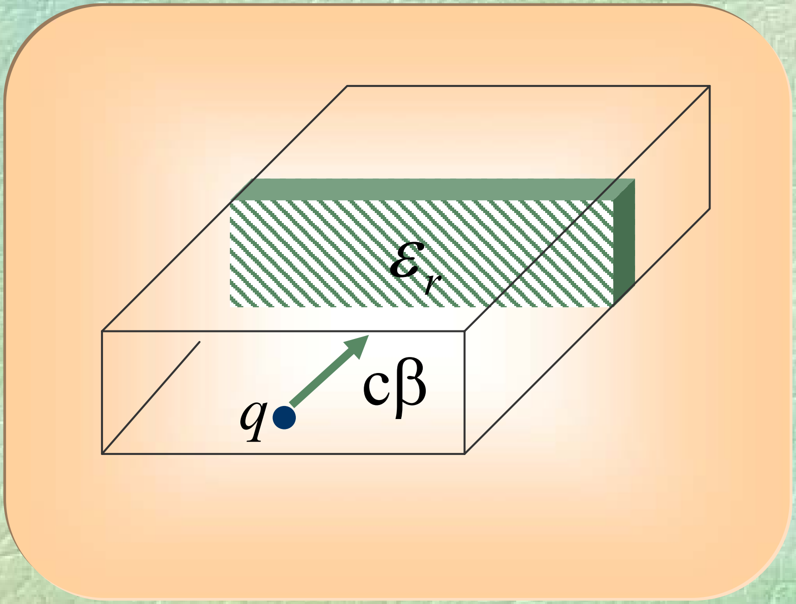


Configurations of Interest



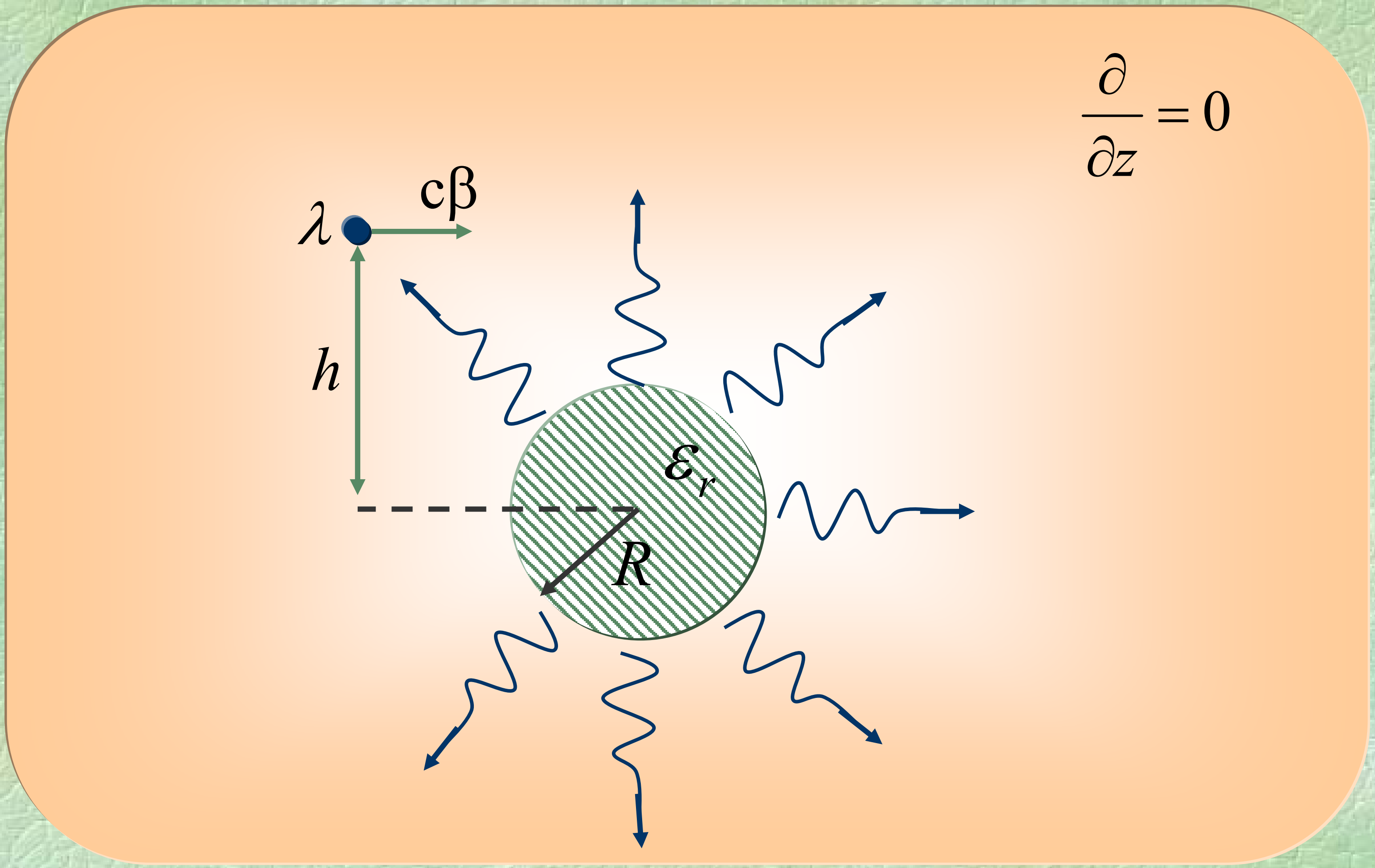
Point Charge & Dielectric Layer

Point Charge & Loaded Wave-Guide



Configuration:

Charged-Line & Dielectric Cylinder



Formulation: Primary Field

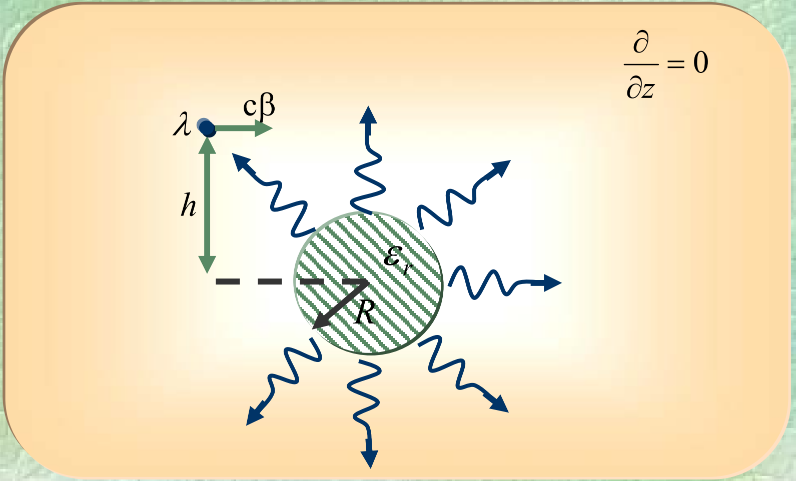
$$J_x(x,y;\omega) = -\frac{\lambda}{2\pi}\delta(y-h)\exp(-j\frac{\omega}{c\beta}x)$$

$$E_\varphi^{(p)} = \frac{-\lambda\mu_0}{4\pi\Gamma}\exp\left(-j\frac{\omega}{c\beta}r\cos\varphi - \frac{\omega}{c\gamma\beta}\left|r\sin\varphi - h\right|\right)\left[\frac{-j\omega}{\gamma^2\beta^2}\sin\varphi + \frac{\omega}{\gamma\beta^2}\operatorname{sgn}(y-h)\cos\varphi\right]$$

$$H_z^{(p)} = \frac{-\lambda}{4\pi}\exp\left(-j\frac{\omega}{c\beta}r\cos\varphi - \frac{\omega}{c\gamma\beta}\left|r\sin\varphi - h\right|\right)\operatorname{sgn}(y-h)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}; \qquad \Gamma = \frac{\omega}{c}\frac{1}{\gamma\beta}$$

