Large Deviations 048944

Hand out 1: proof of (3.1.16), page 66 in D&Z 1st edition (page 80 in 2nd edition).

To show that the expression below equals 0 denote $u_2(i) = \sum_k u(k,i)$. So

$$\sum_{i=1}^{|\Sigma|} \sum_{j=1}^{|\Sigma|} q(i,j) \log \frac{\sum_k u(k,i) q_2(j)}{\sum_k u(k,j) q_1(i)} \tag{0.0.1}$$

$$= \sum_{i=1}^{|\Sigma|} \sum_{j=1}^{|\Sigma|} q(i,j) \left[\log u_2(i) q_2(j) - \log u_2(j) q_1(i) \right] \tag{0.0.2}$$

$$= \sum_{i=1}^{|\Sigma|} \sum_{j=1}^{|\Sigma|} q(i,j) \left[\log u_2(i) + \log q_2(j) - \log u_2(j) - \log q_1(i) \right] \tag{0.0.3}$$

$$= \sum_{i=1}^{|\Sigma|} \sum_{j=1}^{|\Sigma|} q(i,j) \left[\{ \log u_2(i) - \log q_1(i) \} - \{ \log u_2(j) - \log q_2(j) \} \right] \tag{0.0.4}$$

$$= \sum_{i=1}^{|\Sigma|} q_1(i) \{ \log u_2(i) - \log q_1(i) \} - \sum_{j=1}^{|\Sigma|} q_2(j) \{ \log u_2(j) - \log q_2(j) \} \tag{0.0.5}$$

which equals zero since $q_1 = q_2$.