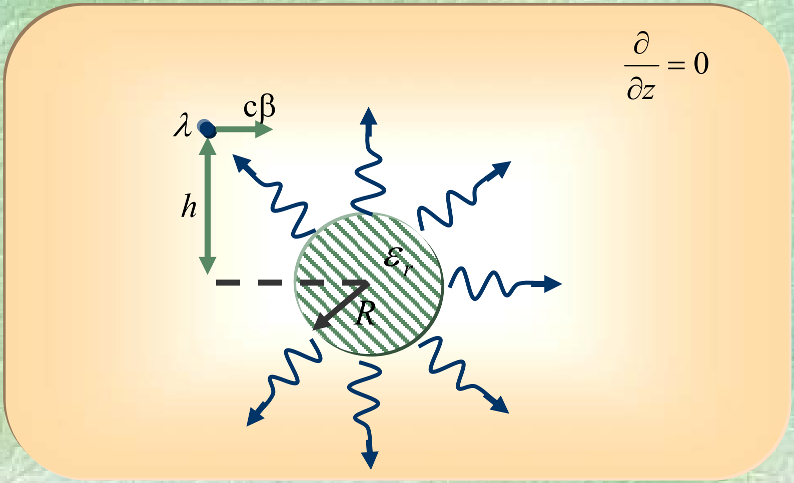
$$x = r\cos\varphi \qquad y = r\sin\varphi$$



Formulation: Secondary Field

$$H_z^{(s)}(r,\varphi;\omega) = \sum_{n=-\infty}^{\infty} e^{jn\varphi} \begin{cases} A_n H_n^{(2)}\left(\frac{\omega}{c} r\right) & r > R \\ B_n J_n\left(\frac{\omega}{c}\sqrt{\varepsilon_r}\, r\right) & r < R \end{cases}$$

$$E_\varphi^{(s)}(r,\varphi;\omega) = -\frac{1}{j\omega\varepsilon_0} \sum_{n=-\infty}^{\infty} e^{jn\varphi} \begin{cases} A_n \frac{\omega}{c} \dot{H}_n^{(2)}\left(\frac{\omega}{c} r\right) & r > R \\ \frac{B_n}{\sqrt{\varepsilon_r}} \frac{\omega}{c} \dot{J}_n\left(\frac{\omega}{c}\sqrt{\varepsilon_r}\, r\right) & r < R \end{cases}$$

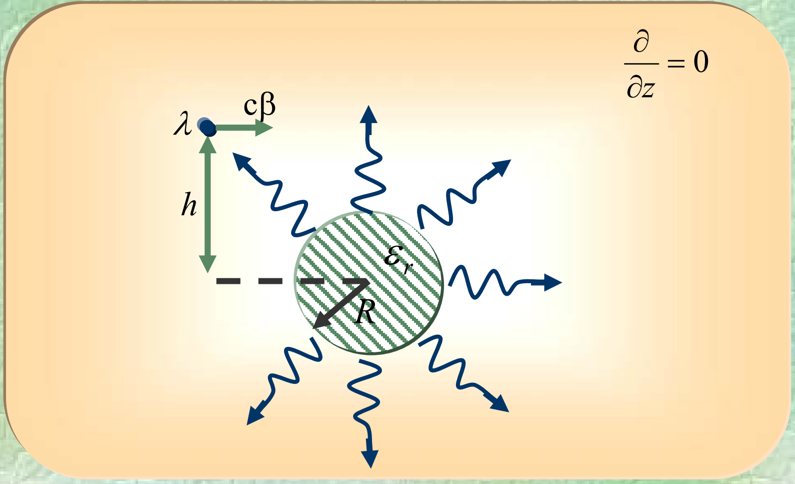


Formulation: Emitted Energy

$$P(r,t) = 2\pi r \frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi \, E_{\varphi}^{(s)}(r,\varphi;t)\, H_{z}^{(s)}(r,\varphi;t)$$

$$W(r) = \int_{-\infty}^{\infty} dt \, P(r,t)$$

$$r >> \lambda \qquad \Rightarrow \qquad W = \frac{16\pi}{\varepsilon_0} \int_0^{\infty} d\omega \frac{1}{\omega} \left[\sum_{n=-\infty}^{\infty} \left| A_n(\omega) \right|^2 \right]$$



Formulation: Field Solution

$$\Omega = \frac{\omega}{c} h$$

$$\overline{A}_n = \frac{A_n}{\lambda / 4\pi}$$

$$\overline{W} = \frac{W}{\lambda^2 / \pi\varepsilon_0}$$

$$\overline{W} = \int_0^\infty d\Omega \frac{1}{\Omega} \sum_{n=-\infty}^{\infty} \left|\overline{A}_n(\Omega)\right|^2 \qquad \frac{d\overline{W}}{d\Omega} = \frac{1}{\Omega} \sum_{n=-\infty}^{\infty} \left|\overline{A}_n(\Omega)\right|^2$$

$$\overline{A}_n(\Omega) = \frac{\left[\frac{V_n}{j\beta}\sqrt{\varepsilon_r} J_n\left(\sqrt{\varepsilon_r}\Omega\frac{R}{h}\right) - U_n \dot{J}_n\left(\sqrt{\varepsilon_r}\Omega\frac{R}{h}\right)\right] e^{-\frac{\Omega}{\gamma\beta}}}{H_n^{(2)}\left(\Omega\frac{R}{h}\right)\dot{J}_n\left(\sqrt{\varepsilon_r}\Omega\frac{R}{h}\right) - \sqrt{\varepsilon_r}\dot{H}_n^{(2)}\left(\Omega\frac{R}{h}\right) J_n\left(\sqrt{\varepsilon_r}\Omega\frac{R}{h}\right)}$$

$$U_n = \left(\frac{\gamma+1}{\gamma-1}\right)^{n/2} J_n\left(\Omega\frac{R}{h}\right) e^{-jn\pi/2}; \qquad V_n = \frac{1}{2}\left(1+\frac{j}{\gamma}\right)U_{n-1} + \frac{1}{2}\left(1-\frac{j}{\gamma}\right)U_{n+1}$$

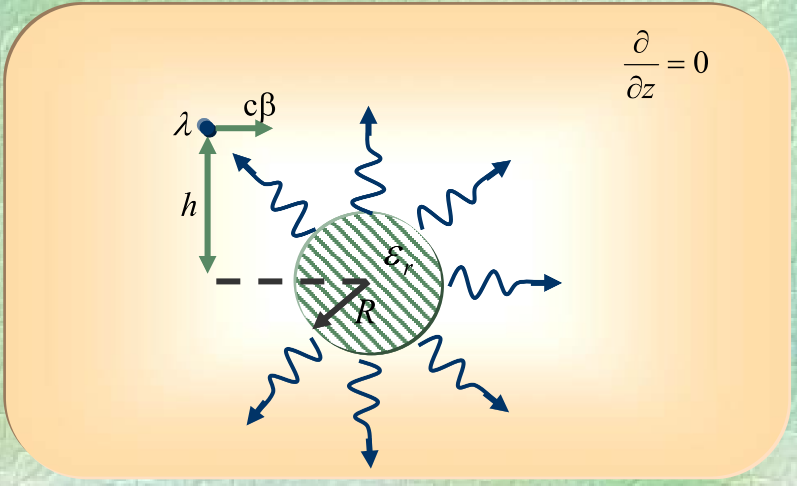
Formulation: Longitudinal Impedance

$$Z_{\parallel} = -\frac{1}{\lambda}\int_{-\infty}^{\infty} dx\, E_x(x,h;\omega)\exp(j\frac{\omega}{c\beta}x)$$

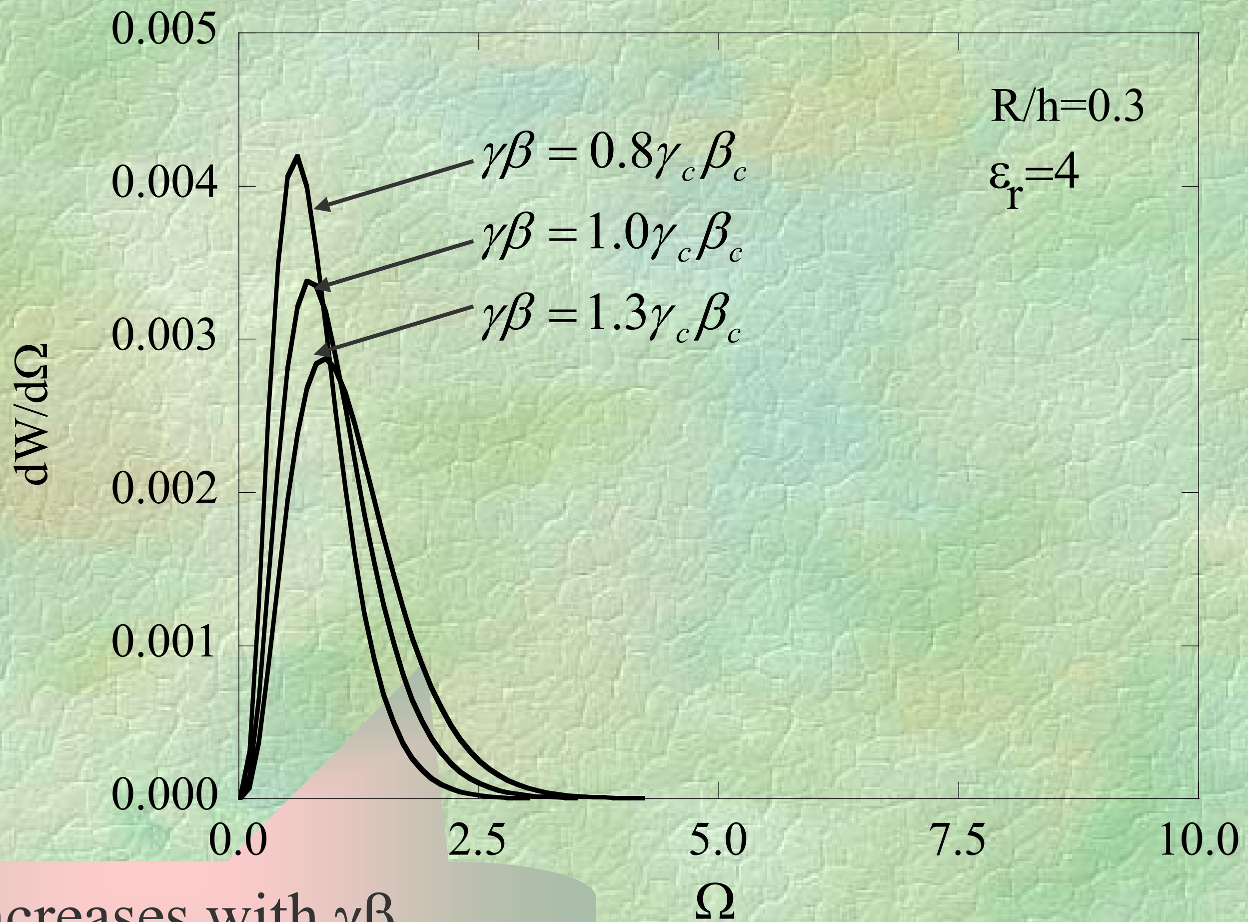
$$W = -\frac{\lambda}{2\pi}\int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} dx\, E_x(x,h;\omega)\exp(j\frac{\omega}{c\beta}x)$$

$$\Rightarrow \qquad \frac{dW}{d\omega} = \frac{\lambda^2}{2\pi} Z_{\parallel}$$

$$\overline{Z}_{\parallel} \equiv \frac{1}{2\eta_0 h} Z_{\parallel} \qquad \Rightarrow \qquad \frac{d\overline{W}}{d\Omega} = \overline{Z}_{\parallel}$$



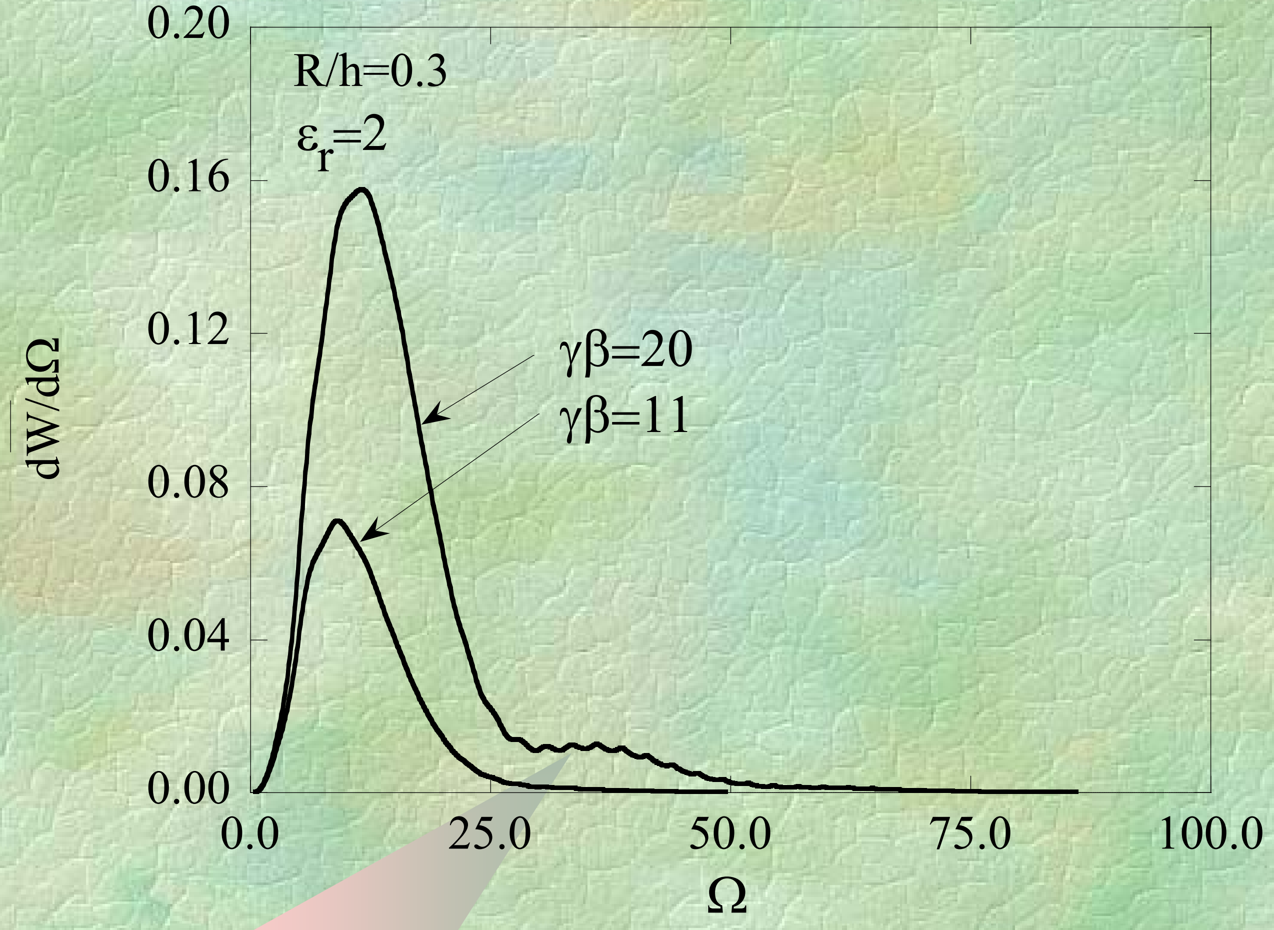
Longitudinal Impedance: Low Energy



Ω_{peak} increases with γβ

Narrow spectrum

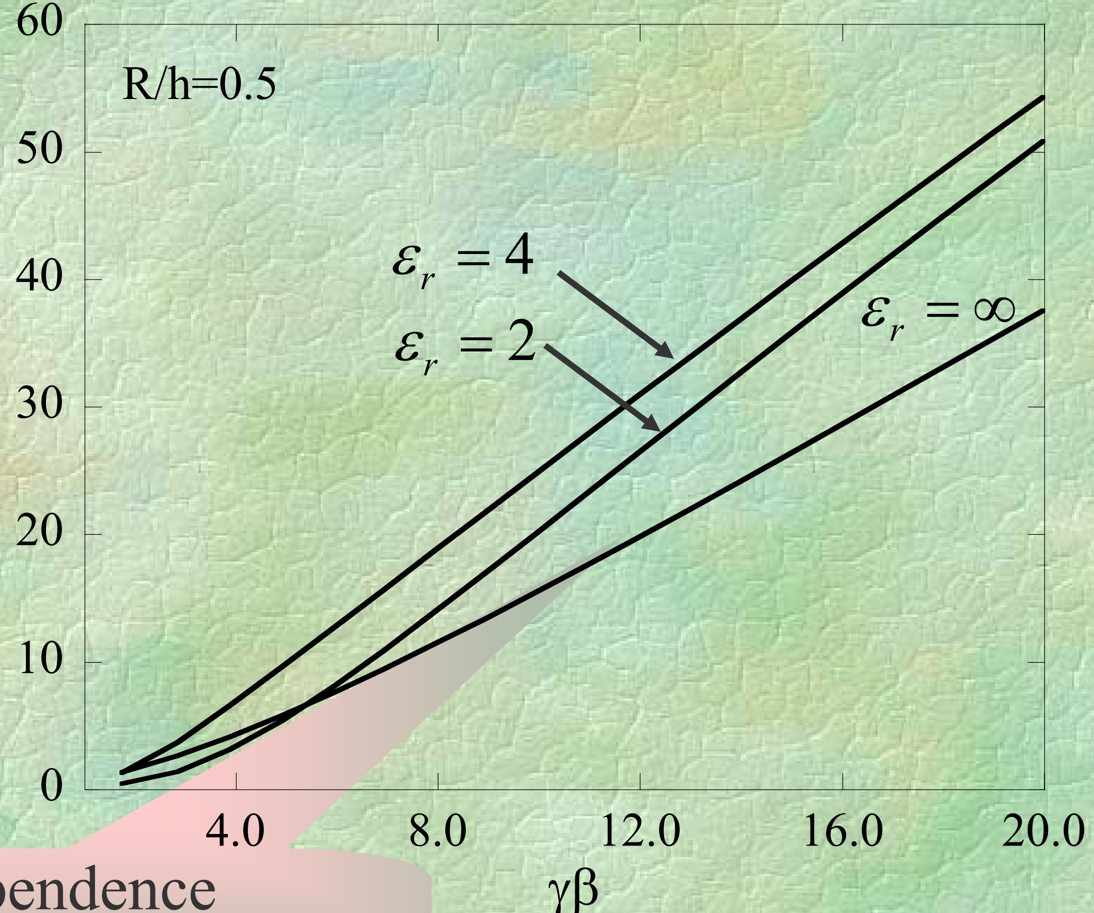
Longitudinal Impedance:High Energy



Ω of peak increases with γβ

Wide spectrum

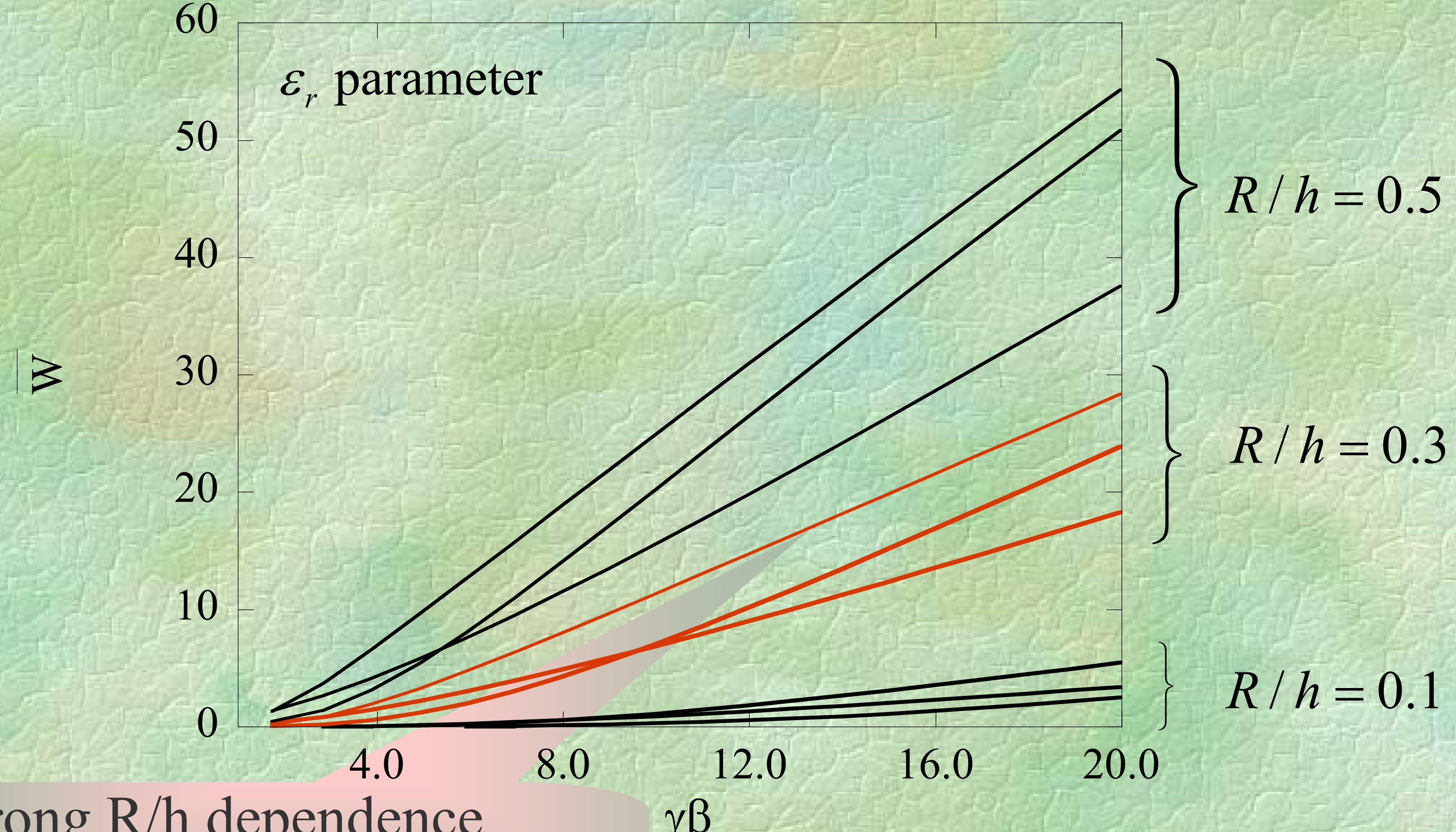
Radiated Energy: Relativistic Case



Linear γβ dependence

$\overline{W}_\infty < \overline{W}$ for γβ >>1

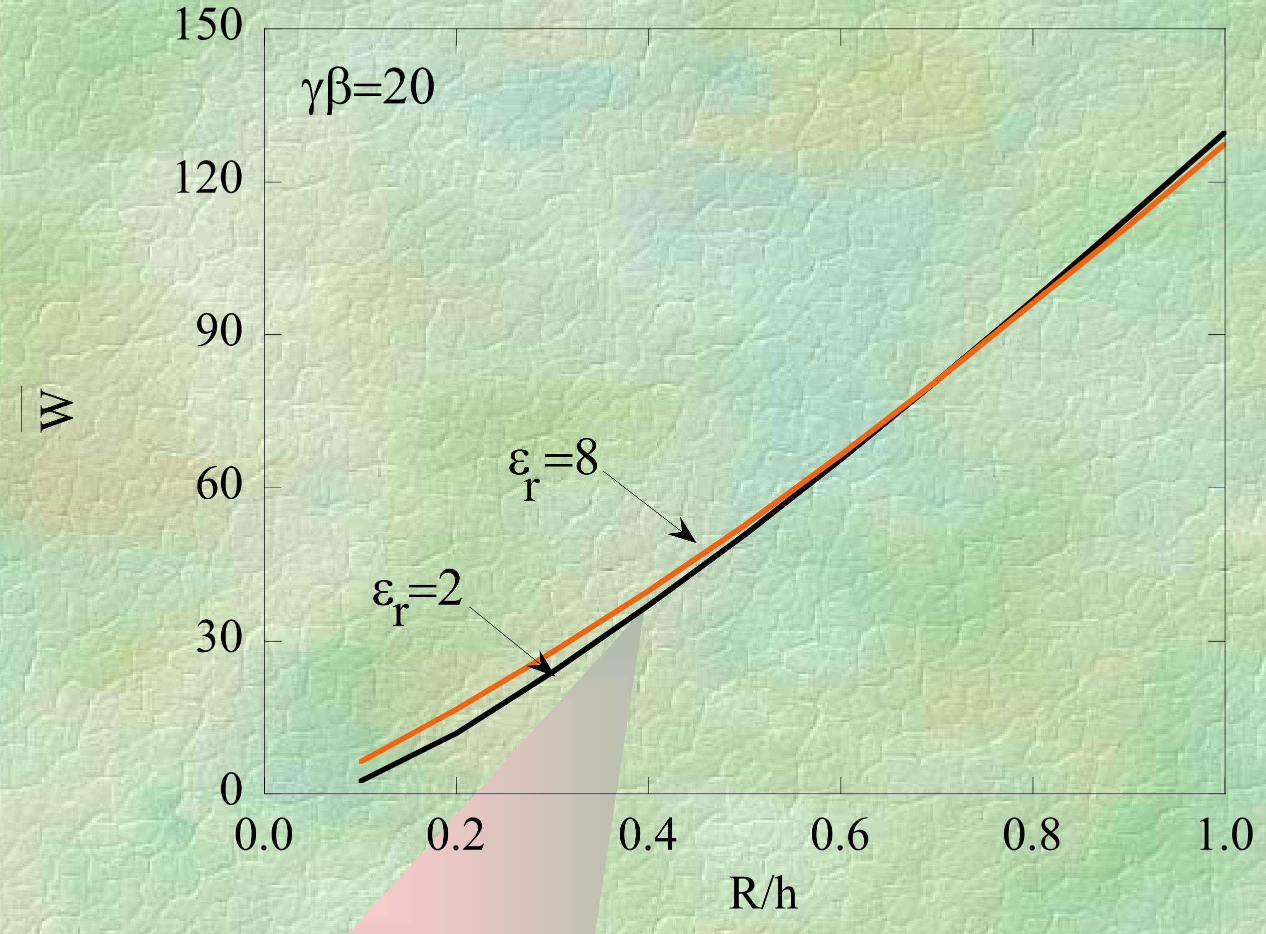
Radiated Energy: Relativistic Case



Strong R/h dependence

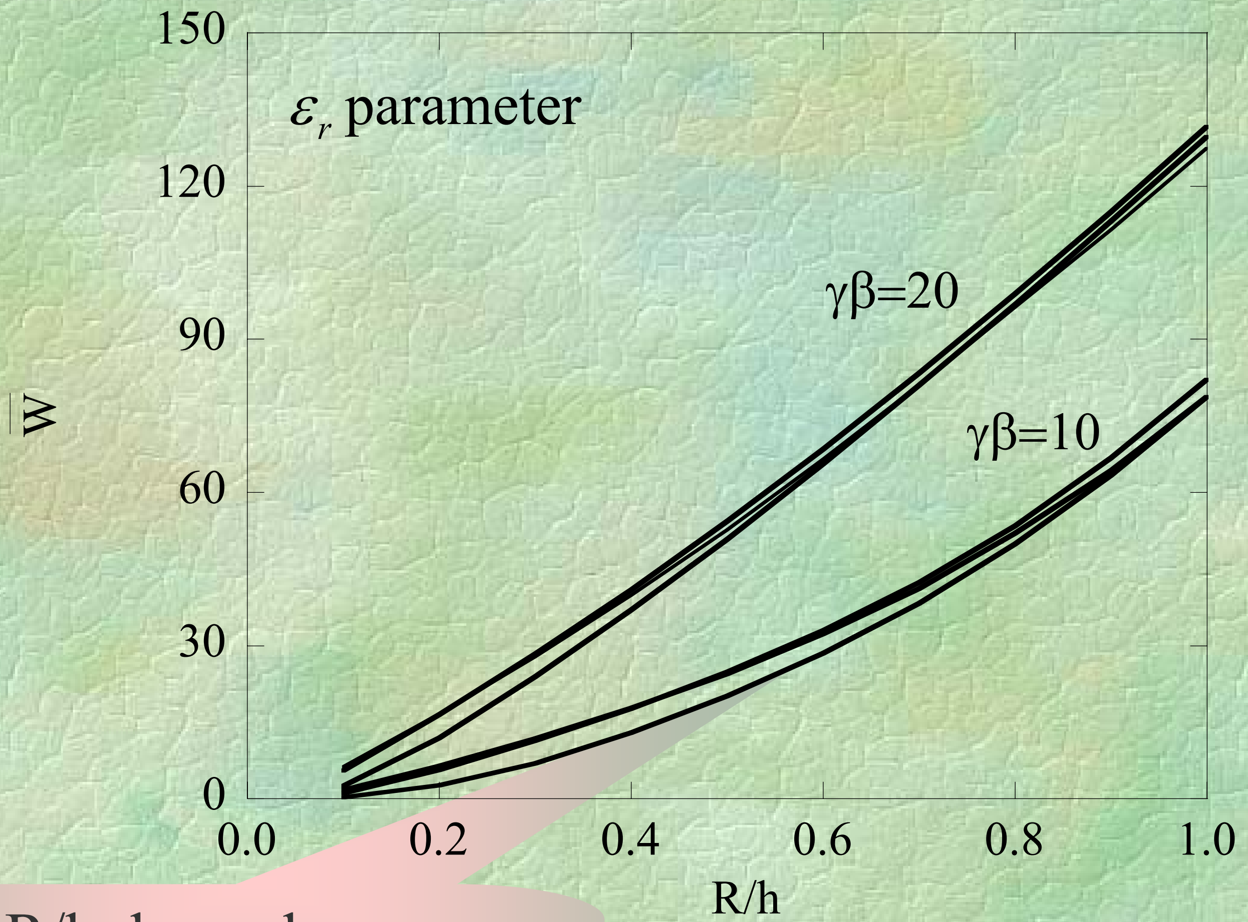
Linear γβ dependence

Radiated Energy: Relativistic Case



Almost linear R/h dependence

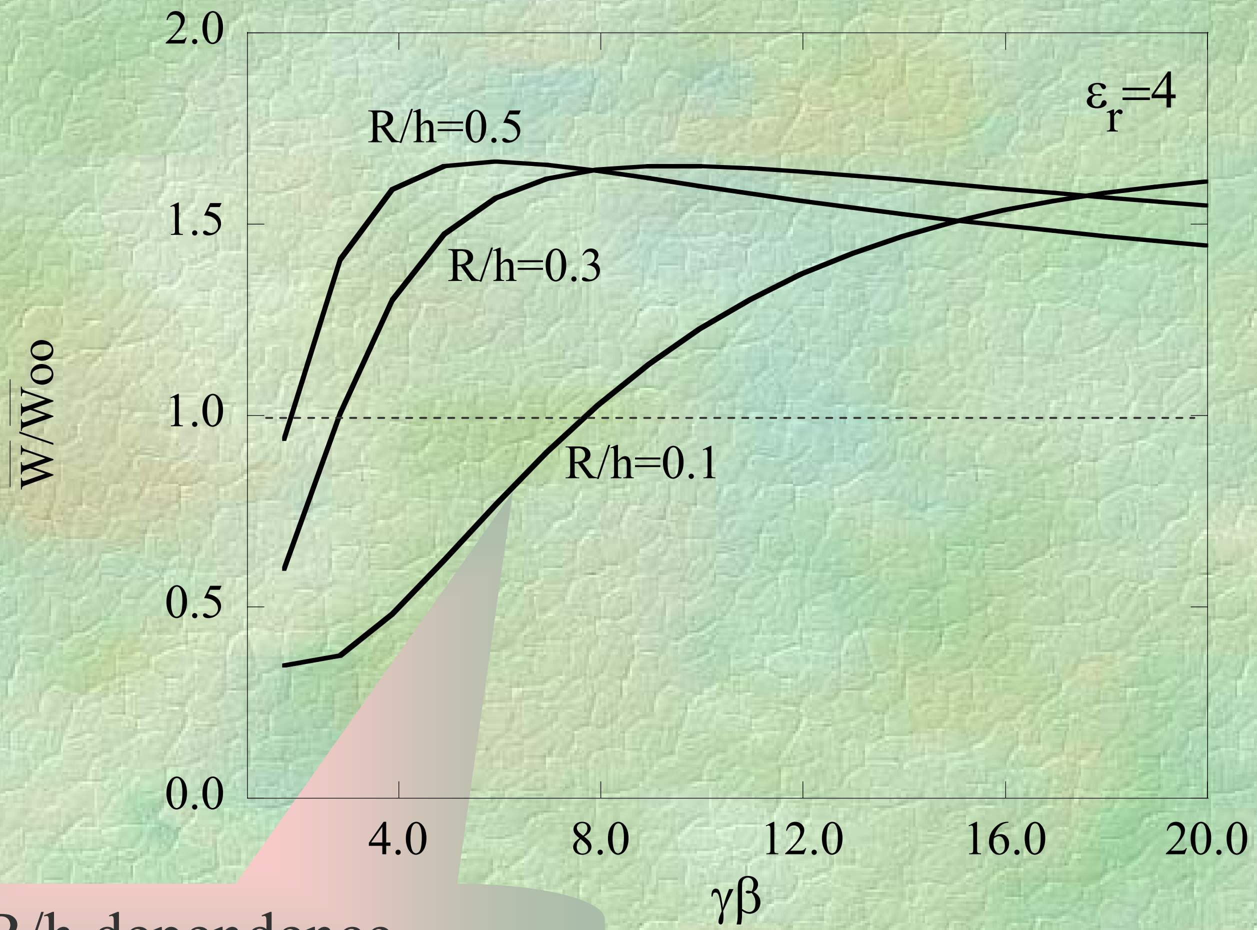
Radiated Energy: Relativistic Case



Strong R/h dependence

Weak ε_r dependence

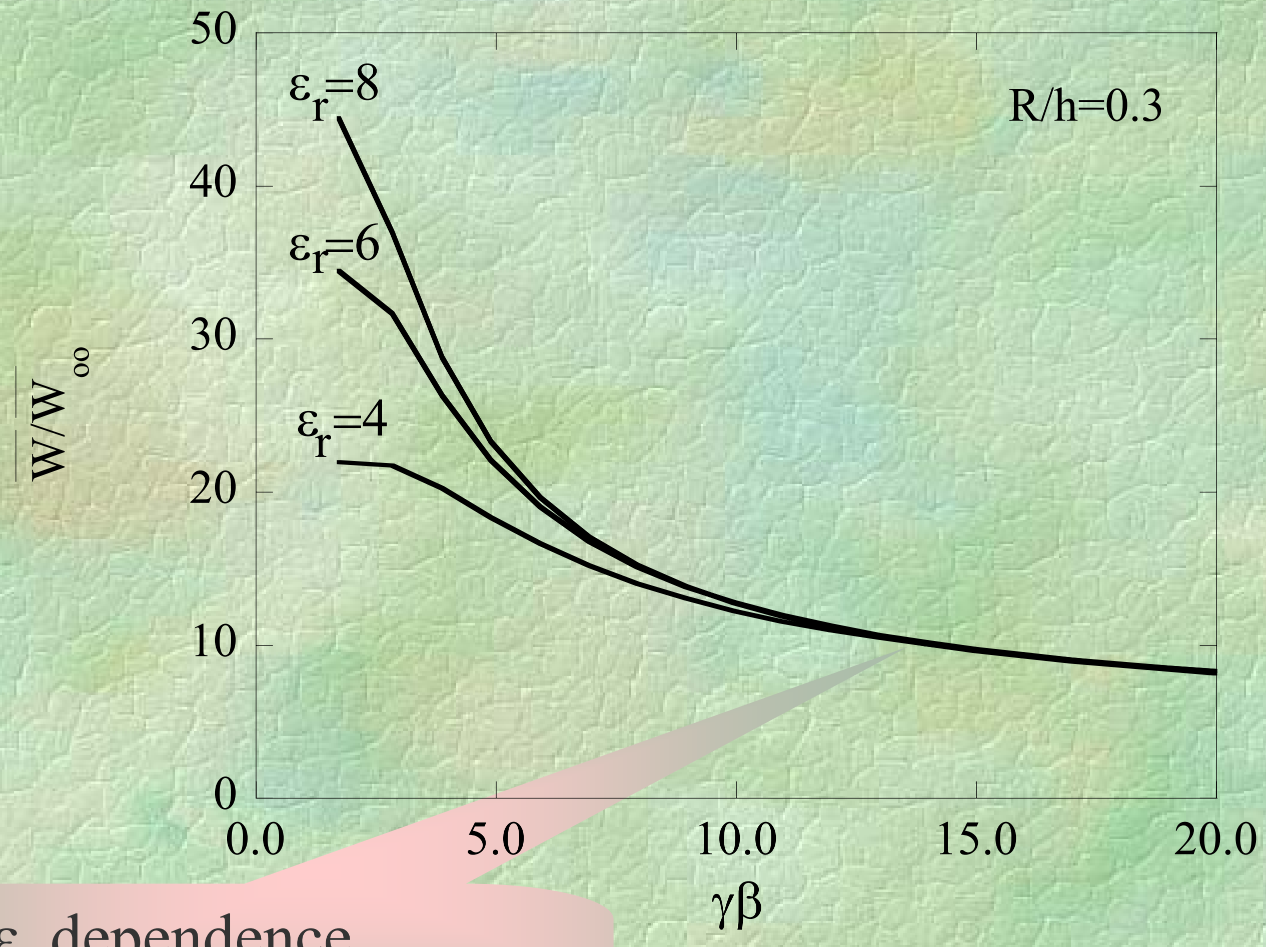
Radiated Energy: Relativistic Case



Weak R/h dependence

Weak γβ dependence

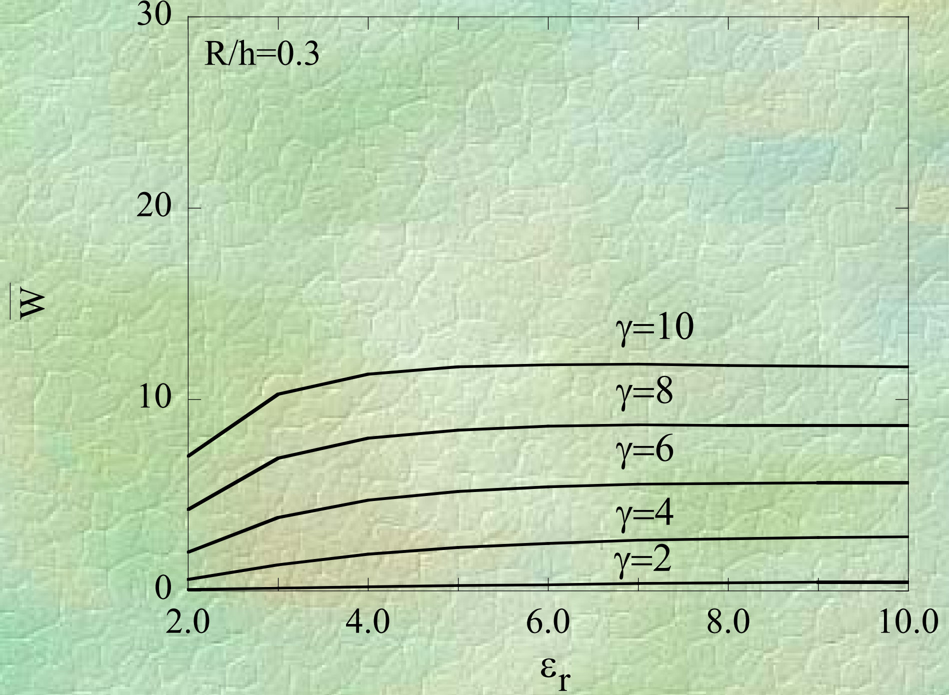
Radiated Energy: Relativistic Case

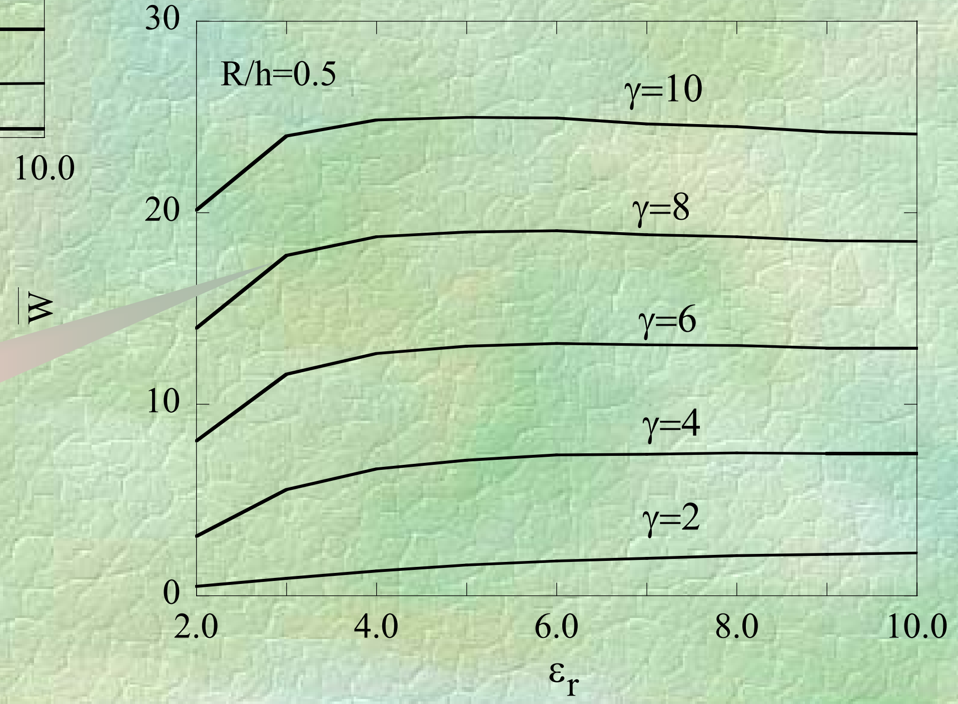


Weak ε_r dependence

Strong γβ dependence

Radiated Energy: Relativistic Case

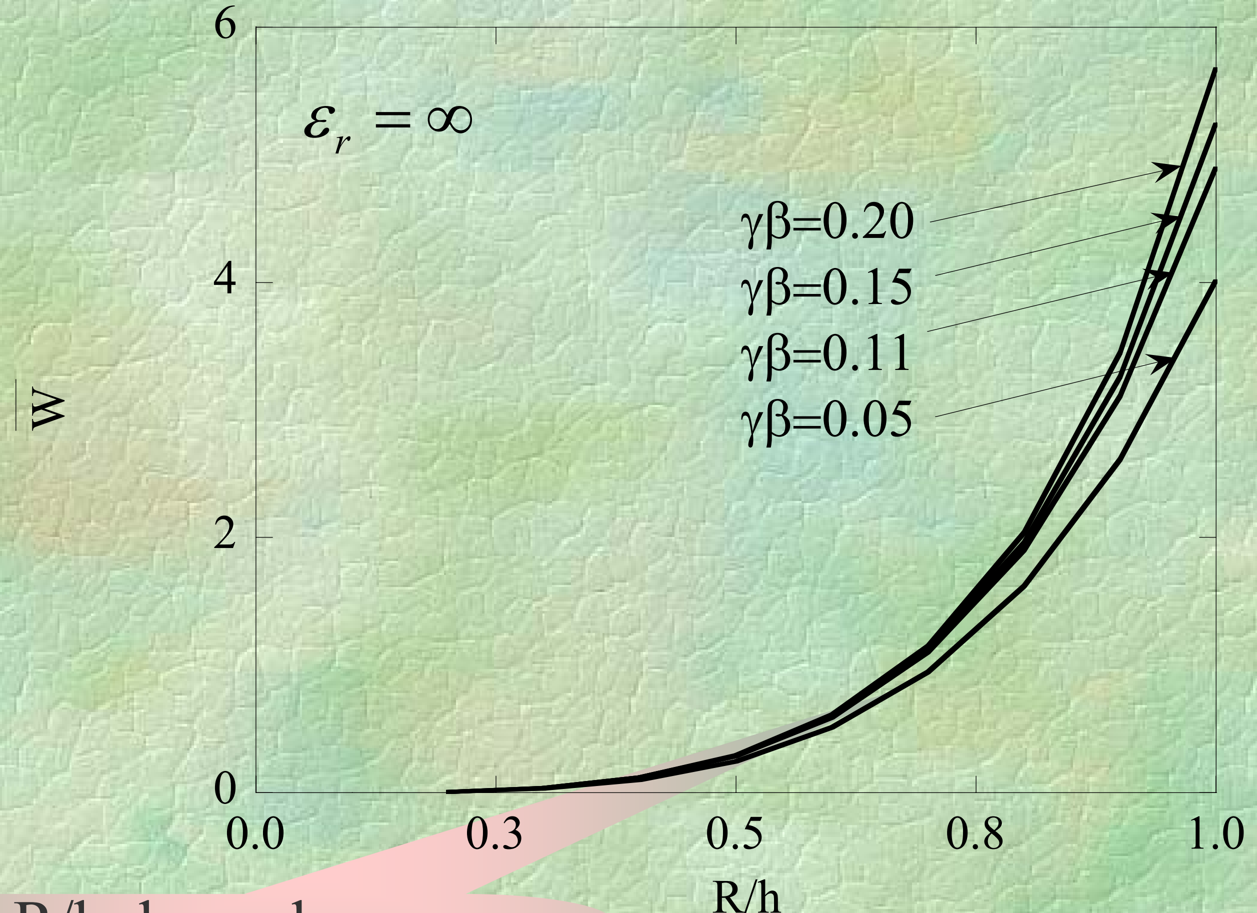




Weak ε_r dependence

Strong $\gamma\beta$ dependence

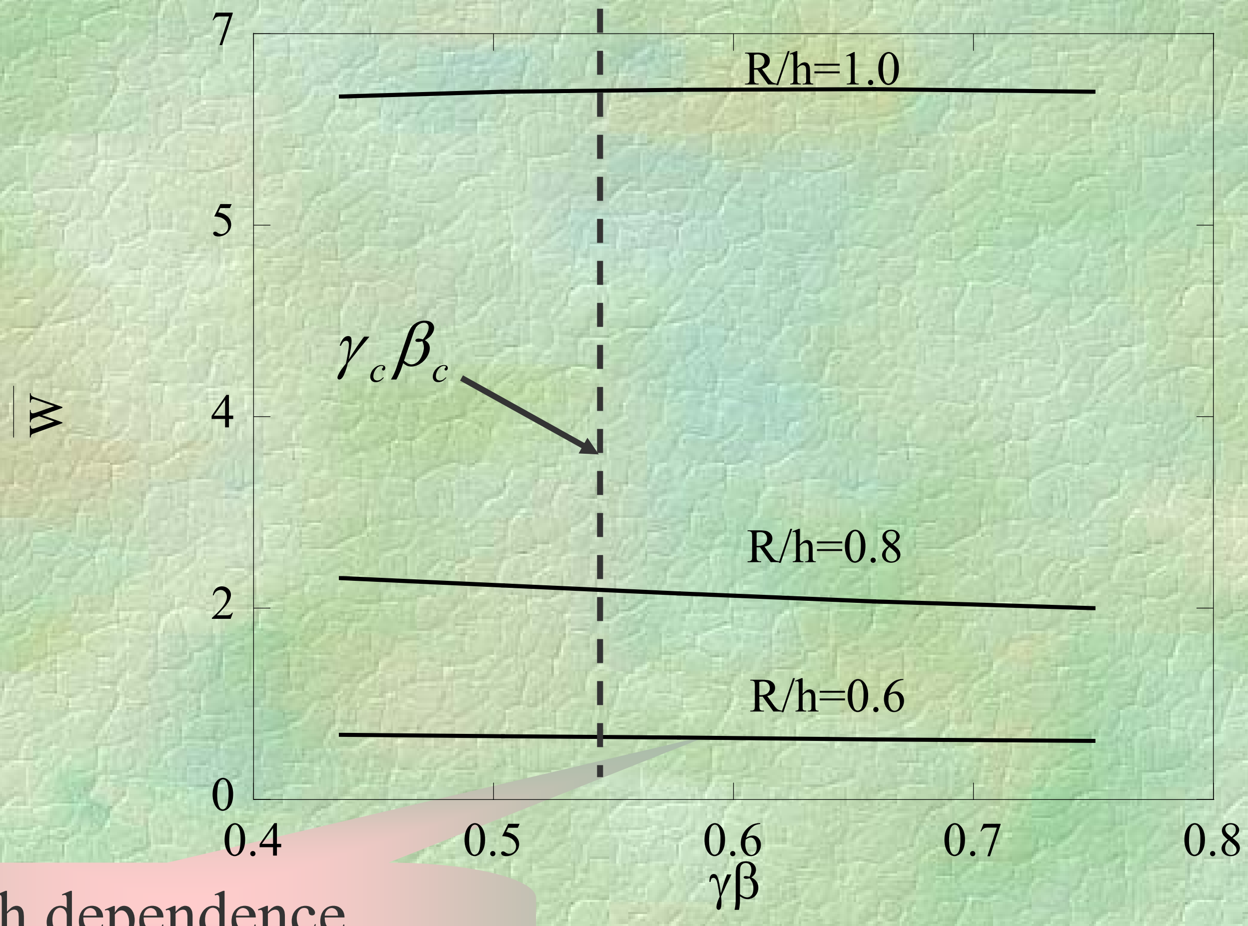
Radiated Energy: Low Energy



Strong R/h dependence

Weak γβ dependence

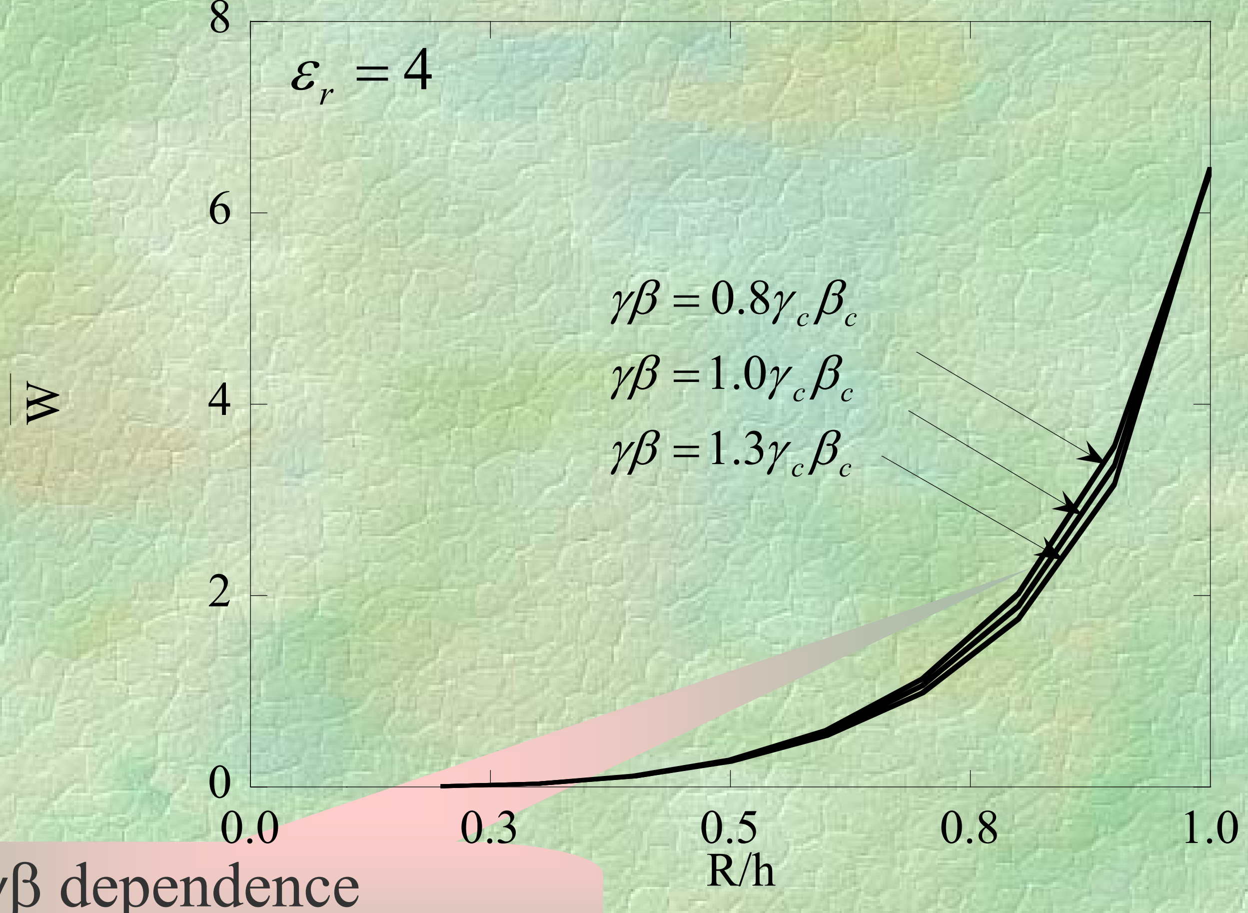
Radiated Energy: Transition & Cerenkov



Strong R/h dependence

Weak γβ dependence

Radiated Energy: Transition & Cerenkov



Weak γβ dependence

Strong R/h dependence

Summary

- Peak of the spectrum (Ω_{peak}) increases with $\gamma\beta$
- Spectrum width ($\Delta\Omega$) increases with $\gamma\beta$
- $\overline{W}$ is linear with $\gamma\beta$ for $\gamma\beta >> 1$
- $\overline{W} > \overline{W}_{\infty}$ for $\gamma\beta >> 1$
- $\overline{W}$ is linear with R/h for $\gamma\beta >> 1$
- $\overline{W}$ virtually saturates as a function of ε_r
- $\overline{W}$ does not change significantly at the Cerenkov point
- $\overline{W} / \overline{W}_{\infty}$ has a maximum as a function of $\gamma\beta$